

# Estimating extremes for the nuclear industry: Some short stories

Lee Fawcett

June 2016

# Motivation



## 1. “Bayesian inference for very rare events: optimal estimation strategies for natural hazards engineers”

- Project team: NCL (LF, DW) & EDF Energy (Hugo Winter, Pietro Bernardara)
- NERC ERIIP grant application
- Translational research using Fawcett & Walshaw (2016)
- Main aim: develop software tools to make state-of-the-art techniques available to practitioners
- April 2016–December 2017
- **One variable, one site**

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- Project team:
  - ▶ NCL (LF, DW) & EDF Energy (Hugo Winter, Pietro Bernardara)
  - ▶ NCL (DW, LF) & Met Office (Simon Brown)
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## 3. “Exploring methodologies for the joint probability analysis of extreme events”

- Project team: NCL (LF, DW), Lancaster (Jenny Wadsworth), EDF Energy (Hugo Winter) & ONR (Tanya McLeod)
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# Outline of this talk

- 1 Modelling extremes: A (very) quick primer
- 2 Problem 1: Some ideas and preliminary results
- 3 Problem 2: Exploratory analyses and open questions
- 4 Problem 3: Description and possible ways forward

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# Modelling extremes: starting point

Let  $\{X_n\}$  denote a stationary sequence of random variables with common distribution function  $F$ , and let  $M_n = \max\{X_1, \dots, X_n\}$ .

It is typically the case that, as  $n \rightarrow \infty$ ,

$$\Pr(M_n \leq x) \approx F^{n\theta}(x),$$

where  $\theta \in (0, 1)$  is known as the **extremal index**; see, for e.g., [Leadbetter & Rootzén \(1988\)](#).

As  $\theta \rightarrow 0$  there is increasing dependence in the extremes of the process; for an independent process,  $\theta = 1$ .

# Modelling extremes: block maxima

The **Generalised Extreme Value** (GEV) distribution is the limiting model for  $F^n$ , with distribution function (d.f.)

$$\mathcal{G}(y; \mu, \varsigma, \xi) = \begin{cases} \exp \left[ - (1 + \xi(y - \mu)/\varsigma)^{-1/\xi} \right], & \xi \neq 0 \\ \exp \left[ -\exp \left( -(y - \mu)/\varsigma \right) \right] & \xi = 0, \end{cases}$$

defined on  $\{y : 1 + \xi(y - \mu)/\varsigma > 0\}$ , where  $-\infty < \mu < \infty$ ,  $\varsigma > 0$  and  $-\infty < \xi < \infty$  are parameters of location, scale and shape.

- It can be shown that  $F^{n\theta}$  is also GEV with d.f.  $\mathcal{G}(y; \mu^*, \varsigma^*, \xi)$ , provided long-range dependence is negligible<sup>1</sup>
- In practice, the GEV is used to model maxima over some convenient calendar unit – usually years

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# Modelling extremes: threshold excesses

Pickands (1975) showed that for  $\theta = 1$  and  $u$  large,  $(X - u | X > u)$  follows a **Generalised Pareto distribution** (GPD) with d.f.

$$\mathcal{H}(y) = \begin{cases} 1 - (1 + \xi y / \sigma)^{-1/\xi}, & \xi \neq 0 \\ 1 - \exp[-y/\sigma] & \xi = 0, \end{cases}$$

defined on  $\{y : y > 0 \text{ and } (1 + \xi y / \sigma) > 0\}$ , where  $\sigma = \varsigma + \xi(u - \mu) > 0$  and  $\xi$  are the scale and shape.

- Simple graphical diagnostics (not discussed here) available for selection of threshold  $u$
- Extremal index  $\theta$  can't be absorbed into the parameters of the GPD; careful consideration of extremal dependence required

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# Modelling extremes: return level estimation

The  **$r$ -year return level  $z_r$**  can then be obtained by inversion of  $\mathcal{G}(z_r)$  or  $\mathcal{H}^\theta(z_r)$ .

For threshold excesses, on equating to  $1 - r^{-1}$  this gives

$$z_r = \begin{cases} u + \sigma \xi^{-1} \left[ (\lambda_u^{-1} w_r)^{-\xi} - 1 \right] & \xi \neq 0 \\ u - \sigma \log (\lambda_u^{-1} w_r) & \xi = 0, \end{cases} \quad (1)$$

where  $w_r = 1 - [1 - (rn_y)^{-1}]^{1/\theta}$ ,  $\lambda_u$  is the rate of threshold excess and  $n_y$  is the (average) number of observations per year.

