SIMPLE GRAPHICS WITH MAPLE

by Carlo F. Barenghi

Making graphs and plots is fun and is also useful to understand mathematical structures. Maple can produce several forms of graphs. This tutorial will give you a simple introduction. Maple can do much more than what is described here, including animation of graphs. If you want to know more, use the on-line help facility which also contains many useful examples.

Login a computer in the Oracle cluster, click the Oracle Software program group and click the maple leaf to fire up Maple. Go through the following tutorial and try the exercises at the end. To exit Maple click the top left button. Do not forget to logout when you have finished.

Maple's prompt is the symbol > so in all the examples below the command which you have to type is the text which follows >. To execute a command, press the return key. If you want to print, click on the print button on Maple's menu bar.

Remember that an efficient way to use Maple, when you have had some initial practice, is not to re-type commands, but rather edit previous commands, change them and execute them again. It is also very useful to copy and modify the examples given by the on-line help.

1. Simple graphs of functions in two dimensions

   It is very simple to plot a function given in explicit form. Try for example

   $$> plot(sin(x), x);$$

   and Maple will plot the function $y(x) = sin(x)$ on the screen.

   Note that the range of both the x and the y coordinates are chosen automatically by Maple. If you want, you can set the range yourself: for example type

   $$> plot(sin(x), x = 0..12);$$

   and Maple will graph the function $y(x) = sin(x)$ in $0 < x < 12$.

   Incidentally, Maple recognizes the symbol Pi for π, so you can try

   $$> plot(sin(x)/x, x = -2 * Pi..2 * Pi);$$

   to plot, for example, the function $y(x) = sin(x)/x$ between -2π and 2π.

   You can set the y range too: for example

   $$> plot(x * sin(x), x = 0..12, y = -12..12);$$

   The ability of Maple to find automatically the y range of a curve is evident if you type

$$> plot(exp(x), x = 0..4);$$

to plot the exponential function $y(x) = exp(x)$.

Note that, in order to show the picture more clearly, Maple has used different scales for the horizontal x axis and the vertical y axis. We say that the plot was 'unconstrained'. If you want Maple to use the same scale on both axes type

$$> plot(exp(x), x = 0..4, scaling = constrained);$$

As said above, you can zoom the graph in the region which you want, for example type

$$> plot(exp(x), x = 1..4, y = 10..60);$$

A title can be added in a simple way, for example type

$$> plot(sin(x), x, title = `SIN(X)`);$$

2. Parametric plots

   Sometimes we want to plot functions given in 'parametric' form. This means that both the x and the y coordinate are functions of a single parameter t. An example is the parabola $y(x) = x^2$; following Section 1, we can be plot it in $0 < x < 10$ by typing

$$> plot(x^2, x = 0..10);$$

   In parametric form we set $x = t$, so $y = x^2$ becomes $y = t^2$,

   and we type

$$> plot([t, t^2, t = 0..10]);$$

   In this way, as t varies from 0 to 10, Maple plots the curve of coordinates $x(t) = t$ and $y(t) = t^2$.

   To apply this idea we can plot a circle. For most of values of x there are two values of y, so the parametric form is convenient. Type

$$> plot([cos(t), sin(t), t = 0..2 * Pi]);$$

   and Maple will graph a circle. A more complicated curve is for example

$$> plot([sqrt(t) + sin(t), t^2, t = 0..20]);$$

3. Multiple plots

   It is possible to show more than one curve on the same graph. For example suppose that we want to see how the slopes of the four curves $y = x$, $y = x^2$, $y = x^3$, $y = x^4$ differ from each others. Type

   $$> plot([x, x^2, x^3, x^4], x = -10..10);$$

   to plot them together.

   Note that Maple likes to work with colours, which is useful to distinguish one curve from the other. In the previous graph Maple used default values, but you can set the colour which you want, for example type

   $$> plot([x, x^2, x^3, x^4], x = -10..10, color = blue);$$

   Now all curves are in blue. You can also set the sequence of colours which you want, for example type

   $$> plot([x, x^2, x^3, x^4], x = -10..10, color = [yellow, orange, red, blue]);$$

   In this way the curve $y(x) = x$ will be yellow, $y(x) = x^2$ will be orange, $y(x) = x^3$ will be red and $y(x) = x^4$ will be blue.

