Dark Solitons, Reconnection Processes and Acoustic Emission in atomic Bose – Einstein Condensates

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Motivation:

- *To study:*

4. The generation of the ***dark solitary wave*** by the ***phase imprinting method***.

6. ***Reconnection*** processes and the concurrent ***acoustic emission***.

8. *Turbulence.*

Outline

- ***Bose – Einstein Condensate***
- ***Gross - Pitaevskii Equation / Nonlinear Schrödinger Equation***
- ***Harmonic trapping potential***
- ***Numerical Method***
- ***Nonlinearity***
- ***A dark solitary wave in a harmonically confined condensate***
- ***Reconnection processes / acoustic emission***
- ***Sound energy***
- ***Turbulence***
- ***Conclusions / Ideas for further work***

Bose – Einstein Condensates (BECs)
offer a possibility of studying nonlinear effects

- T = 0 K
- A pure ***BEC*** of *N* atoms is ***trapped*** *in* ***a stationary potential.***
- Atomic *BEC*s have ***weak interaction*** because they are *dilute gases*.
- The condensate *rotates* at ***angular velocity Ω*** along the ***z*** axis.
- ***Vortex nucleation*** occurs via a *dynamical instability* of the condensate.

The Gross-Pitaevskii equation

$$(i-\gamma)\ \frac{\partial\Psi}{\partial t}=\left[-\frac{^2}{2m}\nabla^2+V_{tr}+g|\Psi|^2-\mu-\Omega L_z\right]\Psi$$

GPE = Gross-Pitaevskii Equation, NLSE = Nonlinear Schrödinger Equation

- The *GPE* governs the time evolution of the (macroscopic) ***complex wave function Ψ(r,t)***
- *Boundary condition:* ***Ψ(x,y,z) = 0***
- The wave function is *normalized*:

$$\int_D |\Psi|^2\, dV = N$$

- Ψ = wave function
- γ = ***dissipation***
-
- μ = chemical potential
-
- Ω = rotation frequency
-
- g = coupling constant

= reduced Planck constant

m = mass of an atom

$-\Omega L_Z$ = centrifugal term

$$(i-\gamma)\ \frac{\partial\Psi}{\partial t}=\left[-\frac{{}^{2}}{2m}\nabla^{2}+V_{tr}+g\left|\Psi\right|^{2}-\mu-\Omega L_{z}\right]\Psi$$

The NLSE,which accurately describes dilute BECs at zero temperature
Support soliton solutions for repulsive / attractive 2 – body interactions

- ☑ $-\frac{{}^{2}}{2m}\nabla^{2}\Psi$: kinetic energy contribution
- ☑ $V_{tr}\Psi$: external confinement of the system
- ☑ $\mu\,\Psi$: chemical potential
- ☑ $g\left|\Psi\right|^{2}\Psi$: strength and form of the nonlinearity
- ☑ $\Psi=\sqrt{n}\,e^{i\,S}$ The wave function is expressed in terms of a ***density*** and a ***phase*** by *Madelung´s transformation*

Basic equations

- **Kinetic energy:** $$E_{kin} = \int d^3x \frac{{}^2}{2m} \left(\sqrt{\rho(x)}\, v(x) \right)^2$$

- **Internal energy :** $$E_{\text{int}} = \int d^3x \, g \left(\rho(x) \right)^2$$

- **Quantum energy:** $$E_q = \int d^3x \frac{{}^2}{2m} \left(\nabla \sqrt{\rho(x)} \right)^2$$

- **Trapenergy:** $$E_{tr} = \int d^3x \, \rho(x) \, V_{tr}$$

- **Total energy:** $$E_{tot} = E_{kin} + E_{\text{int}} + E_q + E_{tr}$$

- **Kinetic energy:** $$E_{kin} = E_{sound} + E_{vortex}$$

The harmonic trapping potential

V_{tr} = harmonic trapping potential

- $$V_{tr} = \frac{1}{2}\left(\left(1+\varepsilon_x\right)x^2 + \left(1+\varepsilon_y\right)y^2 + \left(1+\lambda^2\right)z^2\right)$$

- Q***uantized vortices*** are formed by *rotating* an ***asymmetric trapping*** potential.

- $\varepsilon_x = 0.03, \quad \varepsilon_y = 0.09$, describe small *deviations* of the *trap* from the *axi-symmetry.*

- $\lambda = 5.0$, describes the *shape* of the *harmonic trapping potential*.

The trapping potential

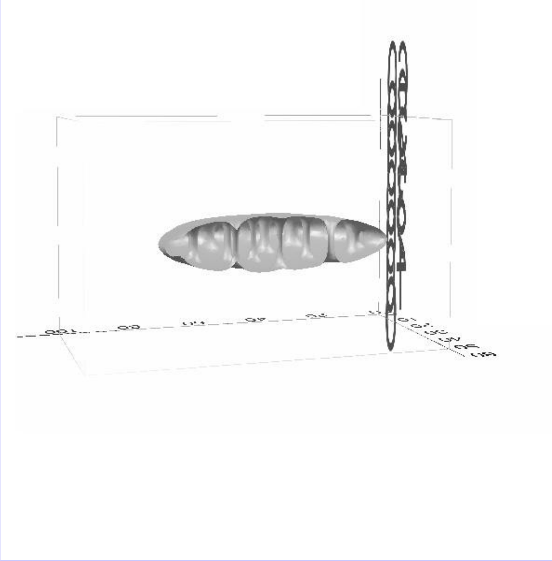
$$V_{tr} = \frac{1}{2}\left(\left(1+\varepsilon_x\right)x^2 + \left(1+\varepsilon_y\right)y^2 + \left(1+\lambda^2\right)z^2\right)$$

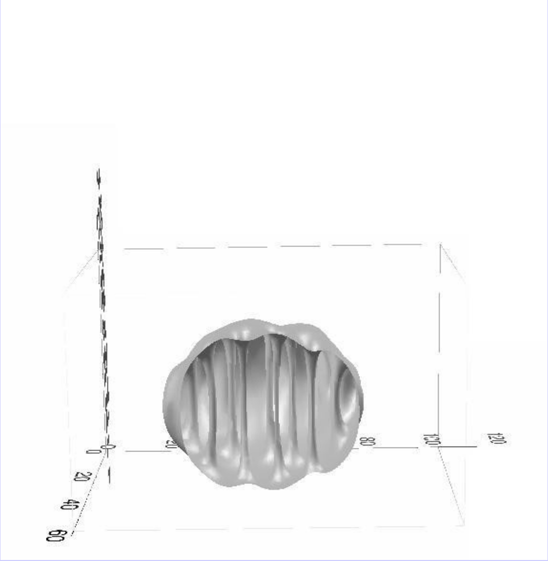
$$\varepsilon_x \approx \varepsilon_y, \quad \Gamma = \frac{\lambda}{\varepsilon}, \Gamma = \textit{the aspect ratio}$$

$$\textit{If}: \quad \varepsilon_x = \varepsilon_y = \lambda = 0 \quad \Rightarrow \quad \textit{Spherical} \quad \textit{trap}$$

$$\textit{If}: \quad \lambda > \varepsilon \quad \Rightarrow \quad \textit{Pancake} \quad \textit{trap}$$

$$\textit{If}: \quad \lambda < \varepsilon \quad \Rightarrow \quad \textit{Cigar} \quad \textit{trap}$$





Numerical Method:

- The solution to the *GPE* is approximated by solving it *numerically* in ***2D/3D.***
  - First in *imaginary time* *steady state*

- The *numerical calculations* are performed using the ***Semi-Implicit (Crank-Nicolson) Method.***
  - Using *Approximate Factorization.*

- *Numerical Domain:*
  - *Box: [-10/15:10/15]x [-10/15:10/15]x [-5/10:5/10]*
  - Mesh points 100 x 100 x 50/100

- *Initial Condition**: Thomas – Fermi Approximation***

$$\Phi(x, y, z) = \sqrt{\frac{\mu - V}{C}}$$

  - Without $\frac{\delta}{\delta t}$ *and* Δ in the GPE.

$$C = \frac{4\pi Na}{L} = const.$$

a = scattering length
N = total number of condensate atoms
L = size of the system along z -axis

Nonlinearity

From:

1. Effectively ***repulsive*** atomic interactions.

5. The ***scattering*** properties between the atoms of the condensate

Support:

9. The appearance of ***solitary waves***, which propagate without dispersion

The scattering amplitude:

$$g = \frac{4\pi\ {}^{2}a}{m}$$, m = the mass of an atom

a = s-wave ***scattering length*** for binary collisions between atoms

a > 0, effective ***nonlinearity*** is ***repulsive*** ***dark soliton***

a < 0, effective ***nonlinearity*** is ***attractive*** ***bright soliton***

Dark soliton

- Absence of the external confinement ***1D objects***
- Propagate ***without spreading*** in a nonlinear medium
- ***The wave function***: $\Psi(x,t) = e^{-i\mu t}(\lambda \tanh[\lambda(\frac{x-vt}{\xi})] + i\frac{v}{c})$
- (x-vt) = the position, $\xi = \frac{\;}{\sqrt{\mu m}}$, healing length
- v = the soliton speed,
- $\lambda = \sqrt{1-\left(\frac{v}{c}\right)^2}$
- $c = \sqrt{\frac{\mu}{m}}$, c = speed of sound, μ = chemical potential,

Instability of dark solitons, solitary waves and Decay mechanism

- Embedded in ***2D /3D geometry*** ***additional energy*** contributions in the transverse directions.

