1. (a) For the GPD, we have:

$$P(X > x) = \lambda_u \left[1 + \xi\left(\frac{x-u}{\sigma}\right)\right]_+^{-1/\xi}.$$

Thus, for location A, we have:

$$P\left(X_A > 32\right) = 0.009\left[1 + 0.184\left(\frac{32-30}{7.440}\right)\right]^{-1/0.184} = 0.006923.$$

Similarly, for location B we have $P\left(X_B > 32\right) = 0.004051$. Hence, assuming independence, we have

$$P\left(X_A, X_B > 32\right) = 0.006923 \times 0.004051 = 0.00002805.$$

(b) (i) We must transform the margins to be unit Fréchet-distributed.

(ii) We would differentiate $G(x, y)$ with respect to x only (where x represents threshold excesses at location A). We would then substitute the transformed threshold exceedance into this derivative for the rainfall total at location A (i.e. $\tilde{x}_A$ for x) and the transformed threshold for location B (i.e. $\tilde{u}_B$ for y).

(iii) [There was a typo in the question here - should be "32mm = ..." and NOT "50mm = ..."]

We have

$$P\left(X_A, X_B > 32\right) = 1 - \exp\left\{-\left(986.461^{-1/0.623} + 1605.221^{-1/0.623}\right)^{0.623}\right\} = 0.00128.$$

This value is almost 46 times larger than when we assumed independence. This could result in huge under-protection if a flood defence was built to protect against such a combination of extremes at both locations, and rainfall between the locations was assumed independent.

[Total = 9 marks]