THE GALACTIC DYNAMO: AXISYMMETRIC AND NON-AXISYMMETRIC MODES

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We discuss recent developments in the theory of large-scale magnetic structures in spiral galaxies. In addition to a review of galactic dynamo models developed for axisymmetric disks of variable thickness, we consider the possibility of dominance of non-axisymmetric magnetic modes in disks with weak deviations from axial symmetry. Difficulties of straightforward numerical simulation of galactic dynamos are discussed and asymptotic solutions of the dynamo equations relevant for galactic conditions are considered. Theoretical results are compared with observational data.

KEY WORDS: Galactic magnetic fields, dynamo theory.

1. INTRODUCTION

The observational discovery of global magnetic structures in nearby spiral galaxies represents one of the most important achievements in the astronomy of the present decade (Beck, 1986; Sofue et al., 1986). This observational success was accompanied by developments in galactic dynamo theory which also succeeded in making predictions confirmed by observations (for a review, see Ruzmaikin et al., 1988a,b).

The causal connection between global magnetic structures in spiral galaxies and hydromagnetic mean-field turbulent dynamos has by today become commonly accepted. From the physical viewpoint, the hydromagnetic dynamo is a process of conversion (by virtue of Faraday's induction law) of the kinetic energy of a plasma (or conducting fluid) into magnetic energy. For galactic dynamos, the energy source is interstellar turbulence, i.e., ultimately it is drawn from supernova explosions and stellar winds. This process of energy conversion is described by the induction equation
\[ \frac{\partial \mathbf{H}}{\partial t} = \nabla \times (\mathbf{v} \times \mathbf{H}) + \nu_m \Delta \mathbf{H}, \quad \nabla \cdot \mathbf{H} = 0, \]  

(1)

where \( \mathbf{v} \) is the velocity field, \( \mathbf{H} \) is the magnetic field and \( \nu_m \) is the magnetic diffusivity.

Equation (1) should be supplemented by boundary conditions. In the idealized case of infinite space occupied by uniformly conducting fluid these boundary conditions reduce to the requirement of a rapid decrease (at least as \( |r|^{-3} \) typical of the lowest multipole harmonic, the dipole) of the field at infinity (for more subtle details see Arnold et al., 1982). In application to real physical systems, e.g. galaxies, more complicated boundary conditions may be appropriate (see below).

When searching for the origin of the magnetic field, it is natural to consider first the initial stages of field growth during which the Lorentz force in the Navier-Stokes equations is negligible. Thus, the velocity field can be considered to be independent of magnetic field. Thus formulated, the problem is known as the kinematic dynamo problem. Since the coefficients of (1) for a stationary velocity field do not depend on time, the magnetic field evolves exponentially in time.

This allows the problem of the origin of the magnetic fields to be formulated as a closed eigenvalue problem, namely that of solving (1) (or modifications of it) with relevant boundary conditions. The clear physical meaning and the closed mathematical formulation of the kinematic dynamo problem place it in a favourable position, because very powerful methods of modern mathematical physics can be readily applied to solve it.

Equation (1) has a rather simple form but, nevertheless, it has a surprisingly rich diversity of solutions whose properties are very sensitive to the particular physical context. For instance, the mean-field dynamo can generate both oscillatory and non-oscillatory magnetic fields, the generated field can be either even or odd with respect to the equatorial plane of the parent body, the preferred mode can be either axisymmetric or non-axisymmetric. This diversity is due to differences in the velocity field and the geometry of the generation region of the object considered. As a relevant example, we may note that the possibility of generation of non-axisymmetric large-scale magnetic fields in spiral galaxies is very sensitive to the shape of the rotation curve and the thickness and shape of the galactic gaseous disk.

One of the most important and widely encountered types of hydromagnetic dynamos in cosmic objects is the mean-field dynamo (Steenbeck et al., 1966; see also Krause and Rädler, 1980). The key feature of this generation mechanism is the mean helicity of the turbulent flow of conducting fluid; also, in real objects, differential rotation usually plays an important role.

2. THE GALACTIC DYNAMO

As follows from observations of the Milky Way and nearby spiral galaxies, galactic magnetic fields are characterized by the following parameters: a large-scale magnetic field of strength \( B \approx 1-5 \, \mu \text{G} \), a field scale along the radius of order
\( L \approx 5 \text{kpc} \), a chaotic component of magnetic field of strength \( b \approx 2-15 \mu \text{G} \). Figures 1 and 2 schematically show the basic global magnetic structures recently discovered in spiral galaxies. Among the observed structures are those exhibiting axisymmetry and non-axisymmetry (bisymmetry); an axisymmetric regular magnetic field may be concentrated at a certain radius: such is the ring magnetic structure exemplified by M31. Generally speaking, bisymmetric structures can also concentrate into a ring. The presence of ring structures is associated with generation regions displaced from the galactic center; in M31 this separation is 10kpc. So far bisymmetric structures have been observed only in galaxies in which the field generation is effective only in the central part of the disk, and bisymmetric rings have therefore not as yet been discovered observationally.