- This leads to the dominant ***decay mechanism.***

- ***Instabilities*** ***soliton decay accompanied by emission of sound waves*** in atomic condensate ***density waves.***

- ***Solitons*** are ***not*** the ***lowest energy state*** in ***2D / 3D*** it tends to ***decay*** into more ***stable / lower energy*** structures, namely ***vortex /antivortex (2D), vortex rings (3D).***

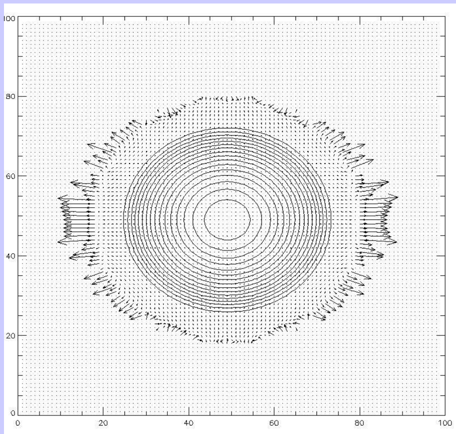
Study of the successive dynamics of the wave functions

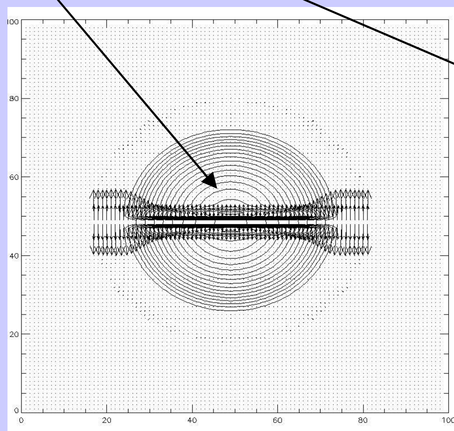
- Monitoring the ***evolution of the density and phase profile.***
- ***Dark soliton*** in ***matter waves*** is characterized by a ***local density minimum*** and a ***sharp phase gradient*** of the wave function at the position of the minimum.
- At t = 200
- Case I: $\Psi' = \Psi \,, y < 0$; $\Psi' = \Psi e^{i\pi} \,, y > 0$
- Case II: $\Psi' = \Psi \,, y < 0$; $\Psi' = \Psi e^{i\pi} \,, y > 0$; $\Psi' = \Psi \,, x < 0$; $\Psi' = \Psi e^{i\pi} \,, x > 0$
- Case III: $\Psi' = \Psi \,, y < 0$; $\Psi' = cc\Psi, y > 0$

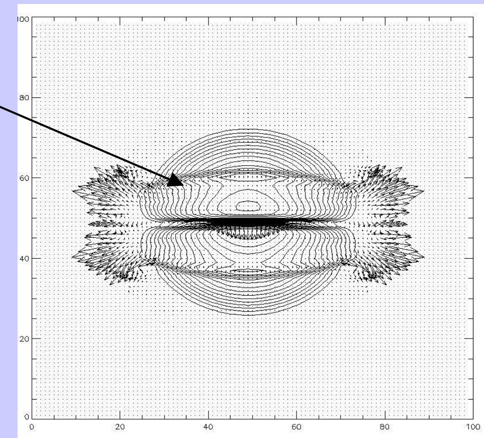
Snapshots of the density profile

The solitron was created from the phase change

The original sound wave

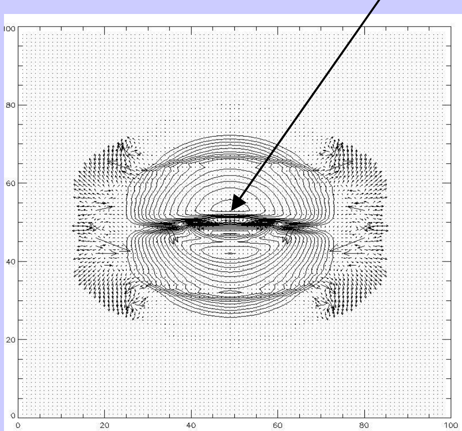


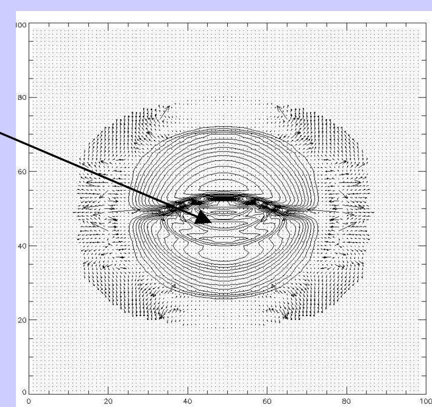




The soliton bends and decays into the vortex pair

Sound waves due to the decay of the soliton

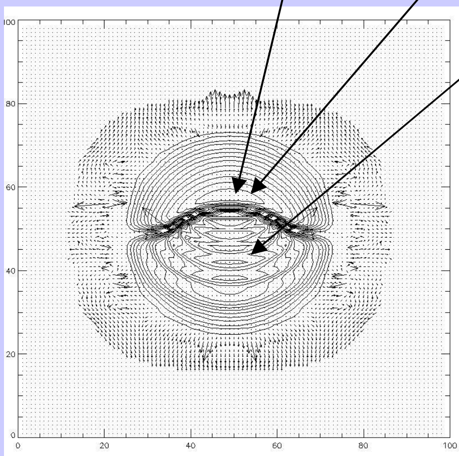




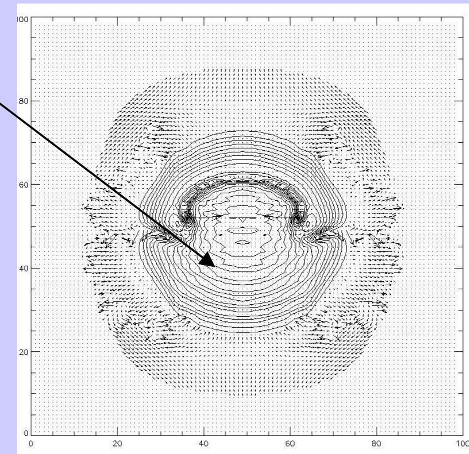
The soliton starts to move and bends because of the difference in the density

Higher velocity

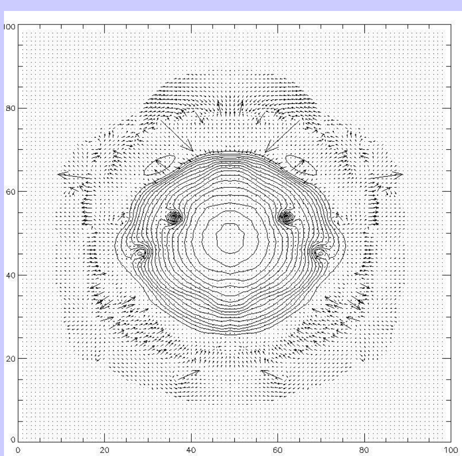
Sound waves due to the vortex pair production

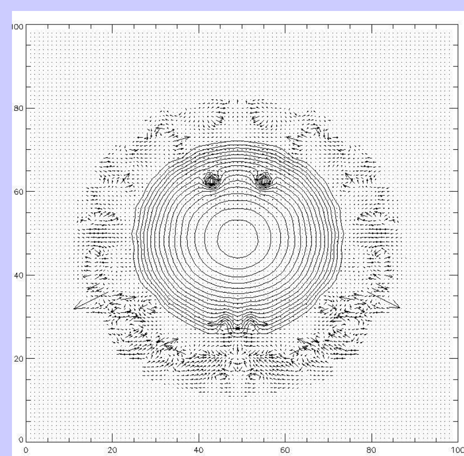


Five pairs of vortices



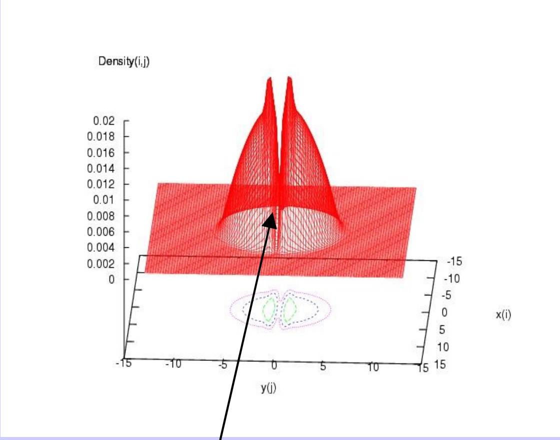
Three pairs disappear due to the interaction with the sound.
The survived two pairs .

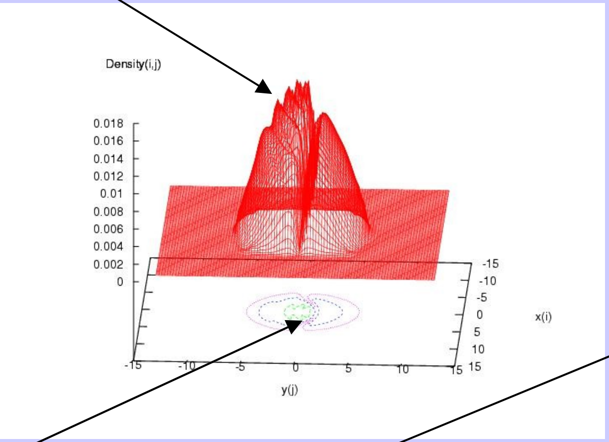


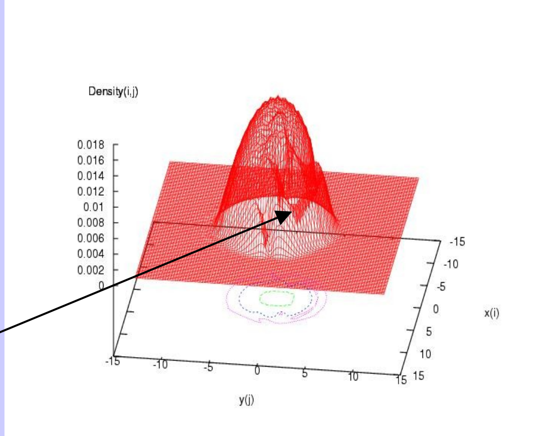


Snapshots of the density profile

Sound waves due to the decay of the soliton.

