UNIVERSITY OF NEWCASTLE UPON TYNE

SCHOOL OF MATHEMATICS & STATISTICS

MOCK
2000/2001

MAS051

Foundation Mathematics (Part 1)

Time allowed: 1 hour 30 minutes

Candidates should attempt all questions. Marks for each question are indicated. However you are advised that marks indicate the relative weight of individual questions, they do not correspond directly to marks on the University scale.

There are TEN questions on this paper.

1. (a) Expand the following expression, and simplify if possible:

$$(a+b)(b-a) \ .$$

(b) Express each of the following in the form a^n (i.e. find n):

(i) a^2a^3 ; (ii) $\dfrac{\sqrt{a}}{a^{-2}}$; (iii) $\left(a^{-2}\right)^{1/2}$.

(c) Rationalise the denominator in the following (and simplify if possible):

$$\frac{2-\sqrt{2}}{1+\sqrt{3}} \ .$$

[10 marks]

2. Show that $x = -1$ is a zero of the cubic

$$f(x) = x^3 - 2x^2 - 13x - 10 \ ,$$

and then use this information to factorise the cubic fully.

Hence list all solutions of the cubic equation $f(x) = 0$.

[10 marks]

3. A triangle has sides of length a, b and c, opposite the angles A, B and C, respectively.

State the cosine rule relating the three sides a, b and c, and the angle C.

Hence solve the triangle (i.e. provide all the missing lengths and/or angles), given that:

$$b = 4, \qquad c = 5, \qquad A = 1.2 \text{ radians} \ .$$

(Give all angles in radians, and all numbers correct to 2 decimal places.)

[10 marks]

4. Solve the following inequalities (giving your answer as the appropriate range of values of x):

$$\text{(a)} \qquad 4 - 3x \leq 5x + 12 \; ;$$

$$\text{(b)} \qquad (2x + 3)(x + 4) > 0 \; .$$

[**10 marks**]

5. (a) Express each of the following in the form $x + \mathrm{i}\,y$, where x and y are real numbers.

$$\text{(i)} \quad (2 + \mathrm{i})(1 - 3\,\mathrm{i}) \; ; \qquad\qquad \text{(ii)} \quad \frac{3 + 2\,\mathrm{i}}{5 - 2\,\mathrm{i}} \; .$$

(b) Solve the quadratic equation

$$z^2 + 4z + 6 = 0 \; ,$$

expressing your answers in the form $x + \mathrm{i}\,y$, where x and y are real numbers.

[**10 marks**]

6. (a) Write the equation (in the standard form $y = mx + c$) for the straight line which passes through the point (2,5) and which is parallel to the straight line $y + 4x = 1$.

(b) Now write the equation (in the standard form) for the straight line that passes through the point (2,5) and is perpindicular to the line $y + 4x = 1$.

[**10 marks**]

7. Consider the two functions $f(x) = x^3 + 3x^2 + 1$ and $g(x) = 5\sin(2x)$.

(a) Calculate $f(g(x))$.

(b) Calculate $g(f(x))$.

[8 marks]

8. Differentiate the following functions. Simplify your answers.

(a) $f(x) = 3x^3 + x^4 - 7x + 5$

(b) $f(x) = \dfrac{x^2 - 6}{x^3 + x^2 + 1}$

(c) $f(x) = (x^5 + 2x + 7)\cos(x)$

(d) $f(x) = \dfrac{5}{\cos(x^2 + 6)}$

[12 marks]

9. Find the stationary points of the function $f(x) = 8x^3 - 6x + 5$ and determine the nature of each stationary point.

[8 marks]

10. Find the significant features of the function

$$f(x) = \frac{x^2}{2x - 4}$$

and use them to sketch the graph of the function.

[12 marks]