Student Number: **Degree Programme:**

NEWCASTLE UNIVERSITY

SCHOOL OF MATHEMATICS & STATISTICS

SEMESTER 1 MOCK EXAM

MAS1202

Number Systems and the Foundations of Analysis

Time allowed: 1 hour 30 minutes

Candidates should attempt all questions. Marks for each question are indicated. However you are advised that marks indicate the relative weight of individual questions, they do not correspond directly to marks on the University scale.

There are EIGHT questions on this paper.

Write your answers on the exam paper, in the spaces provided.
Write rough work on the reverse of the pages.
State carefully where you use any results from the course.

No.	Mark
1	
2	
3	

No.	Mark
4	
5	
6	

No.	Mark
7	
8	
9	

Total	

1. (a) Find the greatest common divisor of 1400 and 37730.

(b) Find integers x and y such that

$$1400x + 37730y = \gcd(1400, 37730).$$

(c) Which of the following equations have integer solutions? In each case either find integer solutions u and v or explain (briefly) why no solution exists.

(i) $1400u + 37730v = 210$;

(ii) $1400u + 37730v = 102$.

(d) Find the general solution for those equations in part (c) above which have a solution.

(e) Find all solutions with $x > -1000$ and $y > 0$.

[25 marks]

2. (a) Let a, b, c and d be integers such that $a|b$ and $c|d$. Prove that $ac|bd$.

(b) Show that

$$5n^2|(5n^2 + 3)^2 - 9,$$

for all $n \in \mathbb{Z}$.

[5 marks]

3. Let a, b and c be non-zero integers such that $\gcd(a, b) = \gcd(a, c) = 1$. Show that $\gcd(a, bc) = 1$.

[5 marks]

4. (a) Show that n^2 has the form $5k$, $5k + 1$ or $5k + 4$, with $k \in \mathbb{Z}$, for all integers n.

(b) Show, using the first part of the question, that if $5|n^2$ then $5|n$.

[15 marks]

5. Prove by induction that:

$$\sum_{k=1}^{n} k(k + 1) = \frac{1}{3}n(n + 1)(n + 2),$$

for all $n \in \mathbb{N}$.

[10 marks]

6. Use Fermat's method of factorisation to find factors of 253, showing all your working. (You may use the fact that $15 < \sqrt{253} < 16$.)

[5 marks]

7. (a) Complete the table below for multiplication modulo 8 using only the integers $0, 1, 2, \ldots, 7$.

$\times$	0	1	2	3	4	5	6	7
0	0	0	0	0	0	0	0	0
1	0	1	2	3	4	5	6	7
2	0	2	4					
3	0							
4								
5								
6								
7								

(b) Which integers have inverses modulo 8?

(c) Compute $13^{23} \pmod 8$.

(d) State how many incongruent solutions there are to the following congruences. Justify your answers. Then find all solutions.

(i) $10x \equiv 6 \pmod{18}$;

(ii) $10x \equiv 9 \pmod{18}$.

[**20 marks**]

8. (a) Let a, b and c be integers such that $a|b$ and $a|c$. Show that $a|b-c$.

(b) Let n be a positive integer and let $S = n! + 1$. Show that if p is a prime divisor of S then $p > n$.

(c) Use the first part of the question to show that there are infinitely many primes. [**Hint:** If there are finitely many primes then set n in the previous part of the question equal to the largest prime.]

[**15 marks**]

THE END