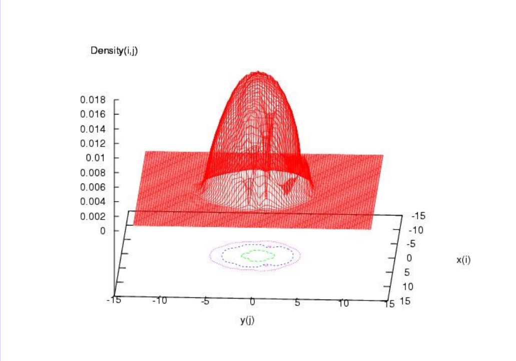


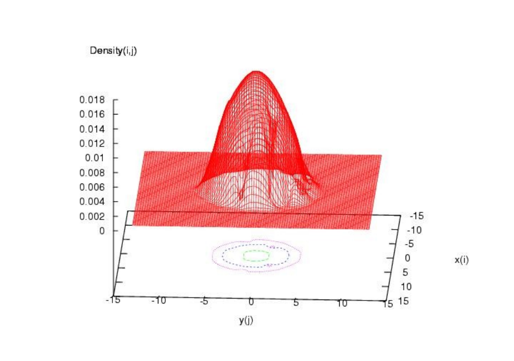


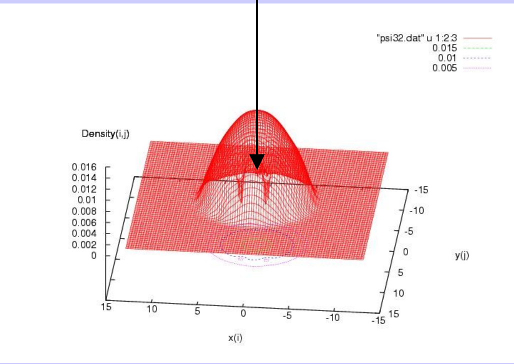


The soliton bends and starts to move. The soliton decays into the vortex pair.

The solitonic wave.

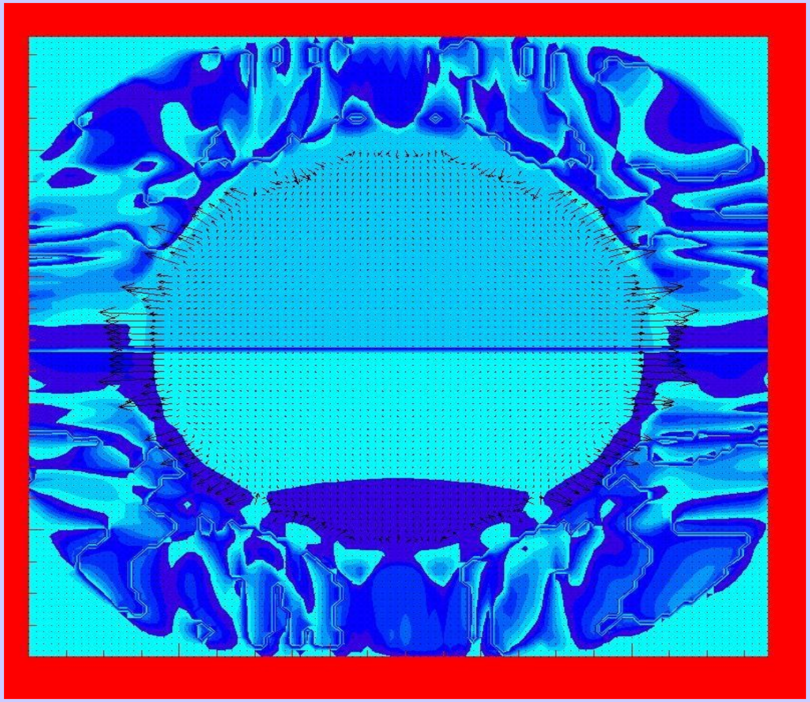


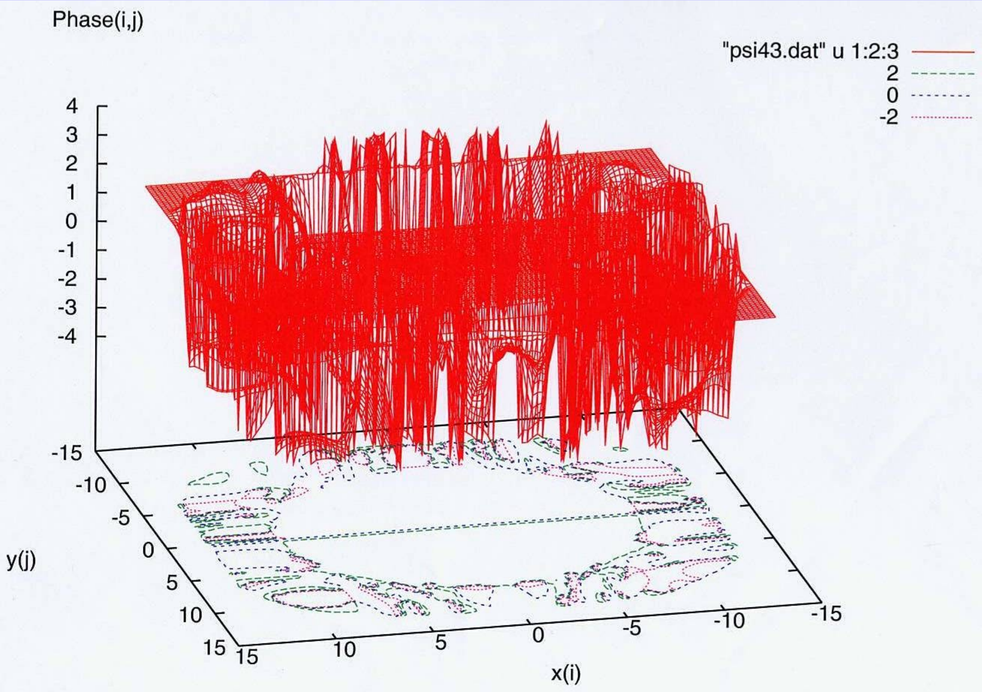




Phase,

$$\Psi' = \Psi \ , y < 0$$
$$\Psi' = \Psi e^{i\pi} \ , y > 0$$





A method to obtain the ground state of the system
- The sound energy -

- By propagation of the wave function in ***imaginary time.*** $\Delta t \rightarrow i \Delta t$

$$i \frac{\delta \Psi}{\delta t} = H\Psi$$

- An ***exponential decay of the wave function*** $\Psi(r, t+\Delta t) = \exp(-i\Delta t H)\Psi(r,t) + O(\Delta t^2)$

- During the imaginary time propagation, the wave function will tend towards the ***ground state*** of the system

Transfer of the energy from the vortices to the sound field

- Divide the energy into a component due to the sound field E_s and a component due to the vortice E_v
- $E_T = E_s + E_v$ (The ***kinetic energy*** consists of a *compressible part* due to ***sound waves*** and an ***incompressible part*** coming from ***quantized vortices***)
- To approximate E_v at a ***particular time***:

7. Take the ***real-time vortex distribution*** and impose this on a separate state with the same a) potential and
   b) number of particles

11. By ***propagating the GPE in imaginary time, the lowest energy state*** was obtained with this ***vortex distribution*** but *without sound.*

13. The energy of this state is E_v.

- The *sound energy* is defined as: $\mathbf{E_s = E - E_v}$

Study of the reconnection of quantized vortices and the concurrent acoustic emission. Transferring of the energy.

- The ***energy*** is generally ***transferred*** between*fer*: ***E_q, E_{sound}, and E_{vortex}.***

- Two ***anti-parallel vortices*** are close within a ***critical distance*** (twice of the healing length), they ***move with ~ c_{sound} ,*** emit ***the sound waves.***

- First stage: The ***quantum energy increases***

- When ***vortex antivortex disappear*** through annihilation, ***quantum energy*** is transferred into ***the kinetic energy*** with ***the acoustic emission***.

- The ***sound waves*** made by the reconnection ***expand***.

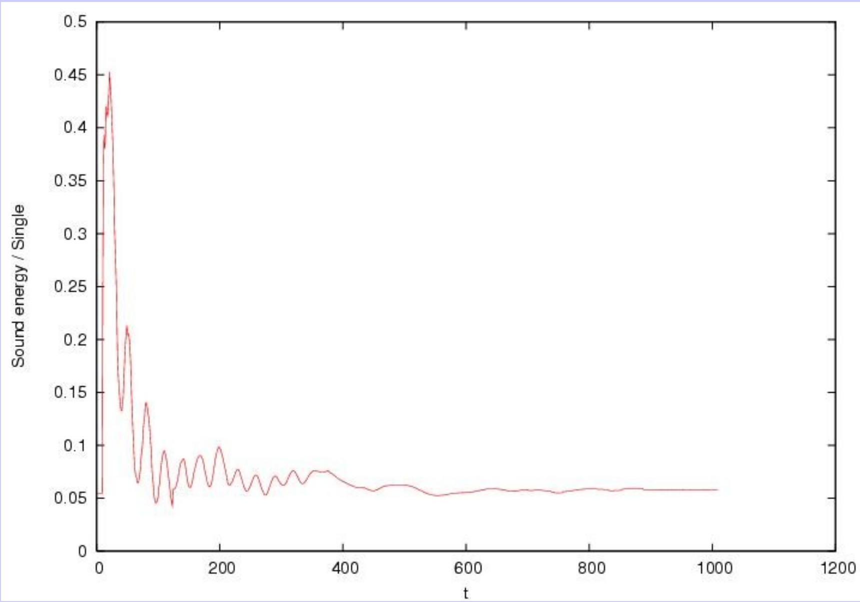
- The ***sound waves*** ***interact continuously*** with the ***vortex core*** even ***after*** the ***reconnection.***

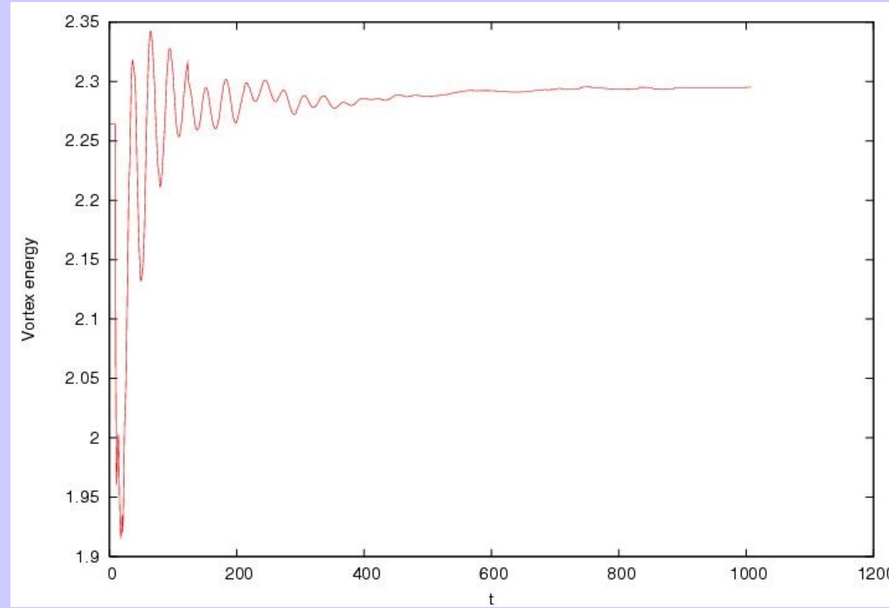
- The ***sound waves*** are ***absorbed*** by the ***vortex core***,which increases the quantum energy

