The Storm of the Century! (Statistics of Extremes)

Lee Fawcett

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Background: Statistics of extremes

So many areas of Statistics are interested in what happens **on average**: Mean, median, mode...

One area that is different is the field of **extreme value theory**.

Why might we be interested in **extremes**, rather than averages?

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Stormy weather in the UK!

- 5 December 2015: **Storm Desmond** – more than a month's rain in parts of Cumbria, flooding in Carlisle
- 22 December 2015: Further flooding in Cumbria
- Christmas Day: **Storm Eva** – 106 flood alerts in UK
- New Year: **Storm Frank** – Scotland, northeast England, North Yorkshire

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We can usually deal with averages here – it is the **extremes** that cause devastation!

Such scientists require the expertise of **mathematicians** to help with **predictions** and **calculations**.

In this talk I will focus on how maths/statistics can be used to help oceanographers and civil engineers to predict extreme **sea-surges**.

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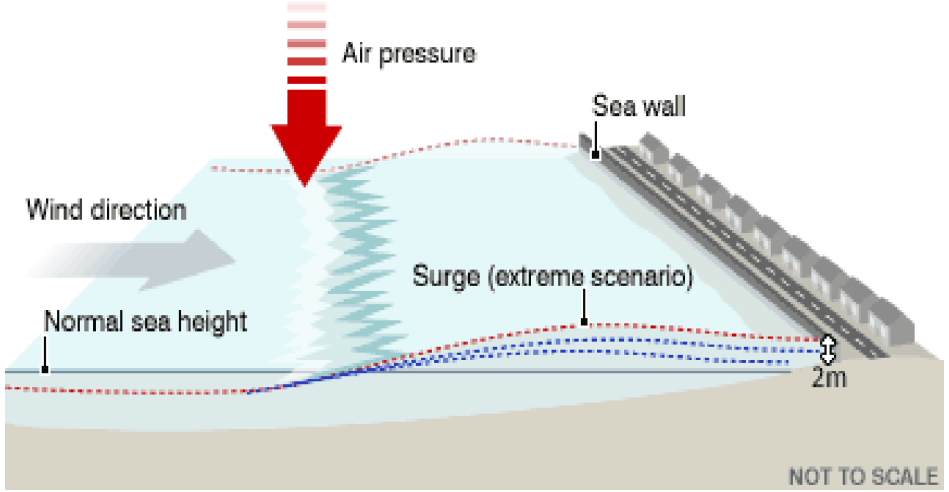
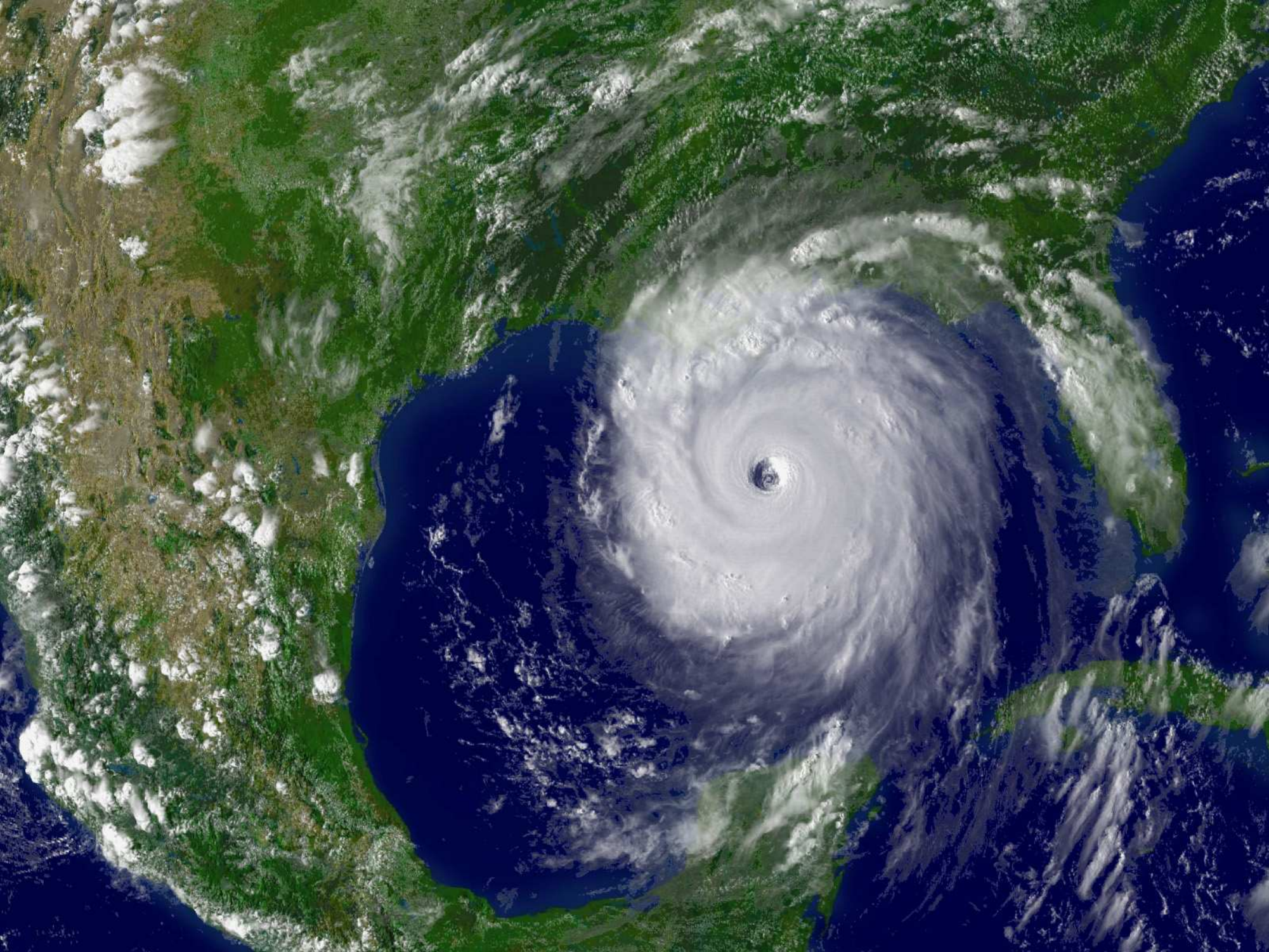
Sea-surge


