MAS1002 Optimisation and Linear Methods: Assignment

AJ Duncan, February 2007

1 Questions Due on Friday 9th March 2007

1.1 A glue consists of resin and hardener which are mixed together when the glue is to be used. The strength of the resulting compound depends on the amount of hardener added. Experimental evidence shows that if x parts of hardener are mixed with 100 parts of resin then the strength $S(x)$ varies according to the following formula

$$S(x) = \begin{cases} x, & \text{if } 0 \le x \le 1 \\ (3-x)/2, & \text{if } 1 \le x \le 2 \\ (x-2)e^{-(x-2)} + 1, & \text{if } 2 < x \end{cases}.$$

How many parts of hardener should be used to maximise the strength of the glue. (In your answer be careful to consider values of x for which S is not differentiable.)

1.2 A manufacturer needs cardboard cartons of 2000 cm^3 capacity. The cartons are to have a square cross section. If the height of the carton is h and the sides measure s centimetres then the area of card required to make each carton is

$$A = (4s+1)(s+h+4)\text{cm}^2.$$

The manufacturer wishes to minimise the amount of card used to make each carton. Given the dimensions above the volume of each carton is s^2h so the requirment on capacity means we must choose s and h so that

$$h = 2000/s^2.$$

(a) Eliminate h from the above expression for A, giving a function $f(s)$ such that $A = f(s)$.

(b) The manufacturers problem is thus to minimise $f(s)$, with $s > 0$. Find $f'(s)$. Show that $f'(s) = 0$ if and only if

$$8s^4 + 17s^3 - 8000s - 4000 = 0.$$

(c) Maple has a command `fsolve`. Use Maple's help facility to find out how to use this command. Use `fsolve` to find an appropriate solution to the above equation. (Your answer to this part of the question should be saved with the answer to Question 1.3 below and handed in using NESS.)

(d) Find the global minimum of f, the dimensions s and h for which A is minimal and the minimal value of A.

1.3 This is exactly the same question as the one on the first computer practical. The answers are to be handed in on the Ness system using the instructions below.

Let $f(x, y) = x^2 - xy + 1$.

(i) Use the Maple command `plot3d` to plot this function in the region of the plane where $-5 \le x \le 5$ and $-5 \le y \le 5$.

(ii) Use `contourplot3d` to make a 3D-contour plot and `contourplot` to make a 2D-contour plot of the same function over the same region. These plots don't look very good at first so to make them more visible insert the option `filled=true` and replot.

(iii) Type in the appropriate command to set $y = 0$ (`y:=0;`). Now plot the function f using the 2D command `plot`. Replot for a few other values of y if you like but your final output should just include the $y = 0$ one.

(iv) First unassign the variable y by giving Maple the `unassign('y');` command. If you don't do this the next part of the question won't work. Now animate the 2D plot, using the command `animate`, with y ranging from -5 to 5.

(v) Now create a sequence of cross sections for fixed values of y from -5 to 5, for this function; using the command `cross_sections:=[seq([x,y,f],y=-5..5)];` as in the Maple example above. Plot all these cross sections at once using the `spacecurve` command. (You'll need to use `op(cross_sections)` rather than plain `cross_sections` as in the Example.)

(vi) Now find the equation of the tangent plane to f at the point $(2, 1, 3)$ and plot both the function and it's tangent at this point on one diagram.

2 Questions Due on Friday 11th May 2007

2.1 Use the method of Lagrange multipliers to find the global maxima and minima (if any) of the function

$$f(x_1, x_2, x_3) = x_1^2 + 2x_2^2 + 3x_3^2$$

subject to the constraints

$$\begin{aligned} x_1 + x_2 + x_3 - 1 &= 0 \\ 2x_1 - x_2 - 3 &= 0. \end{aligned}$$

2.2 Solve the following problem using the simplex method. Maximise

$$5x_1 + 6x_2 + 9x_3 + 8x_4$$

subject to the constraints

$$\begin{aligned} x_1 + 2x_2 + 3x_3 + x_4 &\leq 5 \\ x_1 + x_2 + 2x_3 + 3x_4 &\leq 3 \\ x_1, x_2, x_3, x_4 &\geq 0. \end{aligned}$$

2.3 Submit your answer to this part of the assignment on Ness. Look up the "simplex" command in Maple. Using the "minimize" command of the simplex solve the following problem, which is the diet problem outlined in the notes at the begining of Section 4. Minimise

$$5x_1 + 40x_2 + 20x_3 + 12x_4 + 45x_5 + 40x_6$$

subject to the constraints

$$\begin{aligned} 0 \leq x_1 &\leq 4 \\ 0 \leq x_2 &\leq 3 \\ 0 \leq x_3 &\leq 2 \\ 0 \leq x_4 &\leq 8 \\ 0 \leq x_5 &\leq 2 \\ 0 \leq x_6 &\leq 2, \end{aligned}$$

$$\begin{aligned} 110x_1 + 205x_2 + 160x_3 + 160x_4 + 420x_5 + 260x_6 &\geq 2000 \\ 4x_1 + 32x_2 + 13x_3 + 8x_4 + 4x_5 + 14x_6 &\geq 55 \\ 2x_1 + 12x_2 + 54x_3 + 285x_4 + 22x_5 + 80x_6 &\geq 800. \end{aligned}$$

Save the commands and results and submit them to Ness.

Ness Instructions

To submit your solutions to Maple Exercises do the following.

2.1 Save your Maple worksheet, wherever you like.

2.2 Go to URL https://coursework.cs.ncl.ac.uk/ and log on with your usual university id.

2.3 There is a line of tabs along the top of the page. Click on the "Coursework" tab.

2.4 In the boxes on the top left hand side of the page enter Module "MAS1002" and Coursework "Exercise ?" (? should be 1 for the homework to be submitted in week 6).

2.5 Down the left hand side of the page there are links. Click on the "Submit" link.

2.6 Enter the name of the saved Maple worksheet (use the Browse facility to make sure you get the right file).

2.7 Hit the "Submit Your Work" button at the bottom of the page.

2.8 The receipt number will be emailed to you there is no need to memorise it.