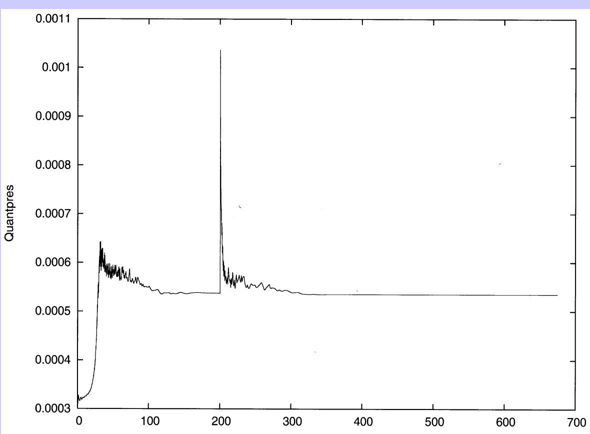
- The ***$\Delta\rho$ never vanishes*** even after the *core structure disappears* the ***energy components are mixed and oscillate.***

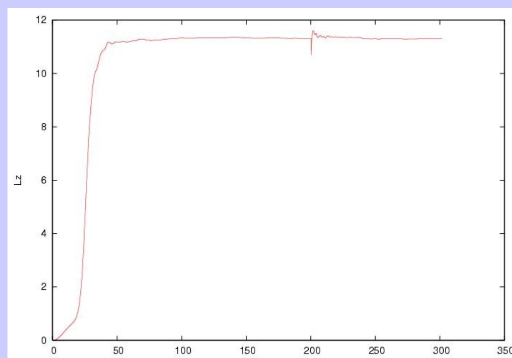
$$\Psi' = \Psi \, , y < 0$$

$$\Psi' = \Psi e^{i\pi} \, , y > 0$$

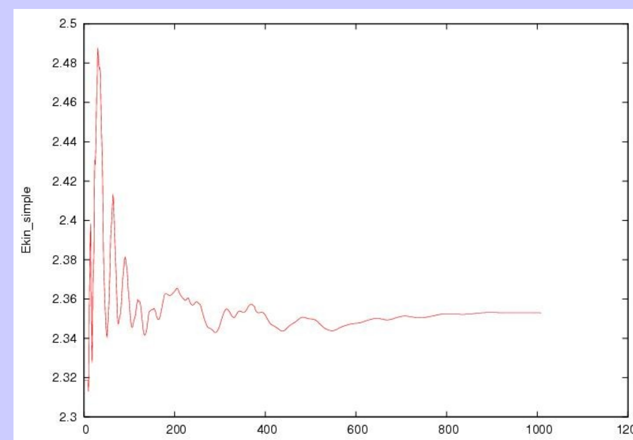








t = 0.15 sec

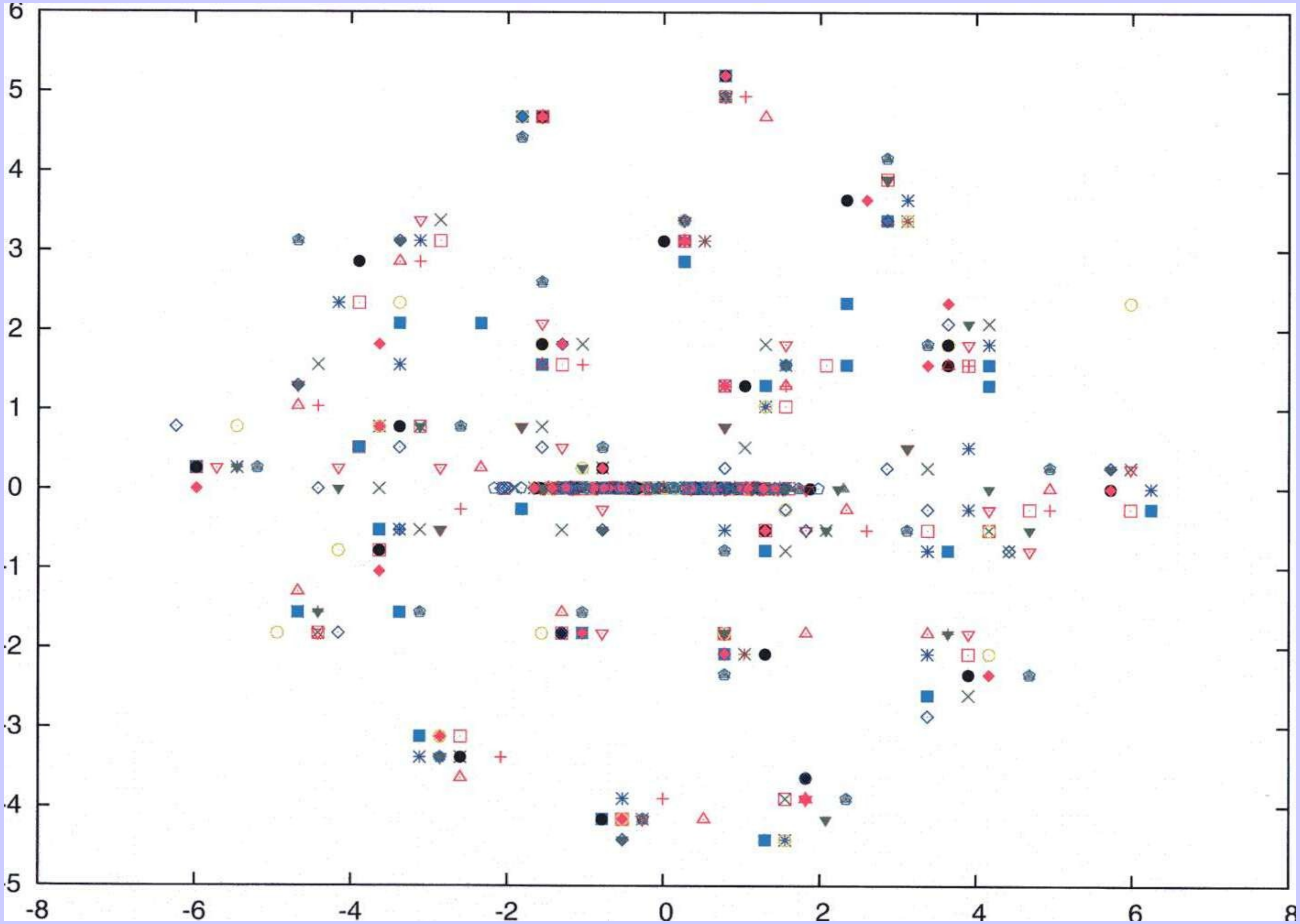


Dimensionless unit:

Time: $\omega_{\perp}^{-1}, \omega_{\perp} = 2\pi \, x \, 219 \, Hz$

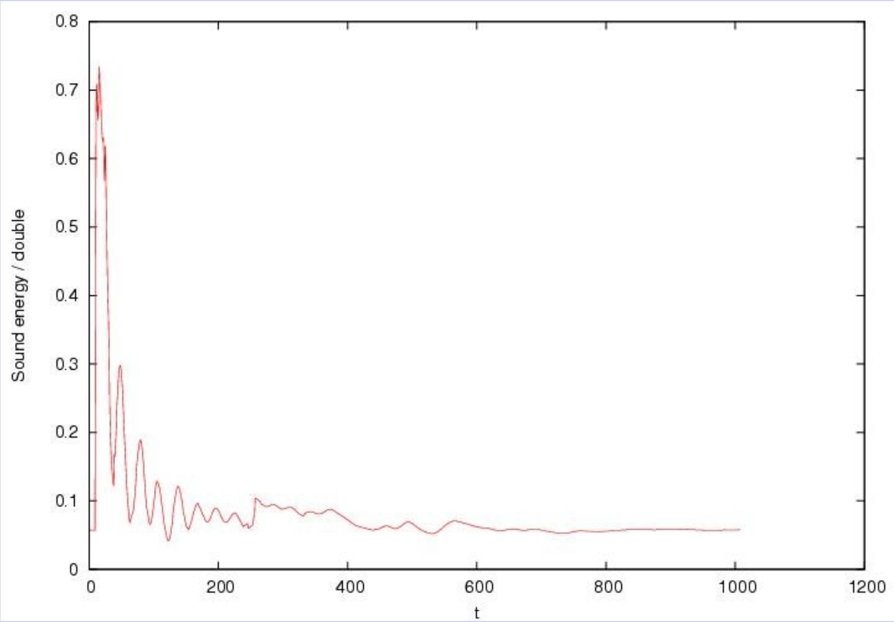
$$\Psi' = \Psi \ , y < 0$$
$$\Psi' = \Psi e^{i\pi} \ , y > 0$$

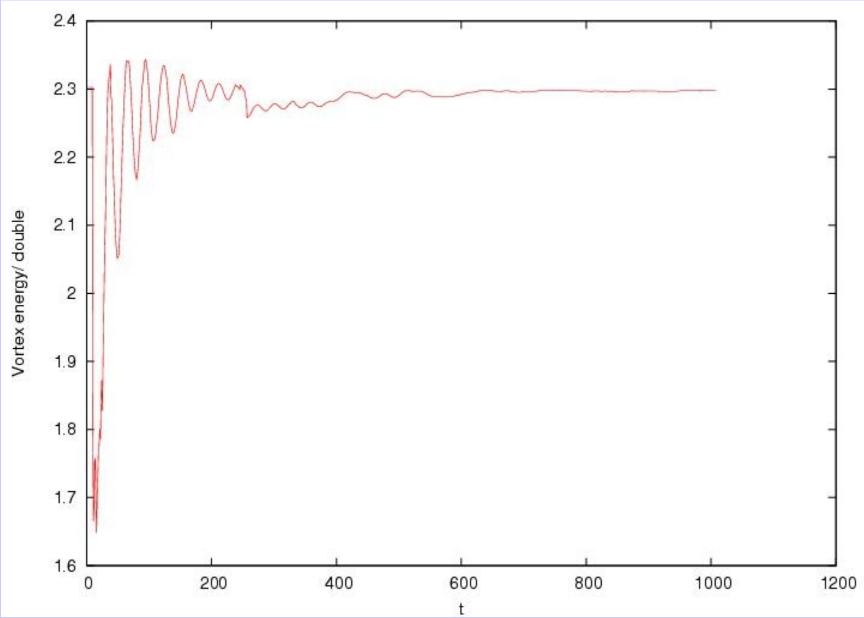
Nr of vortices = 20, Nr of vortex pair = 4