All these large-scale magnetic field structures are generated by mean-field turbulent dynamo action, governed by the following equation:

\[
\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\alpha \mathbf{B}) + \nabla \times (\nabla \times \mathbf{B}) + \beta \Delta \mathbf{B}, \quad \nabla \cdot \mathbf{B} = 0,
\]

(2)

where \( \mathbf{B} = \langle \mathbf{H} \rangle \) is the magnetic field averaged over the turbulent magnetic pulsations. The coefficient

\[
\alpha \approx -\frac{\tau}{3} \langle \nabla \cdot \nabla \times \mathbf{v} \rangle,
\]

is proportional to the mean helicity of the interstellar turbulent velocity field \( \mathbf{v} \). If \( \tau \) is the correlation time of the turbulence,

\[
\beta \approx \frac{\tau}{3} \langle v^2 \rangle \approx \frac{3}{\tau} v,
\]

(3)

is the turbulent magnetic diffusivity, in which \( l \) and \( v \) are the energy-range scale and the root mean square velocity of the interstellar turbulence, respectively. The large-scale velocity field \( \mathbf{V} \) is dominated by differential rotation:

\[
\mathbf{V} = \Omega(r) \times \mathbf{r}.
\]

Inspection of (2) shows which properties of a galaxy are required in order to construct a model of its dynamo. Among them is the rotation curve, i.e. the dependence of the rotation velocity on position. The rotational properties of the spiral galaxies are perhaps the most accessible ingredients of galactic hydrodynamics. Numerous observations provide ample and accurate information about the dependence of \( \mathbf{V} \) on the galactocentric distance \( r \) for hundreds of galaxies. Within galactic disks, variation of \( \mathbf{V} \) with the vertical coordinate \( z \) is weak and can be neglected. However, in the galactic coronae \( \mathbf{V} \) can be strongly dependent on \( z \). The gas rotation law in the coronae is important for the process of field penetration into the surroundings of the disk and for the independent generation of large-scale
Figure 1  Schematic view of the projection of magnetic lines in the axisymmetric magnetic structure on the galactic equatorial plane. Enhancement of the field at some radius is modelled by decrease of the pitch angle of magnetic lines at this radius.

Figure 2  Same as in Figure 1 but for the bisymmetric magnetic mode.
magnetic field in those coronae, which seems to be quite possible (Ruzmaikin et al., 1988a, Section VIII.1).

The turbulent diffusivity $\beta$ is directly related to an observable quantity: the mean square of the turbulent velocity, $\langle v^2 \rangle$. For Kolmogorov turbulence we have $\tau \approx l/u$, and the turbulent diffusivity can be expressed through the observable scale and the root mean square velocity of the turbulence: $\beta \approx \frac{1}{3} l u$. The greatest uncertainty lies in the value of the numerical factor in (3), here taken to be $\frac{1}{3}$. This yields the estimate $\beta \approx \frac{1}{3} \times 100 \text{ pc} \times 10 \text{ km s}^{-1} \approx 10^{26} \text{ cm}^2 \text{ s}^{-1}$, which is probably applicable to all spiral galaxies.

Unlike the mean square of the turbulent velocity, the mean helicity of interstellar turbulence, $\langle \mathbf{v} \cdot \nabla \times \mathbf{v} \rangle$, has not yet been measured directly. For this reason, theorists are here forced into attempting to cross the chasm between observation and theory in two jumps: first, the application of qualitative arguments yields the order-of-magnitude estimate

$$\alpha \sim l^2 \Omega|\nabla \rho|/\rho,$$

(where $\rho$ is the interstellar gas density); this also predicts that $\alpha$ is an odd function of $z$. Second, substitution of the observational orders of magnitude of the relevant parameters gives an estimate of the coefficient $\alpha$. Fortunately, galactic dynamo models are not very sensitive to the particular choice of the dependence of $\alpha$ on $z$, and this permits reliable models of magnetic field generation to be built. Nevertheless, being conscious of the danger of leaping chasms in more than one jump, we emphasize the desirability of a direct observational estimate of the mean helicity of interstellar turbulence. Possibly, the required information is even available in existing HI surveys of interstellar gas (Ruzmaikin et al., 1988a, Section VI.4).

