Vortex Motion in Trapped Bose-Einstein Condensates at Finite Temperature

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Introduction

Numerical methods: **Time dependent simulations of the dimensionless (2D) Gross- Pitaevskii equation was performed using the Crank-Nicholson algorithn**

γ : model of dissipation in atomic BEC [1]

ψ: wave function

μ : chemical potential

g : coupling constan

V_{tr}:trapping potential

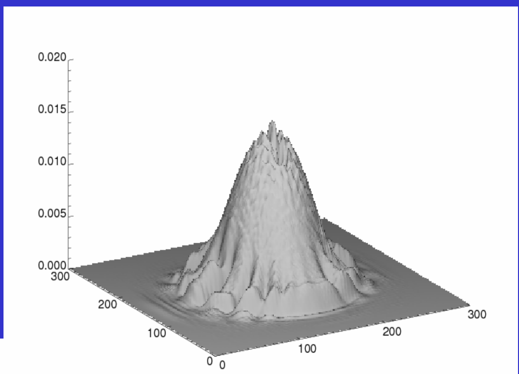
$$(i-\gamma)\hbar\frac{\partial\Psi}{\partial t}=\left[-\frac{\hbar^2}{2m}\nabla^2+V_{tr}+g|\Psi|^2-\mu\right]\Psi$$

$$V_{tr}(r)=\frac{1}{2}m\omega_\perp^2\left\{(1+\varepsilon_x)x^2+(1+\varepsilon_Y)y^2\right\}$$

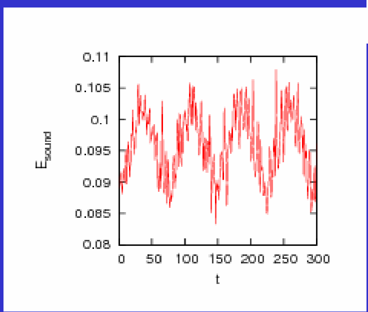
- We describe the ***dynamic of the condensate without (I)*** and ***with (II) dissipation, γ.***
- We want to ***model interaction*** with ***thermal cloud*** in a simple way with the help of ***modified GPE***

1) γ = 0, the energies are constant ***2) γ ≠ 0, the energies are not constant***

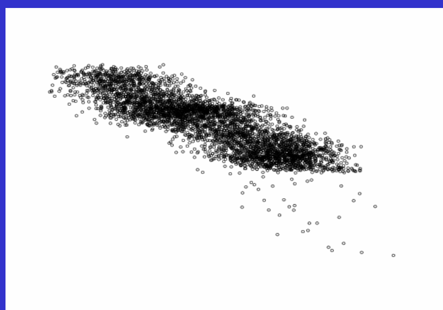
Case I. γ = 0



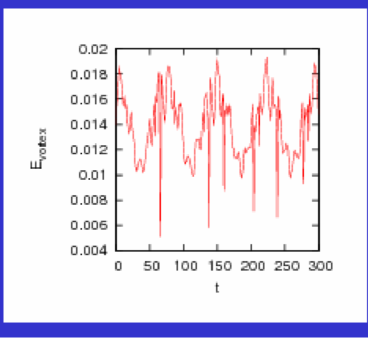
Sound waves on isosurface plot for a *vortex-anti vortex pair* for ***initial separation distance***, **d** = 2



Sound energy for a vortex – anti vortex pair for **d = 2.86**



Correlation between ***vortex energy*** and ***sound energy***. The ***correlation coefficient*** **cc = -0.844**



Vortex energy for a vortex – anti vortex pair for **d = 2.86**

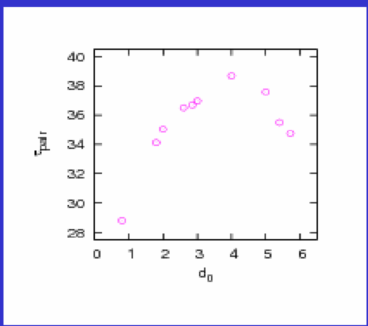
[1] *Tsubota et al, Phys.Rev. A65 023603-1 (2002)*