The GPD scale and shape in (1) can be replaced with their maximum likelihood estimates, or posterior draws; estimating  $\theta$  can be more thorny...

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# Modelling extremes: problems and pragmatic solutions

Threshold-based approaches are now commonplace, given the potential for a huge increase in precision.

- ① **Temporal dependence** and the estimation of  $\theta$ : filter out a set of independent excesses and hence assume  $\theta \approx 1$
- ② **Seasonal variability**: (i) single season approach; (ii) seasonally varying parameters
- ③ **Dependence on covariates**: write the model parameters in the form  $h(X^\top \beta)$
- ④ **Spatial dependence**: very little done in practice

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# Project 1

Current practice within the industry is to provide nuclear plants with protection against the “**once in ten thousand year**” skew-surge event:  $z_{10^4}$ .

- ONR don't mind at all if estimates of  $z_{10^4}$  are overly-conservative. In fact, **this is their recommendation**
- EDF **do** mind: huge cost implications!
- At best, routine threshold analyses performed:
  - ▶ Declustering used to avoid having to estimate  $\theta$
  - ▶ Non-stationarity ignored
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Aim: To allow practitioners to interact with current best practice for univariate return level estimation.

- Making the most of the data to increase estimation precision and avoid unrealistic 95% confidence upper bounds
- Adopting a Bayesian approach to inference to report the **posterior predictive estimate**  $\hat{z}_{10^4, \text{pred}}$
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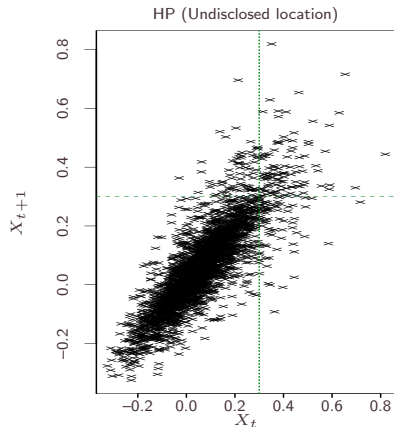
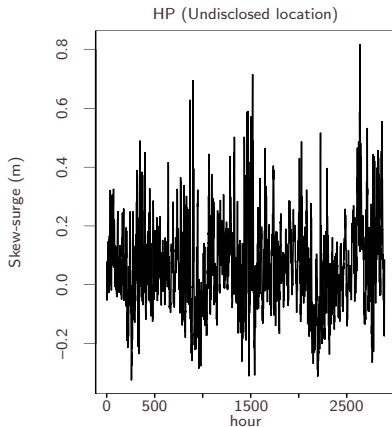
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# Skew-surge analysis: current practice



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- Graphical diagnostics suggest a threshold of  $u = 0.3$  metres
- To account for extremal dependence, current practice is:
  - ▶ Deem a 'cluster' of extremes to have terminated once at least 30 consecutive sub-threshold hourly observations made
  - ▶ Fit the GPD to the set of independent **cluster peak excesses**
- For skew-surge at this location, this gives:

|                | $z_{10}$     | $z_{50}$     | $z_{1000}$   | $z_{10000}$  |
|----------------|--------------|--------------|--------------|--------------|
| MLE (st. err.) | 0.87 (0.11)  | 0.92 (0.12)  | 0.97 (0.20)  | 1.25 (0.31)  |
| 95% PCI        | (0.77, 1.57) | (0.80, 2.09) | (0.82, 3.38) | (0.86, 7.05) |

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# Skew-surge analysis: current practice

- Graphical diagnostics suggest a threshold of  $u = 0.3$  metres
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A large-scale simulation study in [Fawcett & Walshaw \(2016\)](#) shows:

- Estimates of  $z_r$  can be *hugely* sensitive to the choice of declustering interval (set at 30 here)
- Using *all* threshold excesses with the extremal index estimator of [Ferro & Segers \(2004\)](#) can significantly increase precision
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# Skew-surge analysis: improvements (II)