Finally, the models of galactic dynamos require information on the distribution of the thermal ionized gas within the galactic disks. For kinematic dynamo models, the disk thickness and its dependence on galactocentric distance is especially important. For nonlinear models, information on the density distribution is also required. Even in the solar vicinity of the Milky Way, the thickness of the ionized galactic disk is far from being accurately measured; different methods give widely scattered results. Observational information on the shape of the ionized disk is simply absent. Here we are forced to presume that, as for an HI disk, the ionized disks of spiral galaxies thicken outwards on the same radial scale as the neutral gas ($\approx 4 \text{kpc}$). The thickness of the disk is considered to be a free parameter, very weakly restricted by the available observations and qualitative theoretical arguments. It should be stressed that galactic dynamo models are very sensitive to the shape and thickness of the disk. This sensitivity is illustrated by the fact that dynamo models with oblate-spheroidal disks would predict that the magnetic field in the Milky Way is concentrated within the inner $3-5 \text{kpc}$ even when the observed rotation curve is used (White, 1978). Only by taking into account the true shape of the disk, which even thickens toward the edge, can the presence of the field in the solar vicinity be explained (Ruzmaikin and Shukurov, 1981).
One of the important ingredients in the formulation of the galactic dynamo problem is the specification of boundary conditions at the disk surface. The conditions routinely used in dynamo models suppose that the disk is surrounded by a vacuum, i.e. by an insulator with \( v_m \rightarrow \infty \). This is partly justified by the fact that in the galactic corona the turbulent velocity is approximately \( 100 \, \text{km} \, \text{s}^{-1} \) (the sound speed in the \( 10^6 \, \text{K} \) hot gas) and the scale is about 300 pc (McKee and Ostriker, 1977), and that these imply that the magnetic turbulent diffusivity exceeds that in the disk by a factor of 30. However, the inclusion of the galactic coronae into models of galactic magnetic field generation raises fascinating and promising questions.

Models of galactic dynamos predict that both axisymmetric and non-axisymmetric large-scale magnetic fields can be generated in galactic disks. In those galaxies where the theory asserts that only axisymmetric magnetic fields can be generated, the observed magnetic structures also exhibit axial symmetry; when non-axisymmetric (bisymmetric) fields are observed, the theory confirms that in these galaxies they can indeed be generated. In this respect, the agreement between theory and observation is remarkable.

Theoretical estimates of the steady-state field strength in spiral galaxies are based on a supposed balance between the Lorentz and Coriolis forces (the latter produces the mean helicity of the turbulence):

\[
\frac{B_r B_\varphi}{4 \pi h} \simeq \rho \nu \Omega,
\]

where \( h \) is the minimal field scale and \( \rho \) is the gas density. Using the asymptotic relation (Ruzmaikin et al., 1988a)

\[
B_r/B_\varphi = O(R_z/R_\omega)^{1/2},
\]

where \( R_z = z_0 h_0/\beta \) and \( R_\omega = |rd\Omega/dr|_0 h_0^2/\beta \) and the subscript zero denotes typical values of the corresponding quantities; we have \( B_\psi \simeq 7 \, \mu \text{G} \) for \( \rho = 1.7 \times 10^{-24} \, \text{cm}^{-3}, \)
\( v = 10 \, \text{km} \, \text{s}^{-1}, \) \( \Omega = 10^{-15} \, \text{s}^{-1}, \) \( h = 400 \, \text{pc}, \) \( R_z = 1, \) and \( R_\omega = 10. \) In order of magnitude, this estimate agrees with observations of the Milky Way and of external spiral galaxies. Of course, it is necessary to develop a quantitative nonlinear theory that takes into account the back-reaction of the magnetic field generated on \( \alpha \) and \( \Omega. \) The main problem here is associated not merely with that posed by the solution of any complex nonlinear equation but is rather with the formulation of physically adequate models describing the modification of \( \alpha \) and \( \Omega \) under influence of a magnetic field: the physics of this process is still imperfectly understood.

Observations show that in nearby spiral galaxies the large-scale magnetic fields are more or less widely distributed in regions whose radial extent is 5–10 kpc. In contrast, the kinematic eigenfunctions of the dynamo have a radial width of only 2–3 kpc. This difference can be explained by nonlinear effects, as may be confirmed by qualitative estimates. Indeed, in the nonlinear regime, the narrow (kinematic) field distribution spreads because, at the wings of the eigenfunction, the field continues to grow due both to dynamo action and to diffusion from the maximum.
of the eigenfunction. It can be shown that the velocity at which the magnetic field spreads is given by $u \approx (2\gamma B)^{1/2}$, where $\gamma$ is the local growth rate of the kinematic dynamo mode (see below). For the Milky Way $\gamma \approx (5 \times 10^8 \text{ years})^{-1}$ and we have $u \approx 1.2 \text{ km s}^{-1}$. Consequently, in $t_0 = 10^{10}$ years the radial width of the field distribution can become wider by $ut_0 \approx 12 \text{ kpc}$, which is quite sufficient to explain the observed distributions. Of course this is an upper estimate since we have not taken into account the decrease of $\gamma$ with distance from the maximum of the eigenfunction. This spreading cannot occur in regions in which the local growth rate of the field is negative so that, for instance, the ring structure in M31 cannot be dispersed.

