MAS3210 2009/2010 - Revision streamliner

This guide only applies to the January 2010 exam, NOT necessarily to August 2010 or later. When a theorem is said to be examinable you should know its statement and be prepared to answer a question which leads you through its proof. The idea is that you should understand how to prove something (as opposed to memorising it without understanding).

The exam will consist of two sections. **You should answer all questions from both sections.** Section A worth 40 marks will consist of routine questions. Section B worth 60 marks will contain longer or more difficult questions.

1 Combinatorial Designs

Prussian Officers & Kirkman's schoolgirls

Background only.

1.1 A Simpler Problem

Motivation.

1.2 2-Designs

You need to know the notation and definitions: v-set, k-subset, $2-(v,k,\lambda)$ design, $b = |B|$. You should know about the Fano Plane.

1.3 Trivial Designs

You need to know the definition of a trivial design (Definition 1.8).

Theorem 1.7. Is examinable.

Theorem 1.9. Is examinable.

Theorem 1.10. Is examinable.

Corollary 1.11. Is examinable.

1.4 Arithmetic of 2-designs

Theorem 1.12. Is examinable.

Theorem 1.14. Is examinable.

Corollary 1.15. Is examinable.

Theorem 1.16. Is examinable.

1.5 Applications

You will need to be able to answer questions such as those in this section.

1.6 Steiner Systems

You will need to know the definitions of a Steiner Triple Systems of order v (Definition 1.23) and a binary projective space of dimension n (Definition 1.25).

Theorem 1.26. Is examinable.

Theorem 1.29. The proof is not examinable, but the statement and its use is.

Example 1.30. You will not need to know this construction in detail.

Examples 1.31. Examples like this could appear on the exam paper. They could be part of a more general question: given certain parameters, either give an example of a design with those parameters or prove that none exists.

1.7 Projective Planes

You will need to know the definition of a projective plane of order n (Definition 1.32).

1.8 Fisher's Inequality

Theorem 1.34. You will need to know the statement of Fisher's Inequality but you will not be asked for the proof.

You will need to know how to apply FI to examples (usually to show that designs with certain parameters cannot exist).

1.9 Complementary Designs

You will need to know the definition of complementary design (Definition 1.37).

Theorem 1.36. The proof is examinable, as are applications to examples: given certain parameters, either give an example of a design with those parameters or prove that none exists.

1.10 Symmetric Designs

You will need to know the definition of a symmetric design (Definition 1.40).

Theorem 1.42. Is examinable.

Theorem 1.44. Is examinable.

Theorem 1.46. You will know how to use this theorem, but you do not need to know the statement. No proof was given (and you do not need to know one!).

Theorem 1.47. Essentially an example, but you should be able to answer a question that asks you to prove that ....

Example 1.48. You need to be able to apply Theorem 1.46 to this type of problem.

1.11 Matrix Multiplication, J-Matrices and Determinants

You need to know the material in this section in a general sense - you might have to use bits elsewhere - but you won't have to prove the theorems.

1.12 Incidence Matrices

You need to know the definition of an incidence matrix (Definition 1.57) and you need to know what a $\{0, 1\}$ matrix is (but you won't be asked to define it specifically).

Theorem 1.60. You should know the fundamental properties of an incidence matrix, as set out in this theorem, but you won't be asked to prove them.

You should know how to construct an incidence matrix for a design with given parameters (including knowing how to use information on symmetric designs given a bit later). You won't be asked to construct a large matrix from scratch, but you might be asked to do a small one, or to complete a slightly larger one with most of the entries given.

Theorem 1.63. Is examinable.

Theorem 1.64. Is examinable.

Theorem 1.65. Is examinable (statement only, you won't be asked for the proof of this).

1.13 Symmetric Designs

Theorem 1.66. Is examinable.

Theorem 1.67 (Ryser)**.** Is examinable.

2 Geometry Of The Projective Plane

2.1 The Euclidean Plane $\mathbb{E}$

You will not be examined specifically on the Euclidean plane. However you might need to refer to results on the Euclidean plane in proving results on the projective plane.

2.2 The Line At Infinity

2.3 Lines in $\mathbb{P}_2(\mathbb{K})$

You need to know the definition of the projective plane $\mathbb{P}_2(\mathbb{K})$, its points and its lines (Definitions 2.8, 2.9). You need to know the term 'homogeneous coordinates'. You need to know what the line at infinity is (Definition 2.11). You need to know what collinear and concurrent mean.

Theorem 2.12. Is NOT examinable.

Fact 2.13. You should know this, but you won't be asked to prove it.

Theorem 2.14. You should know the statement, but you won't be asked to prove it.

Theorem 2.16. Is examinable.

2.4 Duality

You should know about the Principle of Duality and be able to apply it in various contexts, for example in proving the converse of Desargues' Theorem.

Theorem 2.18. You won't be asked to prove this, but you should know how to find the intersection of two lines.

2.5 The Projective Planes $\mathbb{P}_2(\mathbb{K})$

You need to know the notation $\mathbb{P}_2(p)$.

Lemma 2.21. Is examinable.

Lemma 2.22. You could be asked to prove either statement (or both) and it could be "without using explicitly the Principle of Duality".

Theorem 2.23. Is examinable.

You need to be able to find intersections of lines and equations of lines between two points. You won't be restricted in terms of the techniques you use but you will find it useful to be able to apply a range of techiniques.

Theorem 2.28. Is NOT examinable.

Theorem 2.30. Is examinable.

Example 2.33. This is given as an example, but you should be able to prove statements like "Prove that there are $3p$ points of $\mathbb{P}_2(p)$ lying on the sides of a triangle.".

Definition 2.34. You won't need this.

Theorem 2.35. You won't need to prove this, but you should know it.

2.6 The Triangle of Reference and the Unit Point

You should know what the triangle of reference and the unit point are. You should know the definition of a quadrangle.

Theorem 2.38. You need to know the statement, but no proof was given.

Example 2.39. This would be a reasonable (long) exam question. (Too long for Section A.)

Example 2.40. This example is too long for an exam question, but the ideas are at an appropriate level for a Section B question.

2.7 Desargues' Theorem

You need to know what it means for two triangles to be in perspective from a point.

Theorem 2.42 (Desargues)**.** Is examinable.

Theorem 2.43 (Converse of Desargues)**.** Is examinable.

2.8 Pappus' Theorem

Theorem 2.44 (Pappus)**.** Is examinable.

3 Conics

None of this section is examinable.