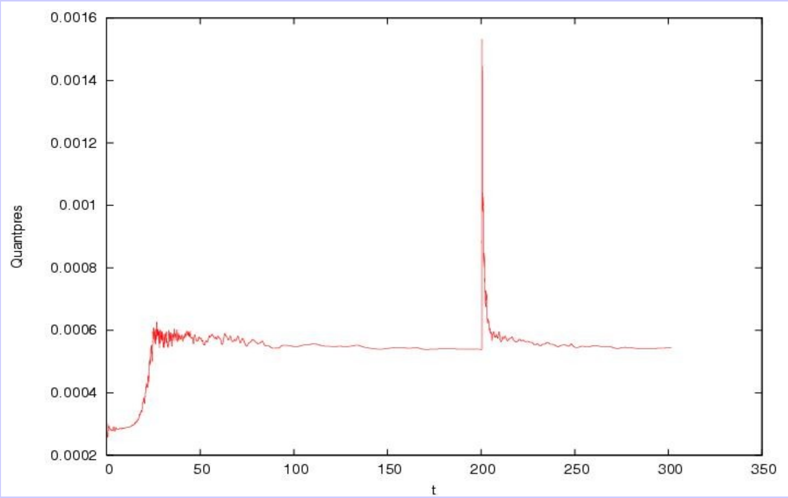


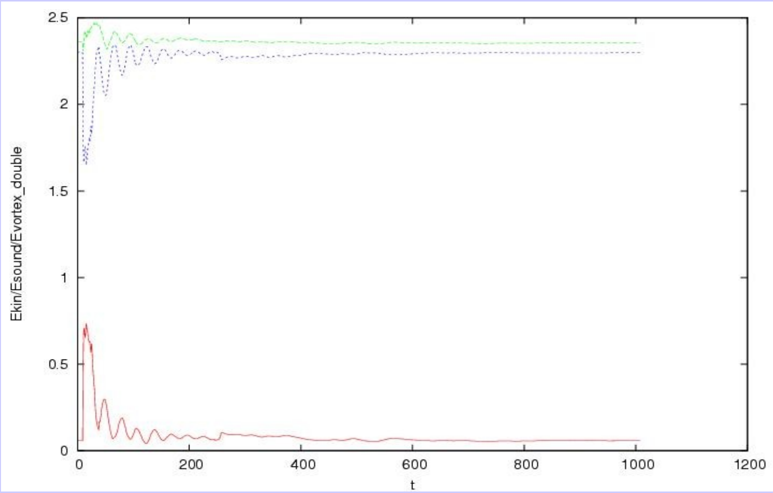
$$\Psi' = \Psi \, , y < 0$$
$$\Psi' = \Psi e^{i\pi} \, , y > 0$$

$$\Psi' = \Psi \, , x < 0$$
$$\Psi' = \Psi e^{i\pi} \, , x > 0$$







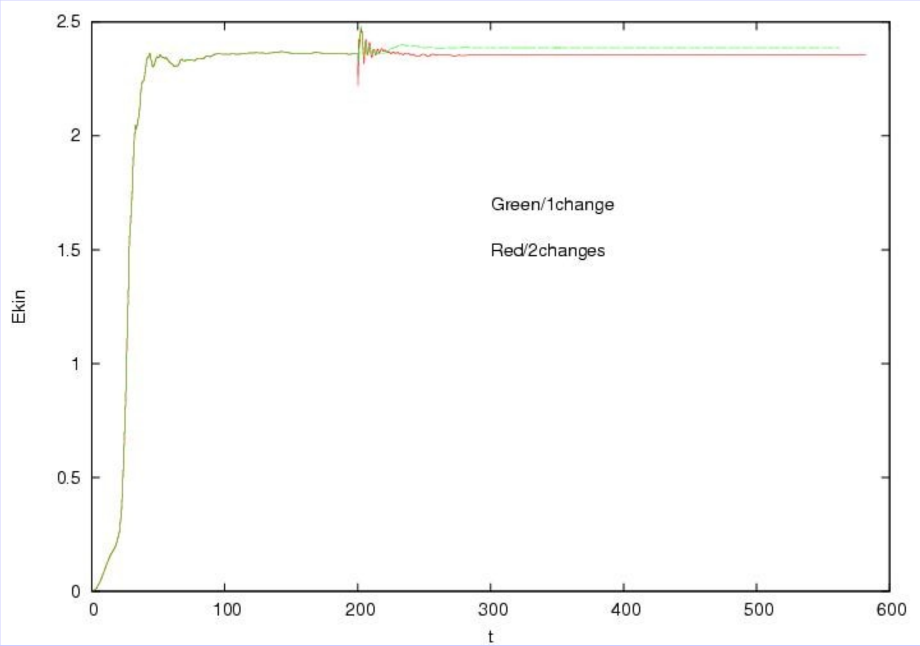


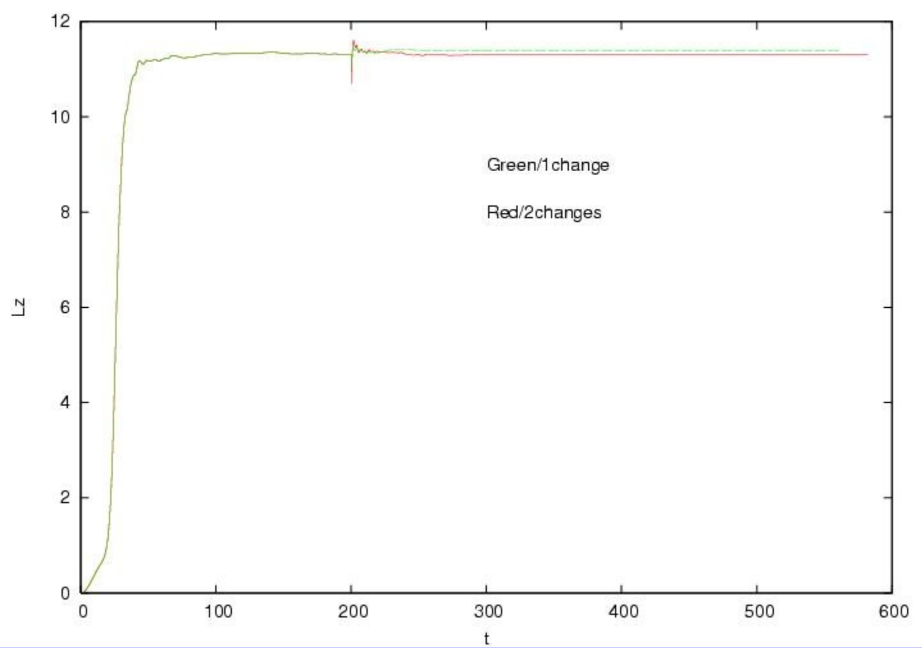
Lz and Ekin for 1 change:

$$\Psi' = \Psi \,, y < 0$$
$$\Psi' = \Psi e^{i\pi} \,, y > 0$$

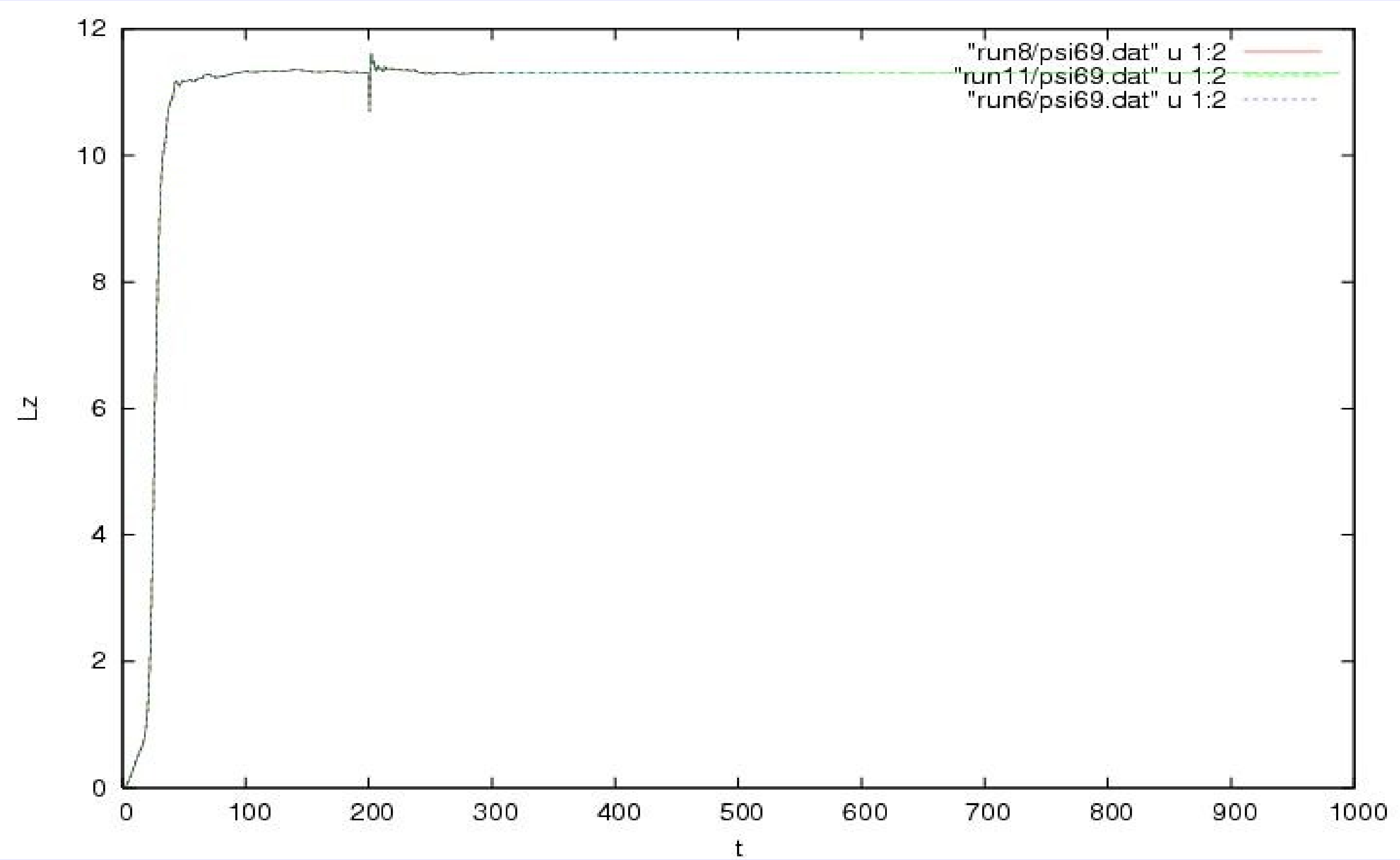
2 changes:

$$\Psi' = \Psi \,, y < 0 \qquad \Psi' = \Psi \,, x < 0$$
$$\Psi' = \Psi e^{i\pi} \,, y > 0 \qquad \Psi' = \Psi e^{i\pi} \,, x > 0$$