Apart from large-scale magnetic fields, the interstellar gas is permeated by a small-scale, chaotic field. There are three sources of chaotic magnetic fields in the interstellar gas: the tangling of the large-scale field by turbulence, the generation by a hydromagnetic fluctuation dynamo, and the outflow from supernovae and young hot stars. The dynamo-generated chaotic fields have a steady-state strength determined by equipartition with the kinetic energy of the turbulence:

$$\frac{b^2}{8\pi} \approx \frac{1}{2} \rho v^2$$

i.e. $b \approx 5 \mu \text{G}$. Again, this estimate agrees with observations of the synchrotron intensity and polarization of spiral galaxies. More delicate is the question of the spectral properties of the chaotic fields. From a theoretical point of view, magnetic energy spectra have been obtained for the kinematic stage of the fluctuation dynamo (see Ruzmaikin et al. (1989a) for a review). From the observational side, recent observations of intrinsic Faraday rotation measures in nearby galaxies approach, in linear resolution, the scale of 100 pc that separates large-scale and chaotic fields; with proper resolution, the investigation of the spectral properties of the fields will become possible. In the Milky Way, there are no difficulties of angular resolution but other specific obstacles prevent rapid progress (cf. Simonetti et al., 1984; Simonetti and Cordes, 1986 and Dagkesamansky and Shutenkov, 1987).

A crucial problem, whose solution would probably allow a further qualitative step to be taken in both observation and theory, is that raised by the dominance of non-axisymmetric magnetic fields in some spiral galaxies. The existing theory developed for axisymmetric disks (Ruzmaikin et al., 1986; Sawa and Fujimoto, 1986; Fujimoto and Sawa, 1987; Baryshnikova et al., 1987; Ruzmaikin et al., 1988a,b; Krasheninnikova et al., 1989 and Starchenko and Shukurov, 1989) predicts that axisymmetric modes are most easily excited in galaxies (they have the greatest growth rates). It is natural to believe that the steady state should inherit the basic symmetry properties of the dominant mode, even though this statement has never been proved rigorously and is currently under debate (Krause and Meinel, 1988; Brandenburg et al., 1988). However, observations do not indicate a dominance of axisymmetric fields amongst observed galaxies. Moreover, even if there exist only a few galaxies having bisymmetric magnetic fields, this fact
requires a special explanation. There is some hope that slightly more complicated dynamo models, e.g. those with weak deviations from axial symmetry, would be able to explain the observations. However, fruitful development of the theory in this direction depends crucially on reliable information becoming available on both the magnetic field structures of individual spiral galaxies and the representative statistical dependence of the dominant field configuration on, for example, the galactic morphological type or on other properties.

3. MODELS OF THE GALACTIC DYNAMO

Despite the apparent simplicity of (2), the papers devoted to models of galactic dynamos are full of cumbersome mathematical equations whose solution requires the application of complicated methods of mathematical physics. This section is an attempt to clarify the physical meanings of some existing models and to explain why the chosen method of solution is appropriate. Detailed assessments of the models can be found, for example, in Ruzmaikin et al. (1988a).

