MAS051 — Example Sheet 10

To be handed in: Friday, November 29th, 2002

Remember to give your Tutorial Group, along with your name, on the assignment. Only the *asterisked* (starred) questions will be marked.

1. Use the quotient rule to differentiate each of the following functions:

(a) $\dfrac{3x+4}{5x-3}$; (b)* $\dfrac{1-2x}{2-x}$; (c) $\dfrac{a+bx}{c+dx}$; (d)* $\dfrac{x^2+4}{x^2-4}$;

(e) $\dfrac{x^2-2x+4}{x^2+2x+4}$; (f)* $\dfrac{x^n+1}{x^n-1}$; (g) $\dfrac{8x}{x^2+1}$; (h)* $\dfrac{\sqrt{x}}{1-x}$;

(i)* $\dfrac{x}{1+\sqrt{x}}$; (j) $\dfrac{1+\sqrt{x}}{1-\sqrt{x}}$; (k)* $\dfrac{ax^2+bx+c}{ax^2-bx+c}$.

2. Find the equation of the tangent line to the curve $y = \dfrac{x^2}{x-3}$ at the point $x = 4$.

3. Differentiate using the product rule: (a)* $x^2 \sin x$; (b) $2x \tan x$; (c)* $x^3 \cos x$; (d) $\sin(x)\cos(x)$.

4. Express each of the following in the form $x + \mathrm{i}\,y$, where x and y are real numbers.
(Here $\mathrm{i} = \sqrt{-1}$.)

(a)* $\mathrm{i}\,(2-3\,\mathrm{i})$ (b) $(1+2\,\mathrm{i})(1-2\,\mathrm{i})$ (c)* $(3-2\,\mathrm{i})(4+7\,\mathrm{i})$

(d) $(a+\mathrm{i}\,b)(a-\mathrm{i}b)$ (e)* $(a+\mathrm{i}\,b)(b+\mathrm{i}\,a)$ (f) $(3a+2\,b\,\mathrm{i})(2a-\mathrm{i}\,b)$

5. Rationalise each of the following complex numbers. (i.e express each in the form $x + \mathrm{i}\,y$, where x and y are real numbers.)

$$\text{(a)}\ \frac{5}{\mathrm{i}}\,; \qquad \text{(b)}*\ \frac{3-4\,\mathrm{i}}{2\,\mathrm{i}}\,; \qquad \text{(c)}\ \frac{4+\mathrm{i}}{4-\mathrm{i}}\,; \qquad \text{(d)}*\ \frac{3-2\,\mathrm{i}}{2+5\,\mathrm{i}}\,.$$

6. Solve the following quadratic equations, giving your answers in the form $x + \mathrm{i}\,y$, where x and y are real numbers.

$$\text{(a)}*\ z^2+2z+3=0\,; \qquad \text{(b)}\ 3z^2-5z+4=0\,; \qquad \text{(c)}*\ 3-z=\frac{4}{z}\,.$$

7. Express each of the following complex numbers in polar form, $re^{i\theta}$, where r and θ are real numbers, $r \geq 0$, and $0 \leq \theta < 2\pi$ (with θ in radians).

$$\text{(a)}\ 2\,\mathrm{i}\,; \qquad \text{(b)}*\ -1+\mathrm{i}\,; \qquad \text{(c)}\ (2\sqrt{3}+\mathrm{i})-(\sqrt{3}+2\,\mathrm{i})\,;$$

$$\text{(d)}*\ 2\,e^{\mathrm{i}\,\pi/3} \times \frac{1}{2}\,e^{\mathrm{i}\,2\pi/3}\,.$$

AJD September 26, 2002