- Extremes are rare – sample sizes generally quite small; informative priors within a **Bayesian framework** can augment the analysis
- In some instances (e.g.  $\xi < -0.5$ ) ML regularity conditions do not hold – not an issue within the Bayesian framework
- **Prediction** neatly handled within a Bayesian analysis
- In practice, easier to make inferences on more complex model structures in the Bayesian world

# Skew-surge analysis: improvements (II)

- We work with a re-parameterised GPD:  $\eta = \log(\sigma - \xi u)$
- Priors:

- ▶ Following [Coles & Tawn \(2005\)](#) we assume

$$\begin{aligned}\tilde{z}_1 &= z_{10} \sim \text{Ga}(g_1, h_1) \\ \tilde{z}_2 &= z_{50} - z_{10} \sim \text{Ga}(g_2, h_2),\end{aligned}$$

where  $(g, h)$  reflect the opinions of a hydrologist

- ▶ Then:

$$\pi(z_{10}, z_{50}) \propto z_{10}^{g_1-1} e^{-h_1 z_{10}} (z_{50} - z_{10})^{g_2-1} e^{-h_2(z_{50}-z_{10})}$$

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# Skew-surge analysis: improvements (II)

- We use the extremal index likelihood of **Süveges and Davison (2010)** based on inter-arrival times of threshold exceedances
- We use a Metropolis-Hastings scheme with independent random walk updating of the marginal/dependence parameters
- We simulate chains of length  $s = 500,000$  after the removal of the burn-in period; convergence 'checked' by initialisation at multiple starting points
- **Posterior predictive return levels** obtained by solving

$$\Pr(Y \leq z_{r,\text{pred}} | \mathbf{x}) \approx \frac{1}{s} \sum_{i=1}^s \Pr(Y \leq z_{r,\text{pred}} | \boldsymbol{\psi}_i) = 1 - r^{-1}$$

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| Predictive                              | 0.83                        | 0.96                        | 1.17                        | 2.95                        |

# Project 1: *Shiny* tools

- A prototype software tool for implementing analyses such as those discussed here is under construction
- Thanks to Keith Newman and several undergraduate interns for working on this!

<https://keithnewman.shinyapps.io/extremeValue/>

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# Project 2

- Extension of project 1, where we have observations on a single variable **at many sites**
- Will form part of EDF's more long-term R&D aims
- EDF work here similar to that currently being undertaken by DW with the Met Office
- Questions:
  - ▶ For our site of interest, can data at other sites inform our analysis and aid the estimation of  $z_{10^4}$ ?
  - ▶ Can we use information at available sites to estimate return levels at locations for which no data exists?
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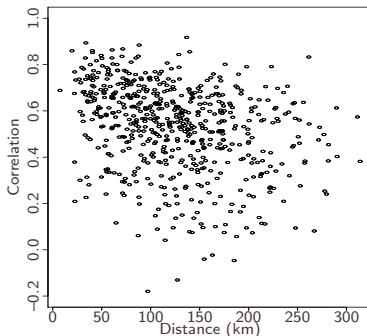
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# Wind speed analysis

- 42 years of annual maximum wind speeds (km/h) at 35 (undisclosed!) locations near a proposed nuclear plant site in France
- New structures must be able to withstand wind speeds with an annual exceedance probability of  $10^{-4}$
- Decaying correlation between extremes with distance:



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- Annual maxima: GEV – location, scale and shape  $\mu$ ,  $\varsigma$  and  $\xi$
- Standard analysis:
  - ▶ Fit separate GEV to each site, obtaining  $\Psi_d = (\hat{\mu}_d, \hat{\varsigma}_d, \hat{\xi}_d)$ ,  $d = 1, \dots, 35$
  - ▶ Could obtain marginal estimates of return levels  $\hat{z}_{r_d}$ , or somehow combine the marginal  $\Psi_d$  to obtain a 'global' return level estimate  $\hat{z}_r$  (e.g. Fawcett & Walshaw, 2012)
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# Wind speed analysis: latent variable model

Suppose  $\{Y(x)\}$  is a random process defined at all points  $x$  of a spatial domain  $\mathcal{X}$ , where  $\mathcal{X}$  is a contiguous subset of  $\mathbb{R}^2$ .