Modern developments of computational physics tempt one to try a direct numerical solution of (2), the more so as this equation is linear in $B$. Other approaches at first appear outdated and at best auxiliary. However, a closer inspection of the problem shows that a straightforward numerical simulation of the mean-field dynamo operating in real galaxies will hardly be fruitful in the foreseeable future. In order to see this, let us estimate the number of mesh points required for an adequate finite-difference approximation of (2) in a galactic disk. Obviously, at least 10 mesh points should be taken across the disk, for $-h \leq z \leq h$. This mesh would allow a modest accuracy of a few percent. In the absence of any prior knowledge gained from the application of analytic methods, one should choose the same spatial step for the radial and azimuthal meshes, as for $z$. Since the galactic radius is approximately 50 times its thickness, one therefore chooses a 500-point mesh along the radius and $2\pi \times 500$ points in the azimuthal direction. Of course, it can be expected that the radial scale of the field is considerably larger than the vertical one, but a confident choice of the minimal radial mesh requires a detailed knowledge of the properties of the eigenfunctions, based on analytical results. (Note that when such results are available, they essentially solve the problem and numerical efforts become superfluous.) Thus, multiplying these numbers together, and taking into account that the magnetic field has three components, we see that the representative finite-difference analogue of (2) consists of $3 \times 10 \times 500 \times 2\pi \times 500 \approx 5 \times 10^7$ algebraic equations. Consequently, the evaluation of the growth rates and corresponding field distributions, by this solution of the spectral problem requires the inversion of matrices with $10^{15}$ elements! Note that this is a lower estimate because we presume that the field outside the disk is known. This problem is out of reach of even the most powerful modern computers, but this is not the principal difficulty. The elements of the matrix would strongly depend on the values of, for example, the function $\alpha$ at $5 \times 10^7$ positions, and it is not clear in advance to which accuracy this function should be prescribed. Calculations of all possible variants and the extraction of important information would require tens to hundreds of runs. An alternative approach
might use Galerkin expansions; this method has no marked advantage since it raises the no easier problem of poorly convergent series. It might seem that the solution of the corresponding Cauchy problem is more promising. However, in this case stability considerations demand that the numerical scheme adopted be implicit, and the resulting procedure also effectively reduces to that of inverting equally large matrices.

Of course, a reasonable approach based on the physics of the process allows some simplification of the problem. For instance, it is natural to solve first the axisymmetric problem and only then to impose deviations from axial symmetry as a weak perturbation. This reduces the number of required mesh points by a factor of 3000. However, it is worth considering the problem more carefully. The difficulties of deriving numerical solutions are associated with the presence of several natural parameters that considerably differ from unity. These are a weak deviation from axial symmetry, the small ratio of the disk thickness to its diameter, and the generation strength characterized by the so-called dynamo number (similar to the Reynolds number in hydrodynamics). Direct application of numerical methods ignoring these parameters strongly hampers any progress. In such circumstances, it is better to turn these large or small parameters into allies, i.e. to adopt asymptotic methods.

The starting point in the application of any asymptotic method consists of an attempt to combine and utilize all a priori physical information about the properties of the solution. For instance, in the outer parts of galaxies the generation sources are relatively weak and we expect the characteristic scale of the field across the disk to be comparable with the disk half-thickness and therefore to be much smaller than its radial and azimuthal scales. Consequently, it is natural to consider first the field generation in an infinite slab, presuming a harmonic dependence of the field on the azimuthal angle, \( \phi \), in an axisymmetric disk. Only afterwards the radial dependence of the field is introduced as a correction that modulates the solution. An idea of this kind has been applied in quantum mechanics to model complex molecules (the adiabatic approximation). Casting this idea into a rigorous procedure, we obtain the algorithm shown in Figure 3 and used by Ruzmaikin et al. (1985, 1988a, b). A distinctive property of the corresponding solutions is their radial scale, which is \( \lambda^{-1/2} \) times the vertical scale of the field, where \( \lambda \approx 0.04 \) is the disk aspect ratio.

Asymptotic methods can be successfully combined with numerical ones: if it turns out that certain intermediate equations can more easily be solved numerically, this should be done. For instance, the radial equation (see Figure 3) is essentially the Schrödinger equation with a complex potential, and it also involves the small parameter \( \lambda \). Nevertheless, this equation can more easily and more accurately be solved numerically, because otherwise one would be compelled to deduce from the observational rotation curves the second derivative of the angular velocity, which is required for the analytic evaluation of the potential whose real part is determined by the local growth rate.

A specific requirement of any asymptotic approach is a clear a priori understanding of the physical nature of the process investigated (this would help also in numerical work, but it is rarely done!). Generally speaking, there are physical
mechanisms that can generate fields that cannot be studied within the framework of the particular scheme of Figure 3. For instance, such are dynamo modes that are essentially non-separable in \( r \) and \( z \) coordinates (Rädler and Bräuer, 1987; Rädler, this volume and Krause, this volume). The theory of the galactic dynamo as it currently exists can be further developed in many directions; the most promising way will be discovered by an analysis of observational results that cannot be explained by existing theory.

We should stress that a consistent asymptotic method cannot be based on arbitrary presumptions about the field structure. The \textit{ansatz} adopted should be consistent with the governing equations, i.e. it should lead to the mutual compensation, to the required accuracy, of relevant terms in (2). Otherwise, the results obtained with such an \textit{ansatz} can be of an unphysical nature even though they may reflect some restricted aspects of reality. For instance, the presumption that magnetic field lines are logarithmic spirals is not based on any physical arguments.