Case I:
No Dissipation: $\gamma = 0$, Vortex – Antivortex Pair

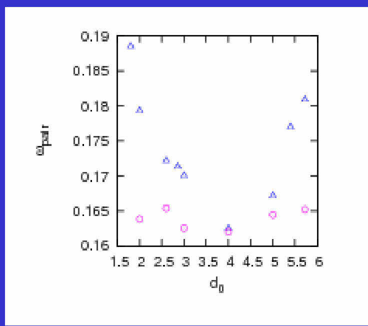
- With help of simulations, we describe the x- and y-coordinates of the vortices and the radius of their trajectory
- In our study, we use a single vortex and vortex-antivortex pairs
- For $\gamma = 0$, the vortex moves in a circular trajectory
- This motion is a result of the non-uniform density of the condensate

Vortex – antivortex pair for different ***initial separation distance d_0***

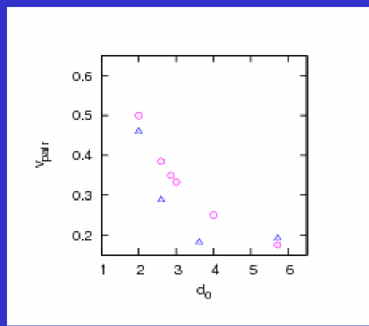
Period of motion



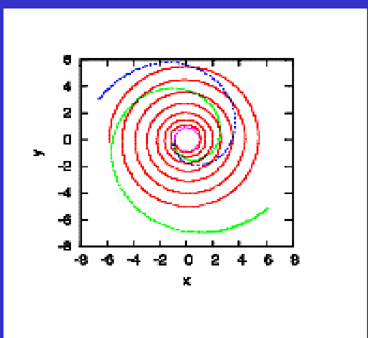
Frequency of motion



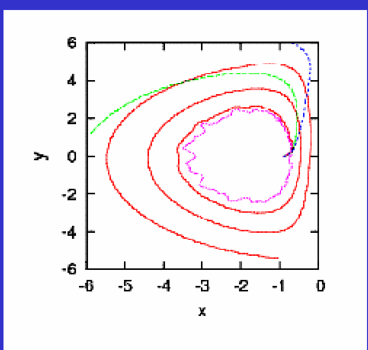
Translation speed



Case II:
With Dissipation γ, Vortex – Antivortex Pair (a Single Vortex)



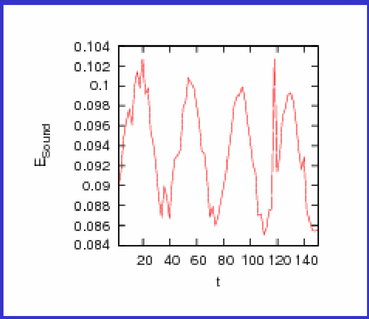
Trajectory for a ***single vortex*** for
$x = 0.9$ and
γ: 0; 0.01; 0.07 and 0.1



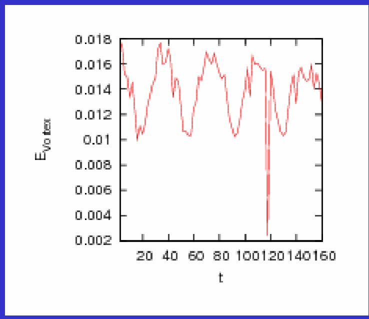
Trajectory of one of the vortices
from the vortex pair for ***$d = 1.8$*** and
γ: 0; 0.01; 0.07 and 0.1

Case II: Motion of vortex – antivortex pair with initial separation distance, d_0 = 2.86 and dissipation γ = 0.003

