Discussion of Riemann manifold Langevin and Hamiltonian Monte Carlo methods by Mark Girolami, Ben Calderhead

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We congratulate Mark Girolami and Ben Calderhead for an interesting paper and an important contribution to MCMC methodology. The authors present two algorithms (manifold Metropolis adjusted Langevin algorithm (MALA) and Riemann manifold Hamiltonian Monte Carlo (RMHMC)) that circumvent problems inherent in Metropolis Hastings methods such as MALA and HMC when sampling high dimensional target densities that may exhibit strong correlations. Such correlations can often occur in state space models (SSMs). We therefore focus our discussion on the utility of MMALA and RMHMC for SSMs.

To fix notation consider a hidden Markov state process \(\{X_n, n \geq 1\}\) with transition density parameterised by \(\theta\) and observed indirectly through \(\{Y_n, n \geq 1\}\). Suppose that \(T\) observations are available on \(\{Y_n\}\) and are conditionally independent given \(\{X_n\}\). The assumed intractable density \(p(\theta, x|y)\) can be sampled by alternating between draws of \(\theta|x, y\) and \(x|\theta, y\). This two step blocking approach is used successfully by the authors for the stochastic volatility model via both MALA and RMHMC. Another possibility is to sample \(p(x|y, \theta)\) via a particle independent Metropolis Hastings (PIMH) scheme (Andrieu, Doucet & Holenstein 2010). Both strategies are likely to perform well provided there isn’t high correlation between \(\theta\) and \(x\). In such scenarios a joint update can alleviate the problem.

It is natural in this case to consider a proposal of the form \(q_1(\theta^*|\theta)q_2(x^*|\theta^*, y)\) where \(\theta\) is the current value of the chain. The particle marginal Metropolis Hastings (PMMH) algorithm (Andrieu et al. 2010) side steps the issue of building an efficient proposal density \(q_2(x^*|\theta^*, y)\) by using a particle filter targeting the intractable density associated with \(\{X_n\}\). It therefore seems appealing to use MMALA or RMHMC in the construction of \(q_1(\theta^*|\theta)\). However, in order to construct the metric tensor \(G(\theta)\), we require a closed form expression for the observed data likelihood \(p(y|\theta)\) which will typically be unavailable. In some scenarios, a closed form approximation of \(p(y|\theta)\) may be used to compute \(G(\theta)\). Alternatively, the metric tensor could be evaluated numerically, but this may be computationally prohibitive.

References


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