One of the possibilities for further progress is associated with a detailed analysis of the boundary conditions on the magnetic field. The boundary conditions at the disk surface have been discussed above. No less important are the boundary conditions at the disk center, especially for the solutions concentrated at moderate galactocentric distances. We intentionally have not shown in Figure 2 how magnetic field lines in a bisymmetric structure behave at the center of the disk. Their structure depends on the precise conditions that are imposed at the disk center. While for axisymmetric modes it is clear that \( B_\phi = B_r = 0 \) for \( r = 0 \), for non-axisymmetric modes \( B_r \) may differ from zero. Presently existing asymptotic thin-disk models assert that \( B_r = 0 \) for \( r = 0 \), which means that magnetic field lines in the center bend to become aligned with the rotation axis. This presumption

Figure 3  This diagram illustrates the procedure of the asymptotic solution of the dynamo equation in a thin disk. At the top the presentation of the solution is shown. The solution can be represented as \( B = Q(r)B(z,r)\exp(\Gamma t + i\phi) \).
agrees with the recent discovery of enhanced vertical magnetic fields in the central regions of spiral galaxies. However, other non-axisymmetric magnetic modes can be generated for which the field does not vanish at the center, $B_r + i m B_\phi = 0$ and $\partial B_r / \partial r = 0$ for $r = 0$. Investigation of these modes requires the rejection of the thin-disk approximation for which $B_\phi$ and $B_r$ similarly depend on $r$. This is partially accomplished by the dynamo model discussed in the next paragraph.

One of the alternative asymptotic models of galactic dynamos (Starchenko and Shukurov, 1989) is based on the fact that in many cases the observed large-scale magnetic field is concentrated in the central parts of galaxies where their disks cannot be considered thin. This asymptotic approximation is based on a large value ($\gg 1$) of the dimensionless dynamo number $D$.

Within several kiloparsecs of the centers of spiral galaxies, $D$ normally exceeds $10^2$. Applicability of this asymptotic method requires that the disk thickness should exceed a certain value. Specifically, the inequality $D^{1/3} \gg \lambda^{-1/2}$ should hold. For $\lambda = 0.04$ this gives $D \gg 100$. This restriction is due to the fact that in any asymptotic method only a single (small or large) parameter plays a decisive role and determines properties of the solution. Thus, for $\lambda \ll D^{-2/3}$ the disk should be considered to be thin, and the thin-disk approximation is therefore appropriate. For $D = 10$, typical of the outer parts of galaxies, this gives $\lambda \ll 0.2$. For parameter values that lie on the boundary between the ranges in which different asymptotic forms are applicable, even a slight adjustment of the parameters can lead to large changes in the solution. This explains the wide diversity of magnetic structures observed in spiral galaxies. For instance, a small difference in disk thickness in galaxies having identical rotation curves can change the dominant axial symmetry of the field.

Within the framework of this asymptotic solution, which may be called maximally-efficient-generation approximation (MEGA, see Ruzmaikin et al., 1989b), the excitation of the lowest non-axisymmetric mode $m = 1$ (here $m$ is the azimuthal wave number of the field) is possible when the parameter

$$Q = (\alpha^2 \beta r |d\Omega/dr|)^{1/3},$$

does not exceed 1 or 2; within this range the $m = 1$ non-axisymmetric mode grows at nearly the same rate as the axisymmetric one. In a thick disk, the $\alpha \omega$ — dynamo can hardly generate non-axisymmetric fields (Rädler, 1986) and asymmetry is due primarily to $\alpha^2$ — dynamo action. Another peculiarity of this asymptotic solution is the strong dependence of the pitch angle of the magnetic field lines on azimuth and radius.

We should mention that for large dynamo numbers the mode with odd parity, with respect to galactic equator, can also be excited, and the properties of the
magnetic fields in the galactic centers can be markedly different from those in the outer regions.

4. MAGNETIC STRUCTURES IN SPIRAL GALAXIES

In this section we summarize the basic results obtained in the thin-disk and strong-generation approximations for the galactic dynamo problem and compare them with observational data.

4.1 Azimuthal Structure

Magnetic structures of bisymmetric type (with the azimuthal wave number \( m = 1 \)) can be excited in galaxies such as M33, M51, M81 and NGC6946, even though they do not dominate when their galactic disks are considered to be axisymmetric. In M31 and IC 342 only axisymmetric fields can be generated. (We have here listed all galaxies for which theoretical models have been constructed.) In the Milky Way, the situation is unclear, because inaccurate observational estimates of the disk thickness preclude definite theoretical predictions; however, the dominance of the axisymmetric field seems to be indicated. The fields observed in M31 and IC 342 are axisymmetric, in M81 the non-axisymmetric field dominates; in other cases the field structure is unclear.