Sound energy

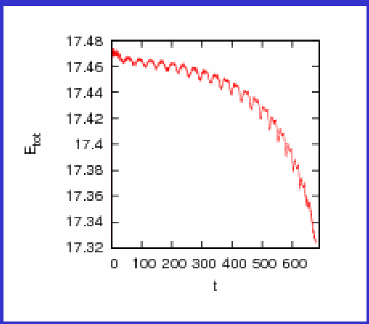


Vortex energy

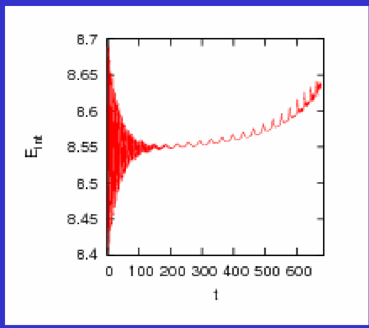


With ***dissipation***
the ***energies decrease***

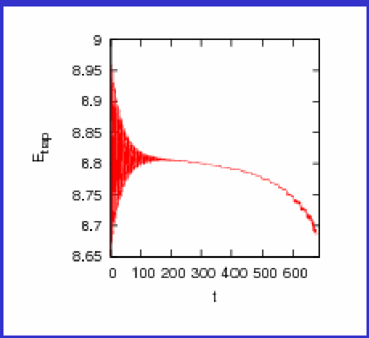
Total energy



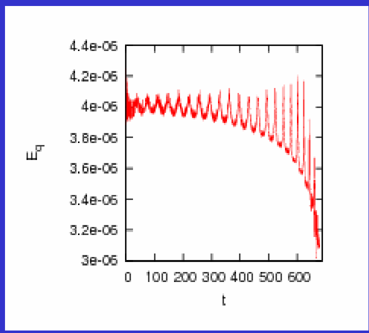
Internal energy



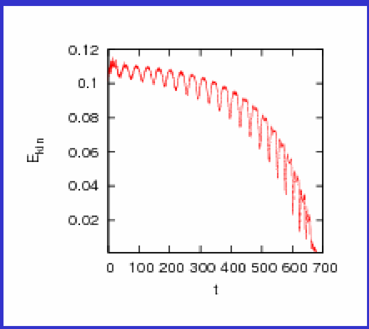
Kinetic energy



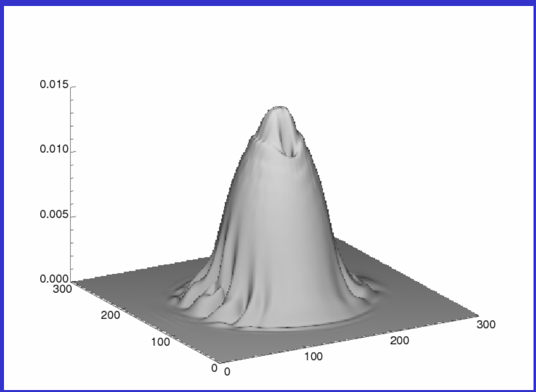
Quantum energy



Trap energy

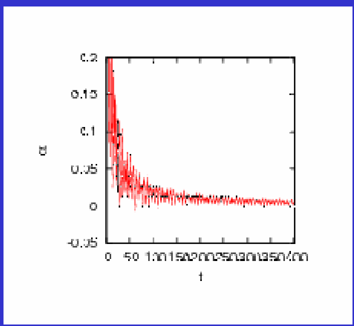


Sound waves on isosurface plot
for a *vortex-anti vortex pair* for ***initial separation distance***, **d** = **1.0** and **γ = 0.01**

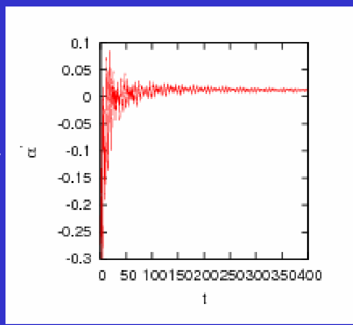


Connection between the ***dissipation*** γ and the ***friction coefficients*** α ***and*** α'