4. Data points

   If you want to plot numeric data, you need to know what is a 'list'. A list consists of two (or more) numbers enclosed in a square bracket. For example [2,8] is a list. Given two (or more) lists, a 'list of lists' is obtained by enclosing these lists within square brackets. For example [[1,1],[2,8],[3,27],[4,64]] is a list of lists. To plot it type

   $$> plot([[1, 1], [2, 8], [3, 27], [4, 64]]);$$

   and Maple will automatically draw a line between the data points. Sometimes it is convenient first to define a quantity as a list, call it q, and then plot it. To do this type

   $$> q := [[1, 1], [2, 8], [3, 27], [4, 64]]; > plot(q);$$

   It is easy to change the style; for example, rather than drawing lines between the points, you can mark the points by typing

   $$> plot(a, style = point);$$

   The default used by Maple before was style=line.

5. Three dimensional plots

   Maple can be used to plot functions of two variables x and y as a surface in three dimensional space. For example type

$$> plot3d(1 - x^2 - y^2, x = -1..1, y = -1..1);$$

   and Maple will plot the surface $z = 1 - x^2 - y^2$ in the region $-1 < x < 1$ $-1 < y < 1$. If you want Maple to draw the axes, type

$$> plot3d(1 - x^2 - y^2, x = -1..1, y = -1..1, axes = framed);$$

   You can also make Maple to draw a full box around the picture: type

$$> plot3d(1 - x^2 - y^2, x = -1..1, y = -1..1, axes = boxed);$$

   It is very convenient to have the graph boxed in this way, because you can rotate the picture to look at it from the direction which you want. To do this, use the mouse to grab the frame of the box, holding the left click down, and turn the box around the way which you like. When the box is in the right position, release the click, and move the prompt to the top menu bar, clicking the button marked 'R' for 'redraw'.

   By default Maple draws pictures as an opaque wire frame, but you can change the style by typing for example

$$> plot3d(1 - x^2 - y^2, x = -1..1, y = -1..1, axes = boxed, style = patch);$$

   which puts colours in the wire frame, or

$$> plot3d(1-x^2-y^2, x = -1..1, y = -1..1, axes = boxed, style = patchnogrid);$$

   which leaves the colours only.

6. Contour plots

   Another way to visualize three dimensional surfaces is to draw a 'contour plot'. A contour plot draw the lines of equal height, like it is done on a map of Ordnance Survey to indicate the height of hills. For example consider the function $z(x, y) = sin(x) * sin(y)$ for $-\pi < x < \pi$ and $-i\pi < y < \pi$. We can draw it first as a surface in three dimensions by typing

$$> plot3d(sin(x) * sin(y), x = -Pi..Pi, y = -Pi..Pi, axes = boxed,$$

$$scaling = constrained);$$

   Maple will show the shape of the function. Rotate it around to make sure you understand the shape, which consists of two 'hills' and two 'valleys'.

Now we are ready to draw the same surface as a countour plot. First we load Maple's high level graphics routines by typing

$$> with(plots);$$

then we type

$$> contourplot(sin(x)*sin(y), x = -Pi..Pi, y = -Pi..Pi, axes = boxed,$$

$$contours = 10);$$

Maple will draw 10 contour lines to indicate the height of the 'hills' or 'valleys'.

7. Parametric plots in 3 dimensions

   The surface of a sphere of radius r has equation $x = r*sin(\theta)*cos(\phi)$, $y = r*sin(\theta)*sin(\phi)$, $z = r*cos(\theta)$ where the angle θ (called the colatitude) varies from 0 to π, and the angle ϕ (called the longitude) varies from 0 to 2π. We can plot the sphere of radius $r = 1$ in parametric form by typing

   $$> plot3d([sin(t)*cos(p), sin(t)*sin(p), cos(t)], t = 0..Pi, p = 0..2*Pi,$$

   $$axes = boxed, scaling = constrained);$$

8. Using Maple quicky

   Remember that an efficient way to use Maple, when you have had some initial practice, is not to re-type all commands, but rather edit previous commands, make the necessary changes and execute them again: this saves you from retyping everything. To do so, you have to go up the window (using the scroll buttons if necessary), find the old command, use the mouse to bring the prompt onto the old command, insert/delete new text, and press the return key again.