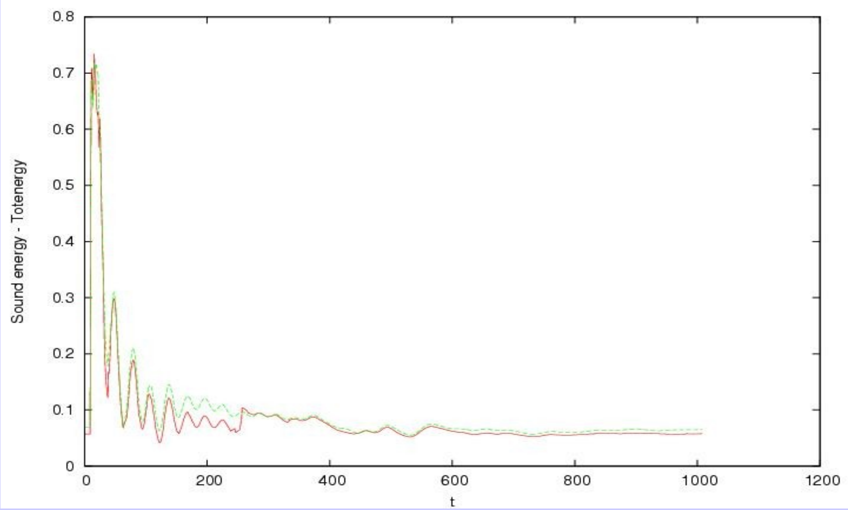
Lz for : run 6 / gamma = 0.0
run 11 / gamma = 1.0
run 8 / gamma = 0.03

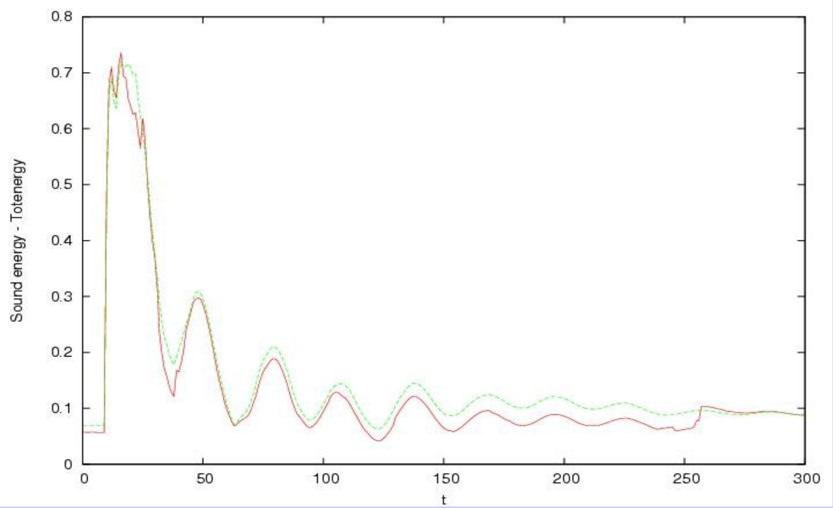


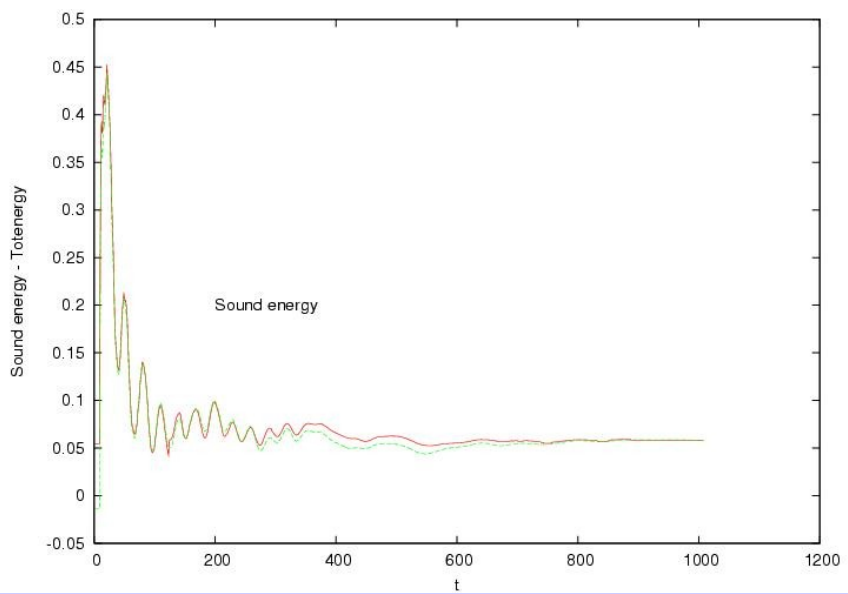
The sound energy in connection with the total energy

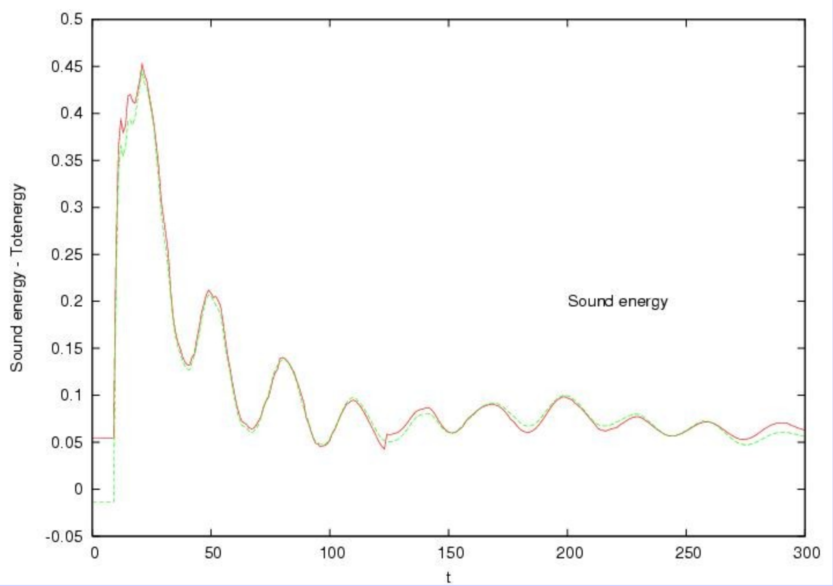
Due to the new level of energy by the discontinuity, the total energy changes.