We have annual maximum wind speeds at a finite subset  $\mathcal{D} = \{x_1, \dots, x_{35}\}$  of sites of  $\mathcal{X}$ ... how do we make inferences for the process elsewhere in  $\mathcal{X}$ ?

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For  $\{\mu(x), \varsigma(x), \xi(x)\}$  assume the following **trend surfaces**:

$$\begin{aligned}\mu(x) &= \beta_{0,\mu} + \beta_{1,\mu}\text{lon}(x) + \beta_{2,\mu}\text{lat}(x) + S_\mu(x; \alpha_\mu, \lambda_\mu) \\ \varsigma(x) &= \beta_{0,\varsigma} + \beta_{1,\varsigma}\text{lon}(x) + \beta_{2,\varsigma}\text{lat}(x) + S_\varsigma(x; \alpha_\varsigma, \lambda_\varsigma) \\ \xi(x) &= \beta_{0,\xi} + S_\xi(x; \alpha_\xi, \lambda_\xi),\end{aligned}$$

where  $S_*$  is a zero mean **Gaussian random field** with covariance function:

$$\alpha_* \exp\{-|h|/\lambda_*\}.$$

Then, conditional on the three Gaussian processes at sites in  $\mathcal{D}$ , we assume that

$$Y_i(x_d) | \{\mu(x_d), \varsigma(x_d), \xi(x_d)\} \sim \text{GEV}\{\mu(x_d), \varsigma(x_d), \xi(x_d)\},$$

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# Wind speed analysis: prior specification

- A joint prior density  $\pi$  must be specified for the parameters  $(\alpha_\mu, \alpha_\varsigma, \alpha_\xi, \lambda_\mu, \lambda_\varsigma, \lambda_\xi, \boldsymbol{\beta}_\mu, \boldsymbol{\beta}_\varsigma, \beta_{0,\xi})$ .
- To reduce computational burden, we use conjugate priors *wherever possible*. For example, for  $\mu(x)$ :
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| $\mu(x)$       | 1          | 12    | 5           | 3     | $\mathbf{0}$           | $v_{i,j} = 100, i = j; 0 \text{ otherwise}$ |
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Given a current value of the Markov chain:

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the next state  $\Psi^{t+1}$  is obtained as follows:

## 1: Updating the GEV parameters at each site

Each component of  $\boldsymbol{\mu}^t = \{\mu^t(x_1), \dots, \mu^t(x_{35})\}$  is updated singly:

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Due to the use of conjugate priors,  $\beta_\mu$  is drawn directly from a MVN having covariance matrix and mean vector

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$$\begin{aligned}\Sigma_\mu &= \left\{ V_\mu^{-1} + \mathbf{X}_\mu^\top (V_\mu^t)^{-1} \mathbf{X}_\mu \right\}^{-1} \quad \text{and} \\ M_\mu &= \left\{ V_\mu^{-1} + \mathbf{X}_\mu^\top (V_\mu^t)^{-1} \mathbf{X}_\mu \right\}^{-1} \left\{ V_\mu^{-1} \mathbf{m}_\mu + \mathbf{X}_\mu^\top (V_\mu^t)^{-1} \boldsymbol{\mu}^t \right\}.\end{aligned}$$

The regression parameters for the GEV scale and shape are updated similarly.

## 3: Updating the sill parameters of the covariance function

Due to the use of conjugate priors,  $\alpha_\mu$  is drawn directly from an inverse Gamma with shape and rate

$$\begin{aligned} A_\mu &= \frac{1}{2}k + a_\mu \quad \text{and} \\ B_\mu &= b_\mu + \frac{1}{2}\alpha_\mu^t (\boldsymbol{\mu}^t - \mathbf{X}_\mu \boldsymbol{\beta}_\mu^t)^\top (V_\mu^t)^{-1} (\boldsymbol{\mu}^t - \mathbf{X}_\mu \boldsymbol{\beta}_\mu^t). \end{aligned}$$

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## 4: Updating the range parameters of the covariance function

Here the situation is non-conjugate and requires a Metropolis step.

