

MAS221 Number Systems and the Foundations of Analysis

Semester 1: Mock Exam

SECTION A

Section A is the same as for MAS121 except that it is out of 40 instead of 50. The marks for Section A questions are A1 16; A2 4; A3 8; A4 8; A5 4.

SECTION B

Each question in this section is worth a total of 30 marks. Section B questions are the same as for MAS121 except that they have the following additional parts: each of which is worth 5 marks.

- B6.** (d) Let a and b be integers and assume that $a + sb = p$ is prime for some integer s . Set $t = s + kp$, where $k \in \mathbb{Z}$. Show that $a + tb$ is composite. Hence show that the sequence

$$a, a + b, a + 2b, a + 3b, \dots$$

contains infinitely many composite numbers.

- B7.** (e) Show that $f_n^2 + f_n f_{n-1} = f_n f_{n+1}$, for all $n \geq 2$. Hence show that $f_n^2 = f_n f_{n+1} - f_n f_{n-1}$ and use this to show that

$$f_1^2 + f_2^2 + \dots + f_n^2 = f_n f_{n+1},$$

for all $n \geq 1$.

- B8.** (e) Let s and t be natural numbers such that $\gcd(s, t) = 1$ and define

$$x = 2st, y = s^2 - t^2, z = s^2 + t^2.$$

Show that $x^2 + y^2 = z^2$. If p is an odd prime such that $p|x$ show that p does not divide y .