After the relaxation the total energy recover the old value



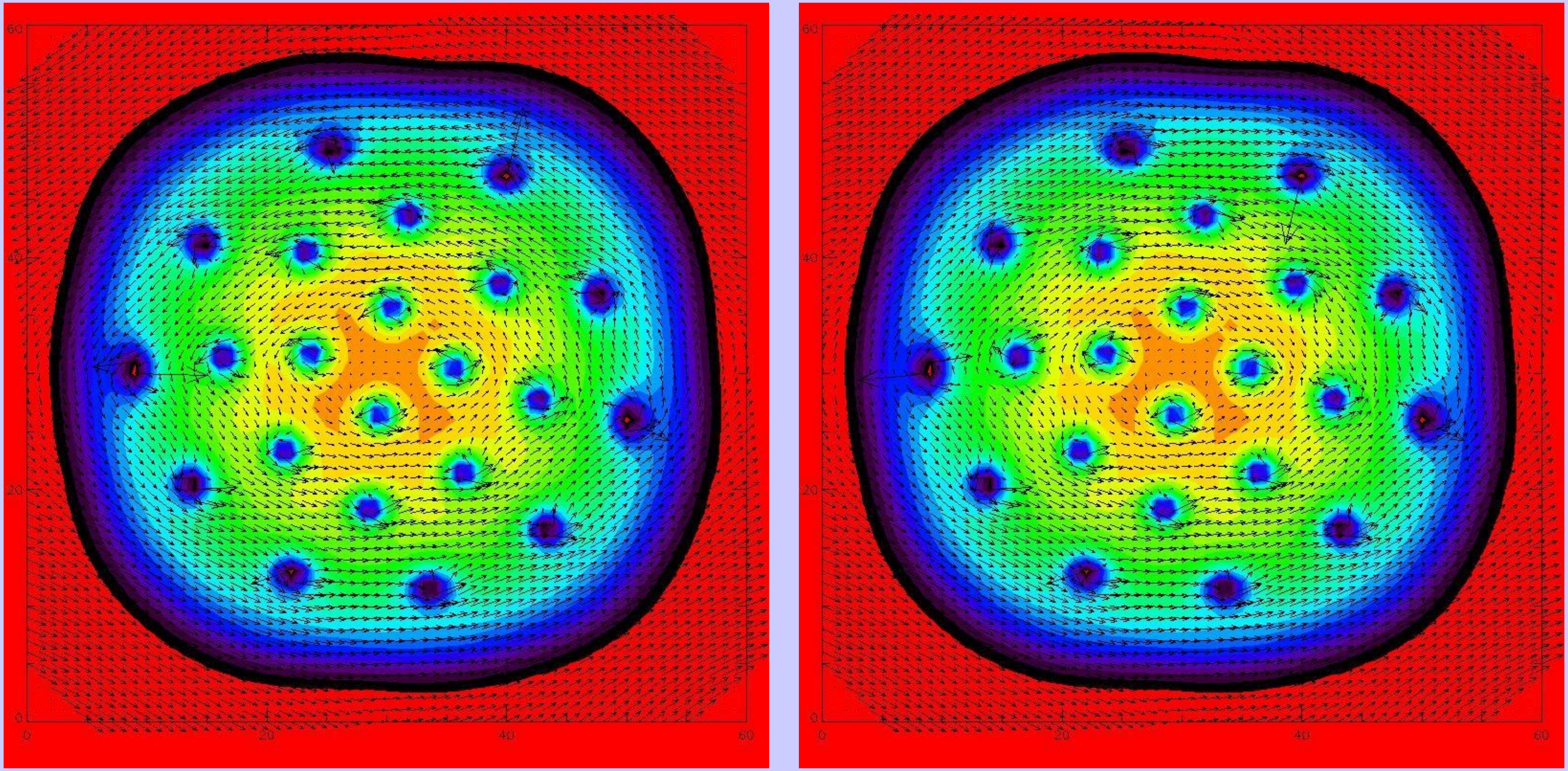




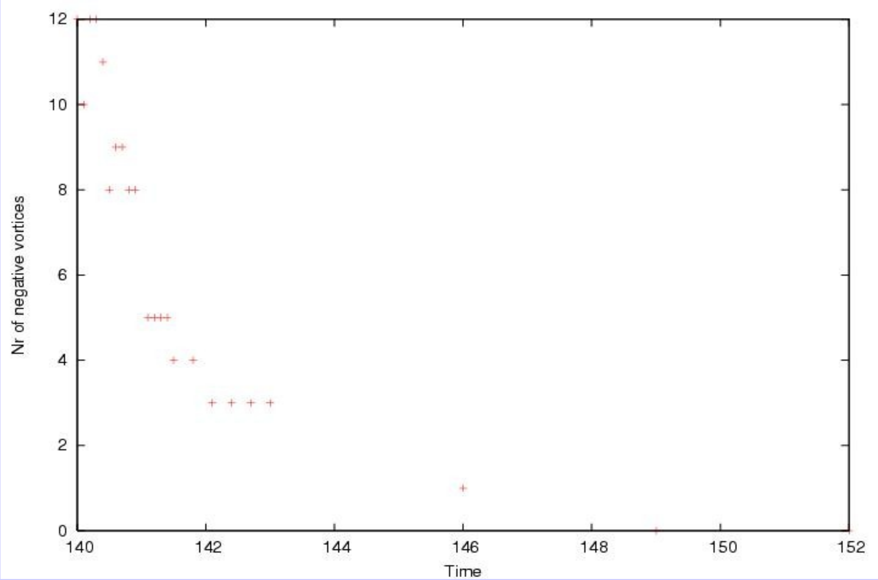


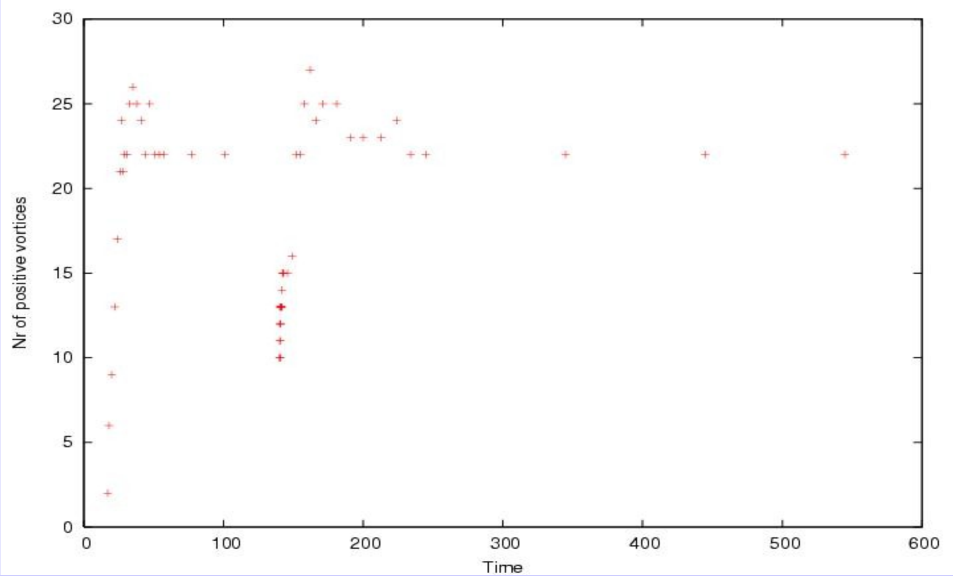
$$\Psi' = \Psi \,, y < 0$$

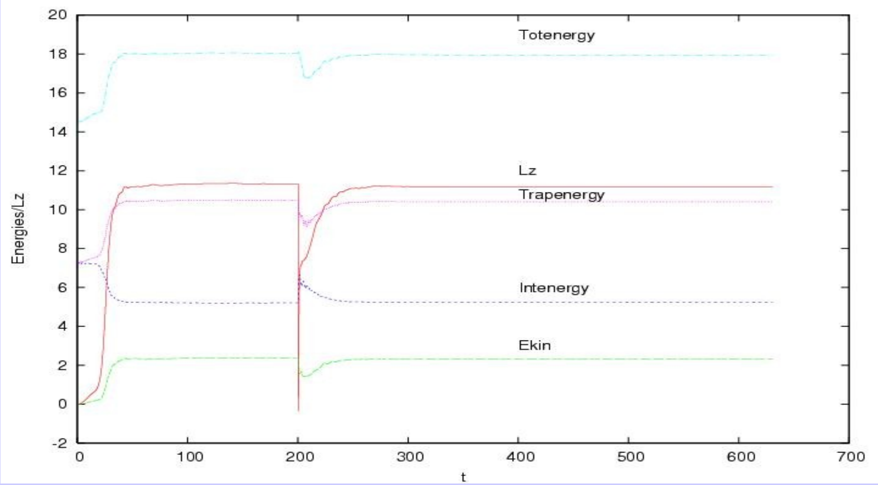
$$\Psi' = cc\Psi, y > 0$$

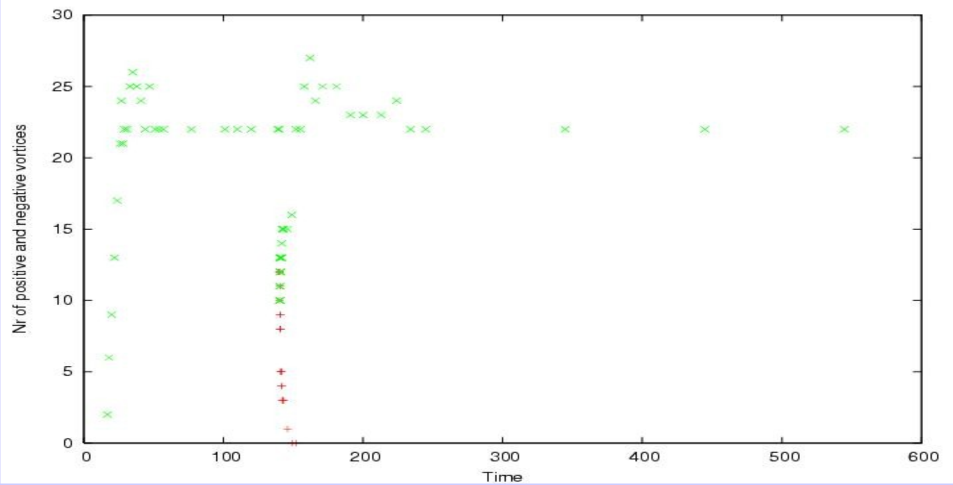


T = 140, Psi = Psi Complex conj

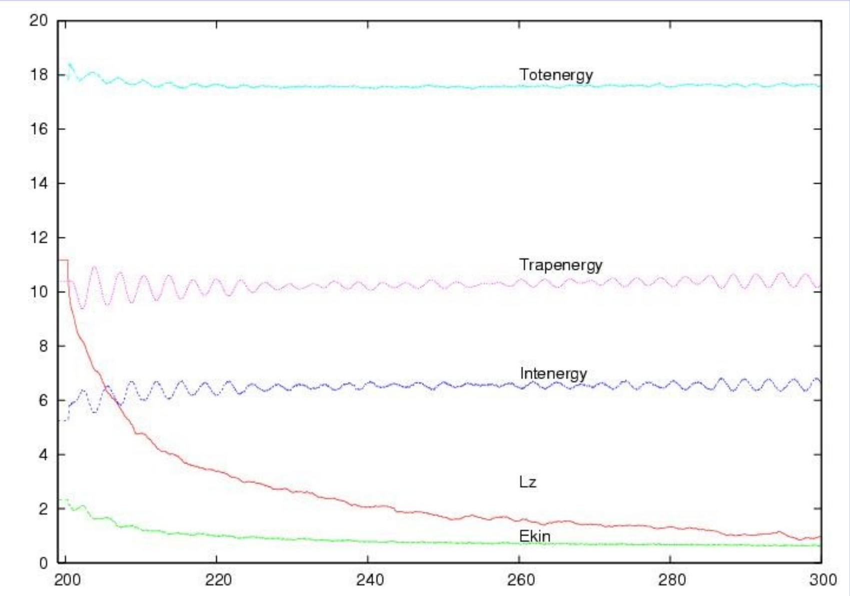




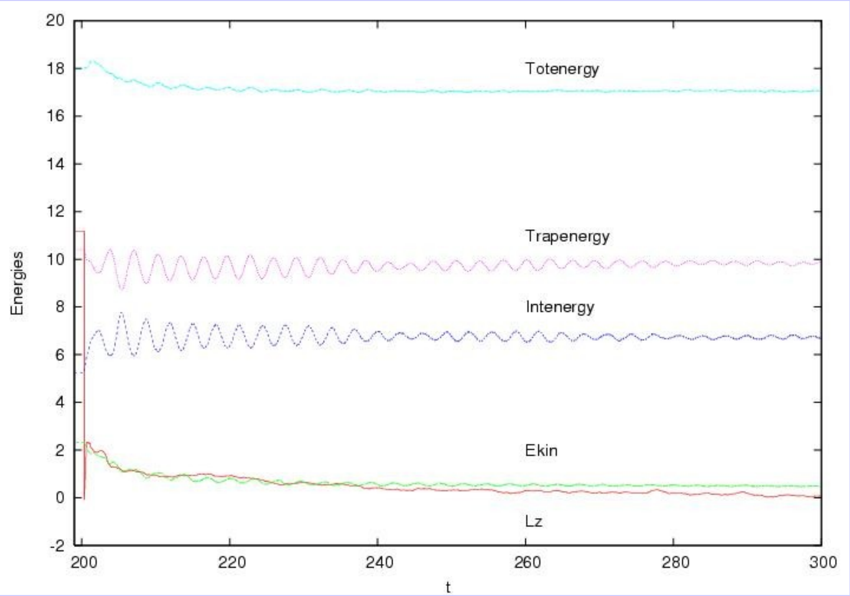




Turbulence



Totenergy
Trapenergy
Intenergy
Lz
Ekin



Energies
t
Totenergy
Trapenergy
Intenergy
Ekin
Lz

The turbulence

- ***No viscosity.***

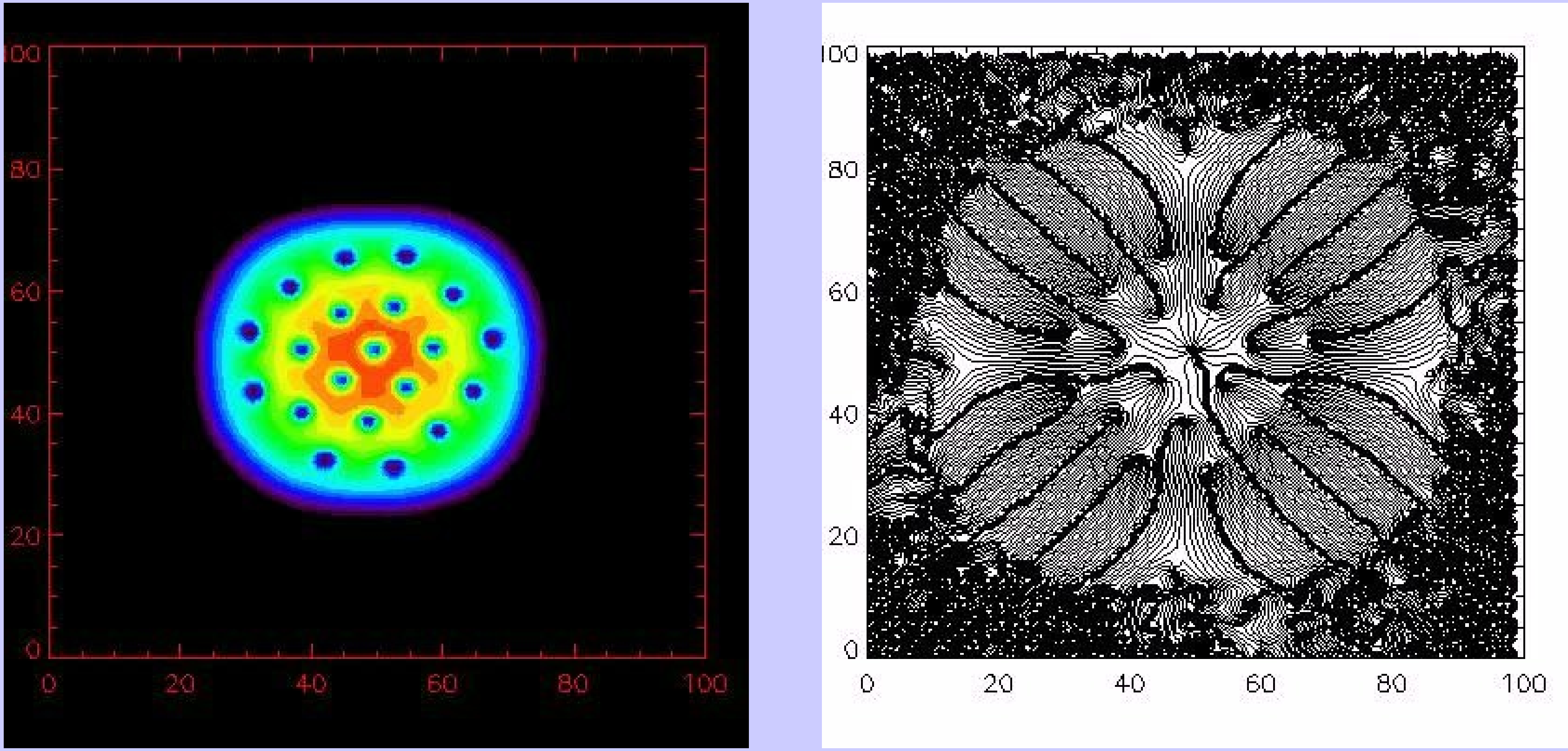
- In ***2D*** this could be considered as ***superfluid analogy*** of ***classical 2D turbulence.***

- The fluid is populated by ***N positive*** and ***negative vortices.***

- The ***turbulent state*** later under certain condition could relax by ***vortex-sound interactions.***

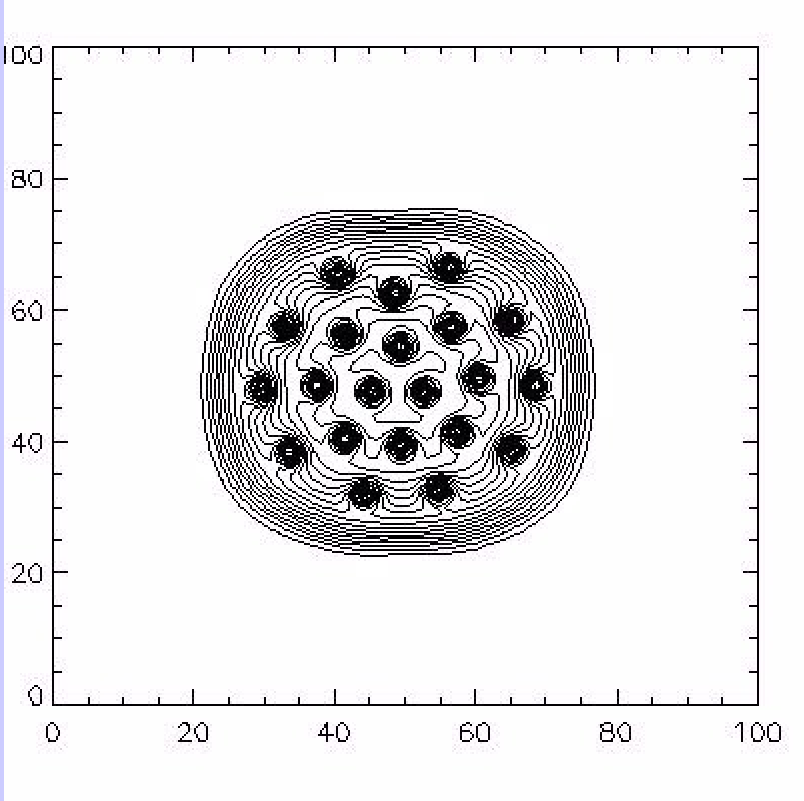
- ***Experiments*** show that ***superfluid turbulence decays even at T   0.***

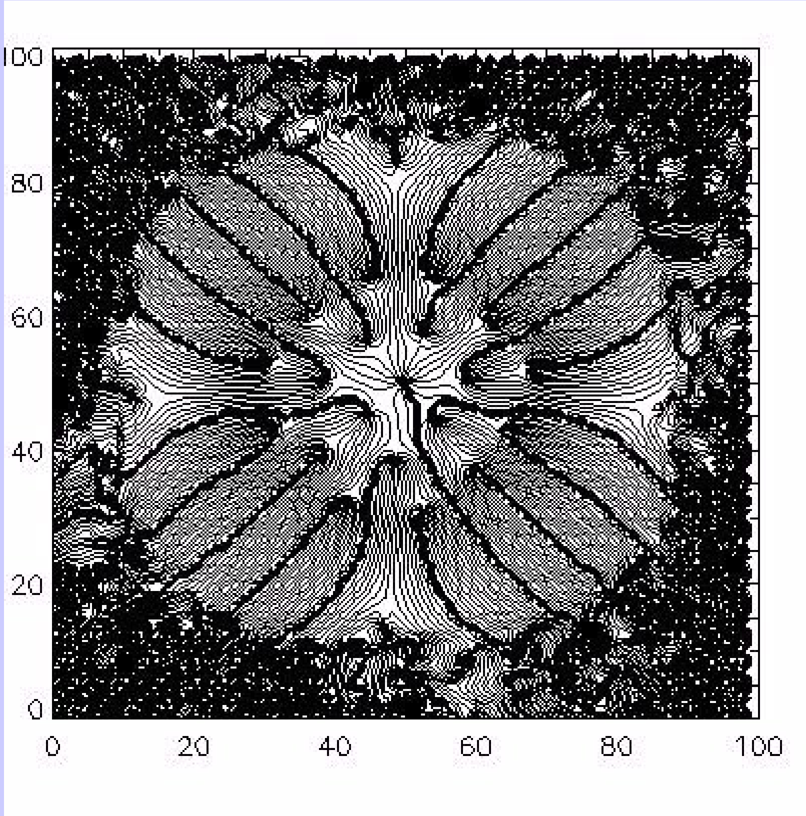
$$\Psi' = \Psi \, , y < 0$$
$$\Psi' = cc\Psi, y > 0$$



$$\Psi' = \Psi \;, y < 0$$
$$\Psi' = \Psi e^{i\pi} \;, y > 0$$

$$\Psi' = \Psi \;, x < 0$$
$$\Psi' = \Psi e^{i\pi} \;, x > 0$$





Conclusions:

- *By generating a discontinuity in the phase, the system try to smooth out this change and generate dark solitary and sound wave .*

- *The sound energy is a part of the total energy and has the biggest contribution to the change of the total energy comes from the sound energy.*

- *Two contributions to the sound energy. First, from the phase change and second from the interaction between vortex-antivortex.*

- *The compressible kinetic energy is directly concerned with the acoustic emission.*

Ideas for further work:

- *To improve the **Laboratory Frame** for the Bose-Einstein Condensate.*
- *To study the connection between **Bose-Einstein Condensates** and **Kelvin Waves.***
- I began to *study the* *TURBULENCE* *—effect both i2D and 3D.*
- ***It is interesting to find out how the number of vortices changes with time as well as what kind of vortex configuations appear as the turbulence decay.***
- ***Such problems we can understand examined the various contributions to the energy, as I have done.***