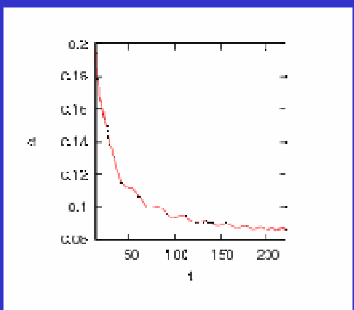
Friction coefficient α and α' for a ***single vortex*** with ***initial position*** $x_0 = 0.9$



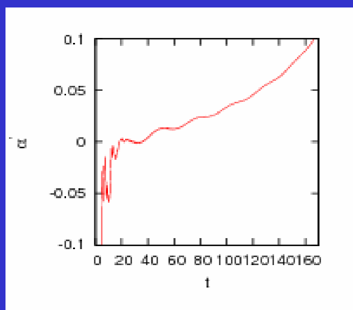
α as a function of ***time, t*** with ***no dissipation***



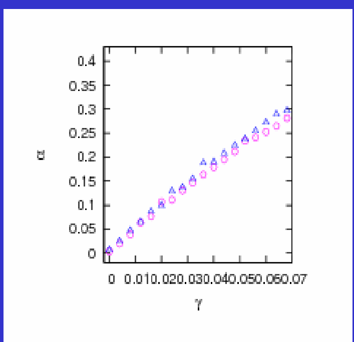
α' as a function of ***time, t*** with ***no dissipation***



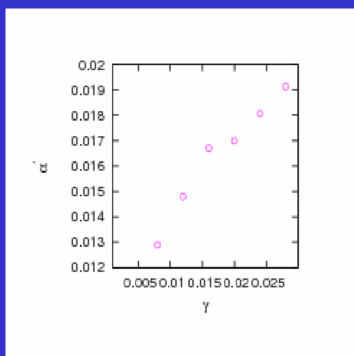
α as a function of ***time, t*** with ***dissipation*** $\gamma = 0.016$



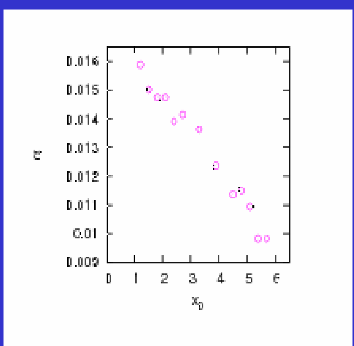
α' as a function of ***time, t*** with ***dissipation*** $\gamma = 0.016$



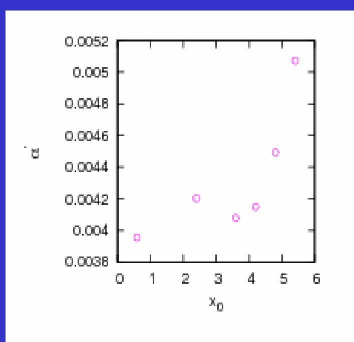
α as a function of ***dissipation*** γ



α' as a function of ***dissipation*** γ



α as a *function* of ***initial position*** x_0



α' as a *function* of ***initial position x0***

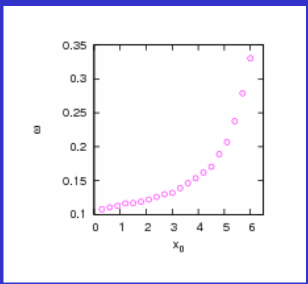
Connection between the dissipation γ and the friction coefficients α and α'

- We convert the dissipation γ, which haas its origin from a model of a thermal cloud into α and α´, which have their origin from the vortex dynamics´s friction coefficients
- We use the Schwarz´s equation [2], which in our case is:

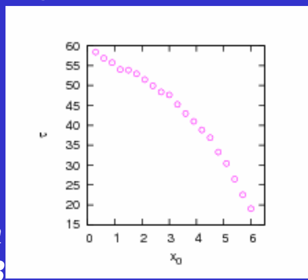
$$\frac{d\vec{r}}{dt} = \vec{v}_{si} - \alpha\, \vec{z} \times \vec{v}_{si} + \alpha' \vec{z} \times \left(\vec{z} \times \vec{v}_{si}\right)$$

