TORSION FREE $\widetilde{A}_2$ GROUPS AND THEIR K-THEORY

Each group has q^2+q+1 generators and $\frac{1}{3}(q+1)(q^2+q+1)$ relations.

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Group A.1
x0*x1*x3 = 1, x0*x2*x6 = 1, x0*x4*x5 = 1, x1*x2*x4 = 1,
x1*x5*x6 = 1, x2*x3*x5 = 1, x3*x4*x6 = 1;
Group A.2
x0*x1*x6 = 1, x0*x2*x5 = 1, x0*x4*x3 = 1, x1*x3*x3 = 1,
x1*x4*x2 = 1, x2*x6*x6 = 1, x4*x5*x5 = 1;
Group B.1
x0*x1*x2 = 1, x0*x2*x4 = 1, x0*x4*x1 = 1, x1*x6*x3 = 1,
x2*x5*x6 = 1, x3*x5*x4 = 1, x3*x6*x5 = 1;
Group B.2
x0*x1*x4 = 1, x0*x2*x1 = 1, x0*x4*x2 = 1, x1*x5*x5 = 1,
x2*x3*x3 = 1, x3*x5*x6 = 1, x4*x6*x6 = 1;
Group C.1
x0*x0*x6 = 1, x0*x2*x3 = 1, x1*x2*x6 = 1, x1*x3*x5 = 1,
x1*x5*x4 = 1, x2*x4*x5 = 1, x3*x4*x6 = 1;
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$q=2:$ Regular Group (A.1) has generators x_j, $0\le j\le 7$, with relations

$$x_jx_{j+1}x_{j+3}=1 \qquad (0\le j\le 6)\,.$$

$q=4:$ Regular Group has generators x_j, $0\le j\le 20$, with relations

$$\begin{cases} x_jx_{j+3}x_{j-6}=1 \qquad (0\le j\le 20),\\ x_jx_{j+7}x_{j+14}=x_jx_{j+14}x_{j+7}=1 \qquad (0\le j\le 6)\,.\end{cases}$$

$q=5:$ Regular Group has generators x_j, $0\le j\le 30$, with relations

$$\begin{cases} x_jx_{j+1}x_{j+6}=1 \qquad (0\le j\le 30),\\ x_jx_{j-7}x_{j-11}=1 \qquad (0\le j\le 30)\,.\end{cases}$$

$q=7:$ Regular Group has generators x_j, $0\le j\le 56$, with relations

$$\begin{cases} x_jx_{j+1}x_{j+8}=1 \qquad (0\le j\le 56),\\ x_jx_{j+6}x_{j-9}=1 \qquad (0\le j\le 56),\\ x_jx_{j+19}x_{j+38}=x_jx_{j+38}x_{j+19}=1 \qquad (0\le j\le 18)\,.\end{cases}$$

$q=8:$ Regular Group has generators x_j, $0\le j\le 72$, with relations

$$\begin{cases} x_jx_{j+1}x_{j+9}=1 \qquad (0\le j\le 72),\\ x_jx_{j+2}x_{j+18}=1 \qquad (0\le j\le 72),\\ x_jx_{j+4}x_{j+36}=1 \qquad (0\le j\le 72)\,.\end{cases}$$

$q=11:$ Regular Group has generators x_j, $0\le j\le 132$, with relations

$$\begin{cases} x_jx_{j+1}x_{j+12}=1 \qquad (0\le j\le 132),\\ x_jx_{j+16}x_{j+59}=1 \qquad (0\le j\le 132),\\ x_jx_{j+40}x_{j+81}=1 \qquad (0\le j\le 132),\\ x_jx_{j+60}x_{j+55}=1 \qquad (0\le j\le 132)\,.\end{cases}$$

The K-group $K_0 = K_1$ for $\mathcal{A}(\Gamma) = C(\Omega) \rtimes \Gamma$.

Notation: $[a, b, ...]$ means $\mathbb{Z}_a \oplus \mathbb{Z}_b \oplus \dots$; $m\,[a, b, ...]$ means $\mathbb{Z}^m \oplus \mathbb{Z}_a \oplus \mathbb{Z}_b \oplus \dots$; $(j)a$ means $a, a, ..., a$ j times. K_0 is the K-group of the crossed product algebra (recall that $K_0 = K_1$), and $K_0/ < [\mathbf{1}] >$ is the K-group modulo the class $[\mathbf{1}]$ of the identity.

	Γ	Γ_{ab}	K_0	$K_0/ < [\mathbf{1}] >$
$q = 2$	A.1	[(3)2,3]	0 [(6)2,3]	0 [(6)2,3]
The $\mathbb{F}_2((X))$ cases	A.2	[2,3,7]	0 [(2)2,3,(2)7]	0 [(2)2,3,(2)7]
$q = 2$	B.1	[3]	0 [3]	0 [3]
The $\mathbb{Q}_2$ cases	B.2	[(2)2,3]	0 [(2)2,3]	0 [(2)2,3]
	C.1	[(2)2,3]	0 [(4)2,3]	0 [(4)2,3]
$q = 4$	Regular	[(6)2,(2)3]	28 [(12)2,(6)3]	28 [(12)2,(6)3]
	Regular′	[(2)2,(2)3,7]	28 [(4)2,(6)3,(2)7]	28 [(4)2,(6)3,(2)7]
$q = 5$	Voskuil	[(2)2,(4)4,3]	62 [(14)2,(5)4,3]	62 [(14)2,(4)4,3]
	Regular	[3,(3)5]	62 [4,3,(6)5]	62 [3,(6)5]
$q = 7$	Voskuil	[(7)3]	190 [2,(21)3]	190 [(21)3]
	Regular	[(2)3,(3)7]	190 [2,(6)3,(6)7]	190 [(6)3,(6)7]
$q = 8$	Regular	[(9)2,3]	292 [7,(18)2,3]	292 [(18)2,3]
$q = 11$	Regular	[3,(3)11]	798 [10,3,(6)11]	798 [3,(6)11]
$q = 13$	Regular	[(2)3,(3)13]	1342 [4,(6)3,(6)13]	1342 [(6)3,(6)13]