4.2 Pitch Angle

Magnetic field lines in both axisymmetric and non-axisymmetric structures are spirals. In the thin-disk model the pitch angle of the magnetic field lines with respect to the circumferential direction is independent of \( \phi \) for both \( m = 0 \) and \( m = 1 \) modes, and shows only a weak dependence on \( r \). Consequently, the ring structures of the magnetic field are associated not with a tighter winding of magnetic field lines as shown in Figures 1 and 2, but rather with a complicated shape of field lines which are nearly aligned with the vertical direction at some positions.

Magnetic fields generated in a thin disk are characterized by a weak decrease of the pitch angle with \( r \). However, there is theoretical evidence for the growth of the pitch angle at great distances from the center of the disk; such an increase is also revealed observationally. This tendency in the calculated field configurations is not very convincing because the accuracy of the asymptotic solutions decreases with distance from the field maximum. The pitch angle of the magnetic field lines generally decreases with \( z \).

In a thick disk the pitch angle varies more strongly with position. For instance, for the \( m = 1 \) mode there exist regions in which the pitch angle is even positive; presumably, these regions are associated with neutral lines. Probably, this peculiarity can explain the behavior of the pitch angle in NGC 6946 reported at this conference.

Note that neither model predicts that the magnetic lines are logarithmic spirals. The visualization of a three-dimensional vector field poses non-trivial problems.
Often magnetic fields are represented by the projections of the magnetic vectors on the equatorial plane. Envelopes of such regular vector arrays can form an ordered pattern which can differ markedly from that of the true magnetic field lines. Therefore, the visualization of solutions by means of arrays of separate magnetic vectors may be misleading.

Existing models of the galactic dynamo yield estimates of the pitch angles that agree reasonably well with observations. However, more detailed comparisons based on both theoretical and observational research are desirable.

### 4.3 Magnetic Age

Nonlinear effects are weak in spiral galaxies in the sense that they only saturate the field growth (and lead to wider steady-state radial distributions of field as compared with the linear distributions. This peculiarity is due to the fact that the growth time of a large-scale magnetic field in a spiral galaxy is only $10^{-100}$ times shorter than the galactic lifetime. For comparison note that for the Sun this ratio is of the order of a few tens of millions. Magnetic ages of galaxies can be characterized by the following dimensionless quantity:

$$T = \Gamma t_0 [\ln (B/B_0)]^{-1},$$

which is the ratio of the galactic lifetime, $t_0 = 10^{10}$ years, to the time required for the field to grow, at the rate $\Gamma$, from the initial strength $B_0$ to the observed one $B$. When a galaxy has two well-separated regions of field generation (e.g., the Milky Way or M31), it is characterized by two different magnetic ages. Presuming that seed magnetic fields are produced by ejection from supernovae and young stars (see Ruzmaikin et al., 1988a,b), we have $B_0 \approx 10^{-9}$ G. The resulting magnetic ages of several spiral galaxies are given in Table 1. The dynamo is still in its kinematic stage when $T < 1$ and nonlinear effects can be pronounced when $T > 1$. An important problem is the estimation of the magnetic age at which essentially nonlinear effects (such as the Maunder minimum in the Sun) can become operative.

### 5. THE PERSPECTIVES: MAGNETIC FIELDS IN A NON-AXISYMMETRIC DISK

The disks of real galaxies are not exactly axisymmetric. A very obvious deviation from axial symmetry is associated with the spiral pattern whose appearance, in addition, seems to be similar to a bisymmetric magnetic structure. It is therefore natural to seek a connection between the prevalence of a non-axisymmetric fields and the effects of the spiral structure on, say, the mean helicity of interstellar turbulence. However, the situation turns out to be more complicated. Indeed, the bisymmetric magnetic field corresponds to the magnetic mode of azimuthal wave number $m = 1$, which has opposite directions at opposite positions on the galactic disk. Meanwhile, the spiral structure inevitably produces a perturbation with $m = 2$. 