The motion of a single vortex with initial position x_0 = 0.9

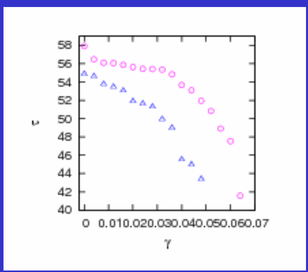
For the first circle like trajectory:



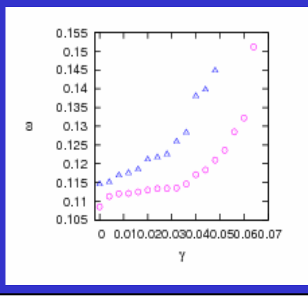
Period of motion, τ as a *function* of ***initial position x_0*** for ***γ = 0.003***



Frequency of motion, ω as a *function* of ***initial positions x_0*** for ***γ = 0.003***

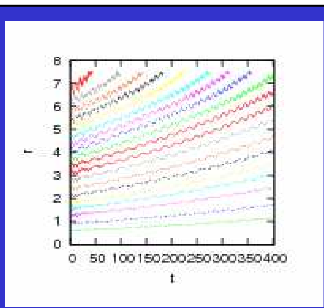


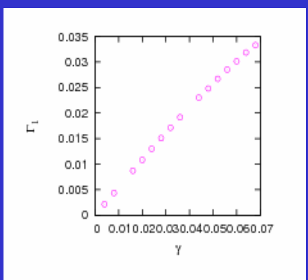
Period of motion, τ for *initial positions* ***x = 0.9*** (circle) and ***x = 1.45*** (triangle)as a *function* of ***γ***



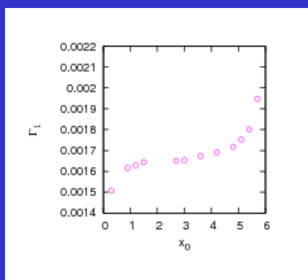
Frequency of motion ω for *initial positions* ***x = 0.9*** (circle) and ***x = 1.45*** (triangle)as a *function* of ***γ***

Radius of the trajectory as a *function* of ***time, t*** for ***different*** *initial positions* ***x_0*** from bottom to top: 0.6; 0.9; 1.2; 1.5; 1,8; 2,1; 2,4; 2,7; 3.0; 3.3; 3.6; 3.9; 4.2; 4.5; 4.8; 5.1; 5.4; 5.7 and 6.0





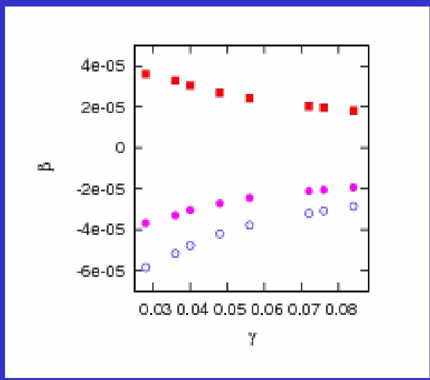
Exponential constant Γ_1 of the ***radius of vortex trajectory*** for a ***single vortex*** with ***initial position x = 0.9*** as a *function* of ***dissipation***



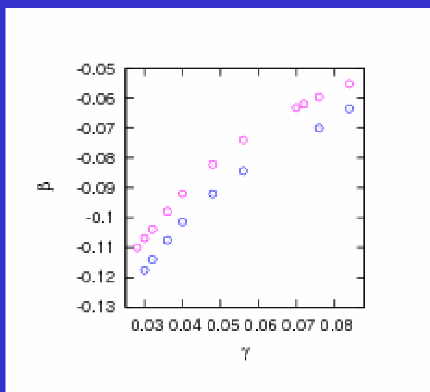
Exponential constant Γ_1 of the ***radius of vortex trajectory*** for ***γ = 0.003*** as a *function* of ***initial positions x_0***