- Generate a proposal  $\lambda_\mu^* \sim U(\lambda_\mu^t - \epsilon_\lambda, \lambda_\mu^t + \epsilon_\lambda)$  and compute the acceptance probability:

$$A(\lambda_\mu^t, \lambda_\mu^*) = \min \left\{ 1, \frac{\pi(\boldsymbol{\mu}^t | \alpha_\mu^t, \lambda_\mu^*, \boldsymbol{\beta}_\mu^*)}{\pi(\boldsymbol{\mu}^t | \alpha_\mu^t, \lambda_\mu^t, \boldsymbol{\beta}_\mu^t)} \times \left( \frac{\lambda_\mu^*}{\lambda_\mu^t} \right)^{c_\mu - 1} \exp \left( \frac{\lambda_\mu^t - \lambda_\mu^*}{d_\mu} \right) \right\}$$

- With probability  $A(\lambda_\mu^t, \lambda_\mu^*)$  the  $\lambda_\mu$  component of  $\Psi^{t+1}$  is set to  $\lambda_\mu^*$ ; otherwise, remains at  $\lambda_\mu^t$
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# Wind speed analysis: results

- MCMC scheme run for 1M iterations, after a burn-in of 10,000
- After around 13.5 hours (as well as a ridiculous amount of time fine-tuning last weekend), the following results were obtained
- Convergence was checked by using multiple start points and eyeballing the chains, nothing more...

|            | $\beta_0$ (kmh <sup>-1</sup> ) | $\beta_1$ (kmh <sup>-1</sup> /km lon) | $\beta_2$ (kmh <sup>-1</sup> /km lat) |
|------------|--------------------------------|---------------------------------------|---------------------------------------|
| $\mu(x)$   | -0.75(-40.72, 38.05)           | -10.20(-32.70, -6.74)                 | 6.46(3.19, 9.77)                      |
| $\zeta(x)$ | 0.21(-38.34, 38.23)            | -2.95(-9.34, 3.45)                    | 0.94(-0.22, 1.93)                     |
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# Wind speed analysis: return level inference

The plot overleaf shows a map of the predictive pointwise posterior mean for the 10,000 year return level wind speeds.

- Generate one conditional simulation of three independent Gaussian processes for each state of the Markov chain given its then-current values of  $\mu$ ,  $\varsigma$  and  $\xi$
- Use this realisation to compute pointwise 10,000-year return levels at ungauged sites
- **Main strength of this approach:**
  - ▶ Use of stochastic processes to model spatial behaviour of the marginal parameters...
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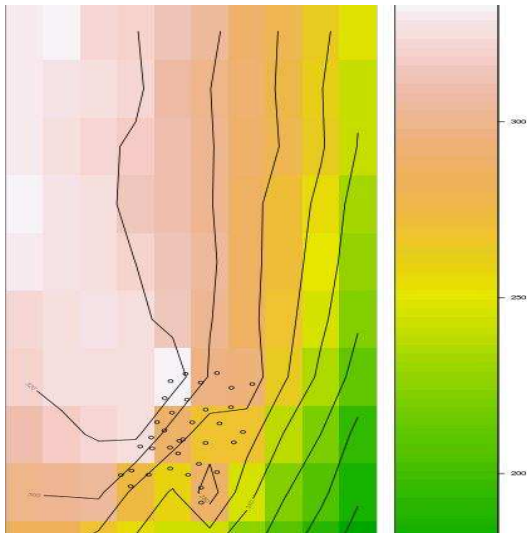
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# Wind speed analysis: return level inference



# Wind speed analysis: potential drawbacks/other avenues

- Conditional independence, given the latent process, probably leads to **unrealistic spatial structure**
- Copula models
- Max-stable processes
- Nature of spatial dependence: **asymptotic**? For example,

$$\Pr(Y > u_y | X > u_x) > 0, \quad u_y \rightarrow \infty?$$

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# Problem 3

- Based on recent discussions at a NERC study group in London
- The ONR are interested in the specification of **high dimensional failure regions** for nuclear structures
- For example, suppose that - for a single location - we have measurements of **skew surge**, **wind speed** and **rainfall**
  - ▶ Can we define, in 3-D space, the region (could be disjoint) for which there is a probability of structural failure of  $10^{-4}$ ?
  - ▶ Need to carefully consider how the variables might interact: a 'dangerous' level of rainfall might depend on how strong the coinciding wind speed or skew surge measurements are
  - ▶ For this, we need to carefully consider the nature of dependence between these variables
- In reality, there are at least 30 such variables which need to be considered, giving rise to lots of dangerous combinations: pairs, triples, ...

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