Table 1  Magnetic ages of several spiral galaxies

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<th>(1) Galaxy</th>
<th>(2) m</th>
<th>(3) Growth time, years</th>
<th>(4) Observed large-scale field, ( \mu G )</th>
<th>(5) Magnetic age</th>
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<td>Milky Way:</td>
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<td></td>
<td>0</td>
<td>( 5 \times 10^7 )</td>
<td>10</td>
<td>21.7</td>
</tr>
<tr>
<td>Solar vicinity</td>
<td></td>
<td>0</td>
<td>( 1.5 \times 10^9 )</td>
<td>2</td>
<td>0.9</td>
</tr>
<tr>
<td>M31:</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Central region</td>
<td></td>
<td>0</td>
<td>( 5 \times 10^7 )</td>
<td>10 (?)</td>
<td>21.7</td>
</tr>
<tr>
<td>Outer ring</td>
<td></td>
<td>0</td>
<td>( 10^9 )</td>
<td>2</td>
<td>1.3</td>
</tr>
<tr>
<td>M51</td>
<td></td>
<td>0</td>
<td>( 5 \times 10^7 )</td>
<td>5</td>
<td>22.5</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>2 \times 10^8</td>
<td>5.9</td>
<td></td>
<td></td>
</tr>
<tr>
<td>M33</td>
<td></td>
<td>0</td>
<td>( 2.5 \times 10^8 )</td>
<td>1</td>
<td>5.8</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>5 \times 10^8</td>
<td>2.9</td>
<td></td>
<td></td>
</tr>
<tr>
<td>M81</td>
<td></td>
<td>0</td>
<td>( 5 \times 10^7 )</td>
<td>3</td>
<td>25.0</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>2 \times 10^8</td>
<td>6.3</td>
<td></td>
<td></td>
</tr>
<tr>
<td>IC 342</td>
<td></td>
<td>0</td>
<td>( 7 \times 10^8 )</td>
<td>3</td>
<td>1.8</td>
</tr>
</tbody>
</table>

Notes: Column (1): Galaxy name. In the Milky Way and M31 magnetic fields are generated independently in the central regions and in the outer rings. Data for these generation regions are given separately. Column (2): The azimuthal wave number of the magnetic structure considered. Estimates of growth times in Column 3 are obtained for axisymmetric models, therefore the \( m=0 \) mode has the greater growth rate. Column (3): Growth time of the dominant mode with \( m \) given in Column (2). Column (4): The observed strength of the large-scale magnetic field. Column (5): Magnetic age with respect to the mode indicated. The strength of the seed field is taken \( B_0=10^{-4} \mu G \) for all galaxies.

For references see Ruzmaikin et al. (1988a); Krasheninnikova et al. (1989) and Starkenko and Shukurov (1989).

(because normally there are two arms) and the perturbations are the same at opposite positions on the galactic disk. However, the disk possesses deviations from axial symmetry other than \( m=1 \) and, consequently, these are able to assist in generating bisymmetric structures. For our purposes, the most interesting deviations are associated with irregular gas deficiencies in the disk, tidal interactions with a galactic satellite and the warping of the galactic disk.

Consider as an example perturbations associated with variations of disk thickness described by \( h=h^0[1+\varepsilon \cos \phi] \), where \( h^0 \) is the unperturbed disk half-thickness and \( \varepsilon \) is the relative magnitude of perturbation. Deviations from axial symmetry result in two effects (see Krasheninnikova et al. (1989) for details). To the first order of perturbation, the growth rates of the modes \( m=0 \) and \( m=1 \) remain unaffected but the eigenfunction \( m=0 \) acquires a non-axisymmetric addition. This means that, when the mode \( m=1 \) is excited more weakly than the mode \( m=0 \) (or even decays), the resulting large-scale magnetic field, which grows at the unperturbed rate of the mode \( m=0 \), is a combination of a strong axisymmetric component distorted by a weaker non-axisymmetric perturbation. Since non-axisymmetric eigenfunctions are concentrated at greater radii than the axisymmetric ones, this perturbation can be prominent in the outer part of a galaxy. This situation is possibly observed in M31 (Beck, 1982).

Second order perturbation theory yields corrections to the growth rates of all modes. For instance, in Figure 4 we show the dependence of \( \Re \Gamma \) on \( \varepsilon \) for \( m=0 \)
Figure 4 The growth rates of the $m=0$ and $m=1$ dynamo modes in M81 (solid lines) and M51 (dashed lines) versus the amplitude of deviation of the disk from axial symmetry for perturbation (4).

and $m=1$ modes in M81 and M51; the effect in M81 is stronger. In M81 the growth rate of the bisymmetric mode even grows with $\varepsilon$. Note that new observational results reported at this conference indicate that (unlike M51) M81 provides a clear example of a bisymmetric magnetic structure.

For sufficiently large values of $\varepsilon$ the growth rate of the $m=1$ mode becomes larger than that of the $m=0$ mode (see Figure 4) (note that the perturbation approach is applicable here, even for rather large values of $\varepsilon$ exceeding unity—see Krasheninnikova et al., 1989). However, for perturbations of the type considered, the bisymmetric mode becomes dominant in M81 only for $\varepsilon > 2.8$; of course, such strong perturbations are unphysical. However, other kinds of deviations from axial symmetry, e.g. the warping of the galactic disk can lead to a stronger effect.

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References