[2] Scwarz. K. W. 1988, PRB, 38, 2398

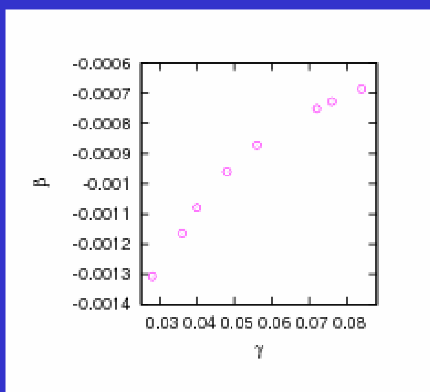
Motion of rndom vortex-antivortex pairs placed in the condensate with ***initial separation distance d_0 = 1.8***



The ***decay/increasing rate*** of the ***total energy*** (purple circle) ***trap energy*** (blue circle) and ***internal energy*** (red square) as a *function* of ***dissipation*** γ



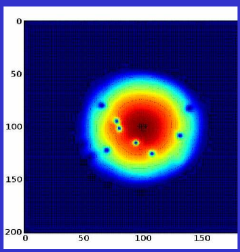
The ***increasing rate*** of the ***kinetic energy*** (purple circle) and the ***z-component of the angular momentum*** (blue circle) as a *function* of ***dissipation*** γ



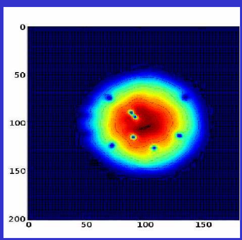
The ***increasing rate*** of the ***quantum energy*** as a *function* of ***dissipation*** γ

Snapshots of the motion of vortex-antivortex pairs and formation of a mini turbulent system

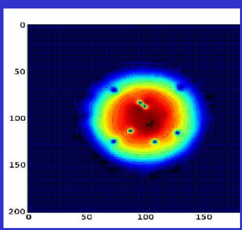
Random *vortex –anti vortex* pairs put in the condensate for ***dissipation, γ = 0.003***
We have *information* about the ***vortex and anti vortex motion*** with the help of the ***evolution*** of the ***density***



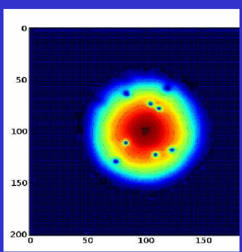
At time ***t = 15.0.***
A *vortex – antivortex pair* begin to *move together*



At time ***t = 21.0***.
The *vortex – antivortex pair moves together*

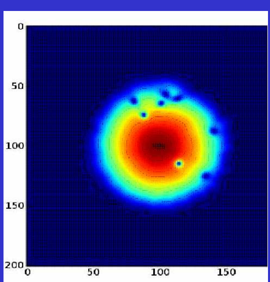
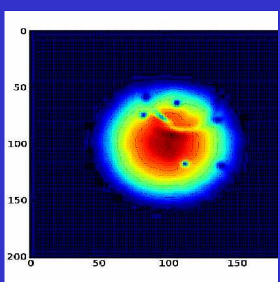
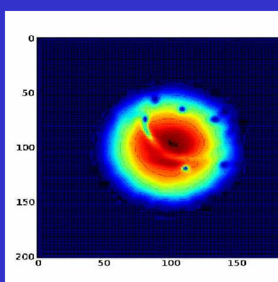
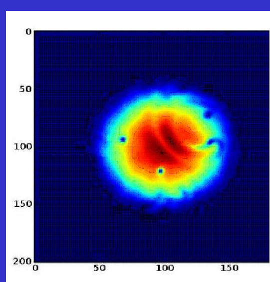
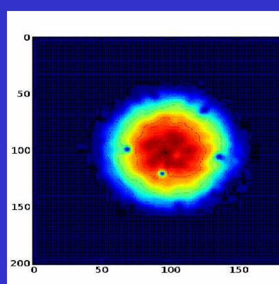