   Another way is to proceed is to cut and paste. Suppose that you want to make minor changes to a previous command. Go back to it, moving the prompt to the beginning of the command at the left; hold down the left button and move to the right to highlight the text which you want, then press Control C. Now go back to where you want, and press Control V to copy the old command. Indert/delete text and execute by pressing return.

   i

9. Using the on-line help

   It was mentioned at the beginning that the help button can be used to learn more commands. It is good practice to copy the examples which are on-line and then modify them. Click 'help' below the main window bar of Maple, select 'topic search', enter for example 'plot' in the box for 'topic', select 'apply' and press return. You will find a description of the 'plot' command, including various options. At the end of the description (use the scroll button to reach it quickly) there are some examples. These examples can be copied onto your Maple's window. as explained in Section 8.

Exercises

1. Exercise 1.

   Plot the function $y(x) = sin(x)/(x * x) - cos(x)/x$ with default axes.

2. Exercise 2. Plot the same function in $0 < x < 20$

3. Exercise 3.

   The derivative of the function $y(x) = sin(x)$ is the function $y'(x) = cos(x)$. Plot the two functions $y(x)$ and $y'(x)$ together to check that when $y(x)$ increases $y'(x)$ is positive, when $y(x)$ decreases $y'(x)$ is negative, and when $y(x)$ has a maximum or a minimum then $y'(x)$ is zero.

4. Exercise 4.

   Consider the function $y(x) = sin(x)/(x * x) - cos(x)/x$. Determine its derivative $y'(x)$. Plot $y(x)$ and $y'(x)$ together, and check that when $y(x)$ increases $y'(x)$ is positive, when $y(x)$ decreases $y'(x)$ is negative, and when $y(x)$ has a maximum or a minimum then $y'(x)$ is zero.

5. Exercise 5.

   You will soon learn that the function $y(x) = sin(x)$ can be approximated in the vicinity of $x = 0$ by the McLaurin series $z(x) = x - (x^3)/6 + (x^5)/120 - (x^7)/5040 + ...$infinite more terms. The more terms you include, the better the approximation is. To check this result, plot the function $y(x) = sin(x)$ together with its four successive approximations $z1(x) = x$, $z2(x) = x - (x^3)/6$, $z3(x) = x - (x^3)/6 + (x^5)/120$ and $z4(x) = x - (x^3)/6 + (x^5)/120 - (x^7)/5040$ To make the graph more clear, restrict it between $0 < x < 4$ and use black for $y(x)$, yellow for $z1(x)$, orange for $z2(x)$, red for $z3(x)$ and blue for $z4(x)$. Note how the yellow, orange, red and blue curves give a better and better approximation to the black curve.

6. Exercise 6.

   Plot the function $y(x) = x^3$ in parametric form.

7. Exercise 7.

   Consider the surface $z(x, y) = x^2 - y^2$. Plot is for $-2 < x < 2$, $-2 < y < 2$. Box the picture and look at it from different directions: which shape is it ?

8. Exercise 8.

   Plot the surface $z(x, y) = sin(x * y)$ in $-2 < x < 2$, $-2 < y < 2$. Rotate it around to understand what it looks like.

Solutions

1. Solution 1

$$> plot(sin(x)/(x^2) - cos(x)/x, x);$$

2. Solution 2

$$> plot(sin(x)/(x^2) - cos(x)/x, x = 0..20);$$

3. Solution 3

$$> plot([sin(x), cos(x)], x);$$

4. Solution 4

   The derivative of $y(x) = sin(x)/x$ is $y'(x) = cos(x)/x - sin(x)/(x^2)$, so type

$$> plot([sin(x)/x, cos(x)/x - sin(x)/(x^2)], x);$$

5. Solution 5

$$> plot([x, x-(x^3)/6, x-(x^3)/6+(x^5)/120, x-(x^3)/6+(x^5)/120-(x^7)/5040], x = 0..4,$$

$$color = [black, yellow, orange, red, blue);$$

6. Solution 6

   Let $x = t$ then $y = x^3 = t^3$ so type

$$> plot([t, t^3, t = 0..2]);$$

7. Solution 7

$$> plot3d(x^2 - y^2, x = -2..2, y = -2..2, axes = boxed);$$

   The surface is a saddle.

8. Solution 8

$$> plot3d(sin * x * y), x = -2..2, y = -2..2, axes = boxed);$$