Air pressure
Sea wall
Wind direction
Surge (extreme scenario)
Normal sea height
2m
NOT TO SCALE



Hurricane Katrina: New Orleans, August 2005

Some facts

- Category 5 Hurricane (Category 3 when it made landfall)
- Killed 1833 people
- Caused $108 bn worth of damage
- Most damage/loss of life caused by **storm surge**
- Storm surge reached 14.4 feet above sea level
- Lowest air pressure 902 mb
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www.KatrinaDestruction.com

Data application: Sea surges at New Orleans

Below are the annual maximum sea-surge levels observed at New Orleans over a 50 year period *before Hurricane Katrina*:

8.5	8.9	9.1	8.9	8.4	9.7	9.1	9.6	8.7	9.3
9.6	9.3	8.7	9.0	8.8	8.9	8.9	12.2	7.8	7.7
8.3	8.1	7.3	6.8	6.7	7.3	7.6	8.2	8.6	9.8
9.5	7.4	7.3	10.2	10.3	10.4	8.8	9.7	10.0	10.8
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Suppose you are the mathematician on the team of scientists working on a new section of flood defence system in New Orleans.

One of the civil engineers asks you to work out some exceedance probabilities for her.

In particular, she wants to know the probability that, this year, the annual maximum sea surge at New Orleans will

1. exceed **8.75 feet**;
2. exceed **11.25 feet**;
3. exceed **14 feet**.

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The relative frequency approach

You will have used this approach to probability in your maths at School.

It helps if the data are re–written in ascending order:

6.7	6.8	7.3	7.3	7.3	7.4	7.6	7.7	7.8	8.1
8.2	8.3	8.4	8.5	8.6	8.7	8.7	8.8	8.8	8.9
8.9	8.9	8.9	9.0	9.1	9.1	9.3	9.3	9.4	9.5
9.6	9.6	9.7	9.7	9.8	10.0	10.0	10.2	10.3	10.4
10.5	10.5	10.8	11.1	11.5	11.8	12.2	12.6	12.7	13.0

Then,

$$P(\text{sea-surge exceeds 8.75 feet}) =$$

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9.6	9.6	9.7	9.7	9.8	10.0	10.0	10.2	10.3	10.4
10.5	10.5	10.8	11.1	11.5	11.8	12.2	12.6	12.7	13.0

Then,

$$P(\text{sea-surge exceeds 8.75 feet}) = \frac{\mathbf{33}}{50} = \frac{66}{100} = 0.66.$$

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6.7	6.8	7.3	7.3	7.3	7.4	7.6	7.7	7.8	8.1
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9.6	9.6	9.7	9.7	9.8	10.0	10.0	10.2	10.3	10.4
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The relative frequency approach

Using the relative frequency approach, work out the other two exceedance probabilities:

- P(sea-surge exceeds 11.25 feet)
- P(sea-surge exceeds 14 feet)

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8.9	8.9	8.9	9.0	9.1	9.1	9.3	9.3	9.4	9.5
9.6	9.6	9.7	9.7	9.8	10.0	10.0	10.2	10.3	10.4
10.5	10.5	10.8	11.1	11.5	11.8	12.2	12.6	12.7	13.0