At time ***t = 25.0***.
The *vortex – antivortex pair moves together*

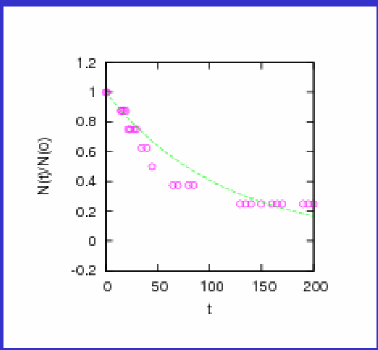


At time ***t = 33.0***.
The *vortex – antivortex pair moves together*

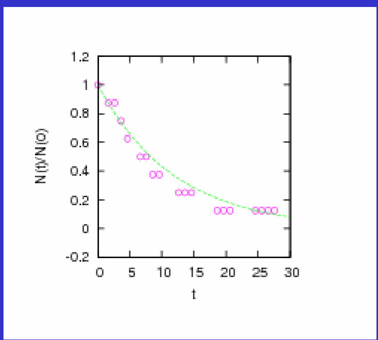
Annihilation and sound wave production
Formation of a mini turbulent system

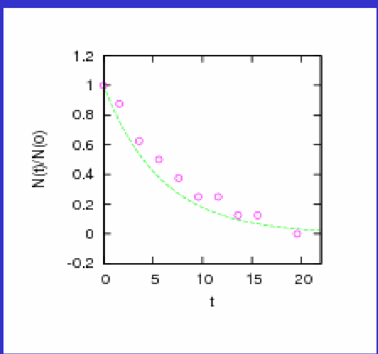
Number of vortices for random vortex – antivortex pairs placed in the condensate with initial separation distance $d_0 = 1.8$ as a function of time for different dissipations γ



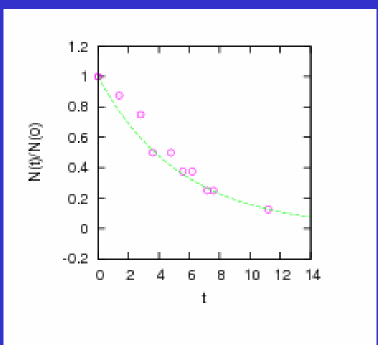
For dissipation γ = 0.003
(The green curve: f(x) = exp(b*t), b = -0.0089)



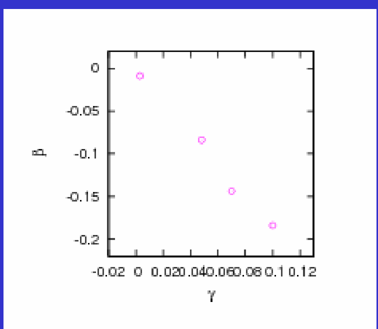
For dissipation γ = 0.048
(The green curve: f(x) = exp(b*t), b = -0.0837)



For dissipation γ = 0.07
(The green curve: f(x) = exp(b*t), b = -0.1437)



For dissipation γ = 0.1
(The green curve: f(x) = exp(b*t), b = -0.184)



Decay rate β *of the* ***number of vortices*** *as a function of* ***dissipation γ***

Conclusions

- Without dissipation the vortex trajectory is a closed curve
- With dissipation the vortex spirals out to the edge of the condensate
- Smaller and larger initial separation distances, d_0 have similar periods
- The cyclical motion and the inhomogeneity of the condensate changes the compressibility and as a result the moving vortex pair produces acoustic emissions
- The sound is reabsorbed
- We study the sound production with help of the sound energy and find an anticorrelation between the sound energy and vortex energy
- We calculate the translation speed, v_{pair}/v_y and the frequency of motion $\omega_{pair}/\omega'_{pair}$ with two methods and show that they coincide very well
- Differenet energies were studied for dissipation $\gamma = 0.003$ in the case of vortex-antivortex
- We describe the friction coefficients, α and α' as a function of dissipation γ, initial position x_0 and frequency of motion ω
- α and α' increase as the dissipation, γ increases
- We discuss the exponential constant of the radius of trajectory and different energies
- We demonstrate with some plots the decrease of the number of vortices with time, which are exponential functions