MAS1002 Optimisation and Linear Methods: Computer Practical

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1 Introduction

In the computer practical work through the maple tutorials described below and hand in the excercises as part of the assignment due in week 6.

The Maple commands that you need to know for these practicals are covered in MAS1001 Maple worksheets which can be found at `www.ncl.ac.uk/math/internal/maple/` by following the "Induction" or "book1" links for basic commands and "book2" for plotting and graphics.

To access files you need to follow the instructions at

`www.ncl.ac.uk/math/internal/coursedata.htm`

to map the file

`\\campus\software\mathematics and statistics\course data`

to a drive letter. It is assumed below that this file is mapped to the Q: drive.

2 Functions of Two Variables

In the lectures two methods of visualising functions of 2 variables are covered. This section of the worksheet investigates these visiualisation methods using Maple.

Contour Plots

Let f be a function from $\mathbb{R}^2 \longrightarrow \mathbb{R}$ and suppose that we plot the graph of f using coordinates x, y and z. For a fixed real number c the *contour* of *height* c is then the set of points in the (x, y)-plane which satisfy the equation $c = f(x, y)$. If we draw contours at regular intervals then the resulting collection of curves in the plane gives a picture of the 3 dimensional graph of $f(x, y)$.

Example 2.1. Let f be given by $f(x) = \sin(x^2 + y^2)$. Open the Maple worksheet practical_1.mw on Q:1002 and run through the section entitled "Contour Plots"

Cross Sections

With f again a function from $\mathbb{R}^2 \longrightarrow \mathbb{R}$ fix a value b and set $y = b$. As before we can plot the points $z = f(x, b)$ in the (x, z) plane. Do this for several values of b. Then for each b we have a plot on the plane $y = b$. The union of these plots gives a model of the surface.

Example 2.2. Let f be given by $f(x) = \sin(x^2 + y^2)$. Open the Maple worksheet practical_1.mw on Q:1002 and run through the section entitled "Cross Sections"

Planes

If $f(x, y) = mx + ny + c$, where m, n and c are constants then the graph $z = f(x, y)$ is called a plane. For example the set of points satisfying $z = 2x + y - 1$ is a plane. On the same worksheet as before run through the section on planes. Notice that there are planes with equations not involving z, which are not the graphs of functions as above.

Notice that a plane is determined by m and n, which are it's *gradient* as well as by c which determines how far from the origin it is.

Tangents

The tangent to the graph of a function $f(x)$ of one variable at a point $(x, f(x))$ is the line which best approximates f at that point. From calculus we know that the gradient of this line is $f'(x)$. Similarly, the tangent plane to a function $f(x, y)$ of 2 variables, at the point (x, y, z) is the plane which best approximates f at this point. It turns out that this is the plane through the given point with gradient $m = \partial f/\partial x$ and $n = \partial f/\partial y$ (in the terminology above).

Example 2.3. Find the equation of the tangent plane to $h(x, y) = x^2 - y^2$ at the point $(2, 1, 3)$. First note that this point is on the graph. We have $\partial h/\partial x = 2x$ and $\partial h/\partial y = -2y$, so at the gradient at $(2, 1, 3)$ is $m = 2x = 4$, $n = -2y = -2$. The plane $z = 4(x - 2) - 2(y - 1) + 3$ has gradient $m = 4, n = -2$ and passes through the point $(2, 1, 3)$ so should be the tangent. Run through the remainder of the worksheet to see that this is the case.

3 Exercises

The following will form part of the first assignment to be handed in week 6. The Maple output is to be handed in on the Ness system using the instructions below.

Let $f(x, y) = x^2 - xy + 1$.

(i) Use the Maple command `plot3d` to plot this function in the region of the plane where $-5 \le x \le 5$ and $-5 \le y \le 5$.

(ii) Use `contourplot3d` to make a 3D-contour plot and `contourplot` to make a 2D-contour plot of the same function over the same region. These plots don't look very good at first so to make them more visible insert the option `filled=true` and replot.

(iii) Type in the appropriate command to set $y = 0$ (`y:=0;`). Now plot the function f using the 2D command `plot`. Replot for a few other values of y if you like but your final output should just include the $y = 0$ one.

(iv) First unassign the variable y by giving Maple the `unassign('y');` command. If you don't do this the next part of the question won't work. Now animate the 2D plot, using the command `animate`, with y ranging from -5 to 5.

(v) Now create a sequence of cross sections for fixed values of y from -5 to 5, for this function; using the command `cross_sections:=[seq([x,y,f],y=-5..5)];` as in the

Maple example above. Plot all these cross sections at once using the `spacecurve` command. (You'll need to use `op(cross_sections)` rather than plain `cross_sections` as in the Example.)

(vi) Now find the equation of the tangent plane to f at the point $(2, 1, 3)$ and plot both the function and it's tangent at this point on one diagram.

4 Ness Instructions

To submit your solutions to Maple Exercises do the following.

1. Save your Maple worksheet, wherever you like.
2. Go to URL https://coursework.cs.ncl.ac.uk/ and log on with your usual university id.
3. There is a line of tabs along the top of the page. Click on the "Coursework" tab.
4. In the boxes on the top left hand side of the page enter Module "MAS1002" and Coursework "Exercise ?" (? should be 1 for the homework to be submitted in week 6).
5. Down the left hand side of the page there are links. Click on the "Submit" link.
6. Enter the name of the saved Maple worksheet (use the Browse facility to make sure you get the right file).
7. Hit the "Submit Your Work" button at the bottom of the page.
8. The receipt number will be emailed to you there is no need to memorise it.