Arriving at the Magnetic Induction Equation

Using Maxwell's equations (1) and (3) given previously, we form

$$\begin{aligned}\frac{\partial \mathbf{B}}{\partial t} = -\nabla \times \mathbf{E} &= -\nabla \times [\frac{1}{\sigma}\mathbf{J} - (\mathbf{v} \times \mathbf{B})] \\ &= -\nabla \times [\frac{1}{\sigma\mu_0}(\nabla \times \mathbf{B}) - (\mathbf{v} \times \mathbf{B})]\end{aligned}$$

Now we introduce $\eta = \frac{1}{\mu_0\sigma}$ = **the Magnetic Diffusivity**, where μ_0= Permeability and σ= Electrical Conductivity.

Now, $\nabla \times \nabla \times \mathbf{B} = \nabla(\nabla \cdot \mathbf{B}) - \nabla^{\mathbf{2}}\mathbf{B}$ but we know from our previous equation (4) that $\nabla \cdot \mathbf{B} = \mathbf{0}$ (a result which indicates the absence of monopoles). Thus we are left with $\nabla \times \nabla \times \mathbf{B} = -\nabla^{\mathbf{2}}\mathbf{B}$

Therefore, altogether we have

$$\frac{\partial \mathbf{B}}{\partial t} = \eta\nabla^2\mathbf{B} + \nabla \times (\mathbf{v} \times \mathbf{B}) \tag{1}$$

This is called the **Magnetic Induction Equation**.