$P(\text{sea-surge exceeds 11.25 feet}) =$

The relative frequency approach

6.7	6.8	7.3	7.3	7.3	7.4	7.6	7.7	7.8	8.1
8.2	8.3	8.4	8.5	8.6	8.7	8.7	8.8	8.8	8.9
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9.6	9.6	9.7	9.7	9.8	10.0	10.0	10.2	10.3	10.4
10.5	10.5	10.8	11.1	11.5	11.8	12.2	12.6	12.7	13.0

$P(\text{sea-surge exceeds 11.25 feet}) =$

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$$P(\text{sea-surge exceeds 11.25 feet}) = \frac{6}{50} = \frac{12}{100} = 0.12$$

The relative frequency approach

6.7	6.8	7.3	7.3	7.3	7.4	7.6	7.7	7.8	8.1
8.2	8.3	8.4	8.5	8.6	8.7	8.7	8.8	8.8	8.9
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$$P(\text{sea-surge exceeds 11.25 feet}) = \frac{6}{50} = \frac{12}{100} = 0.12$$

The relative frequency approach

6.7	6.8	7.3	7.3	7.3	7.4	7.6	7.7	7.8	8.1
8.2	8.3	8.4	8.5	8.6	8.7	8.7	8.8	8.8	8.9
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10.5	10.5	10.8	11.1	11.5	11.8	12.2	12.6	12.7	13.0

$P(\text{sea-surge exceeds 14 feet}) =$

The relative frequency approach

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8.2	8.3	8.4	8.5	8.6	8.7	8.7	8.8	8.8	8.9
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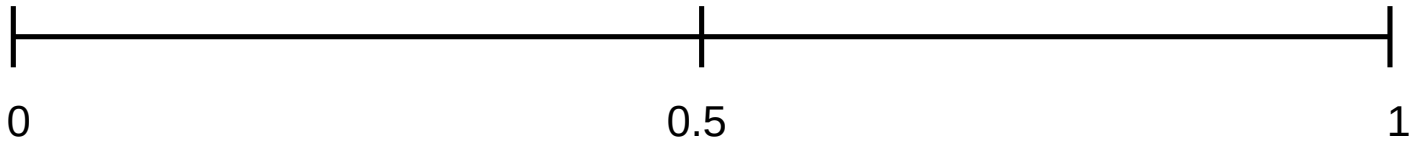
9.6	9.6	9.7	9.7	9.8	10.0	10.0	10.2	10.3	10.4
10.5	10.5	10.8	11.1	11.5	11.8	12.2	12.6	12.7	13.0

$$P(\text{sea-surge exceeds 14 feet}) = \frac{\mathbf{0}}{50} = 0$$

The relative frequency approach

Think about the **probability scale**:



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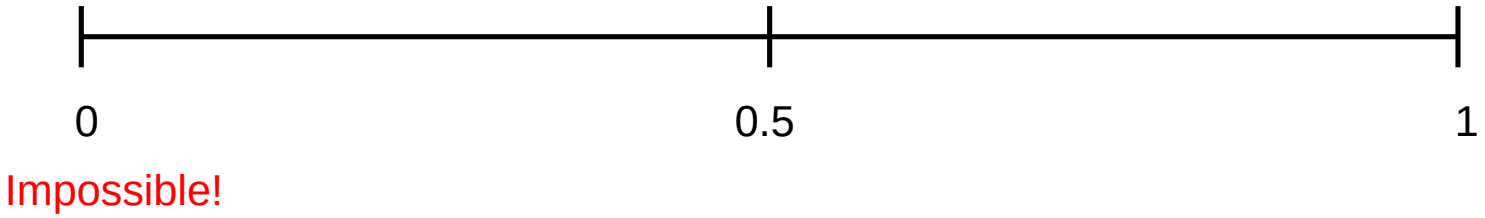

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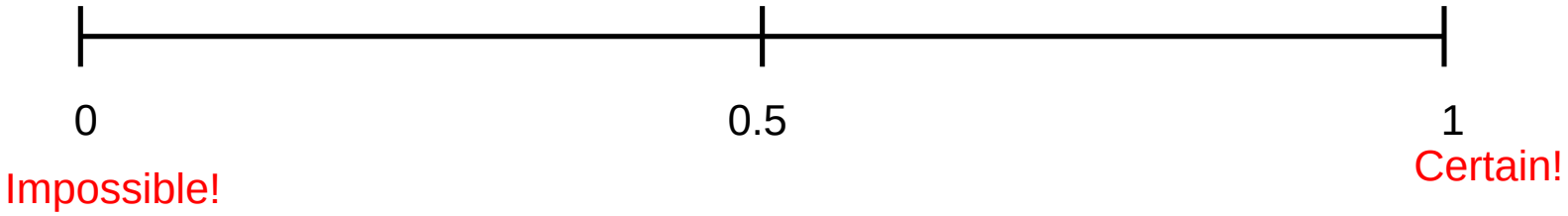

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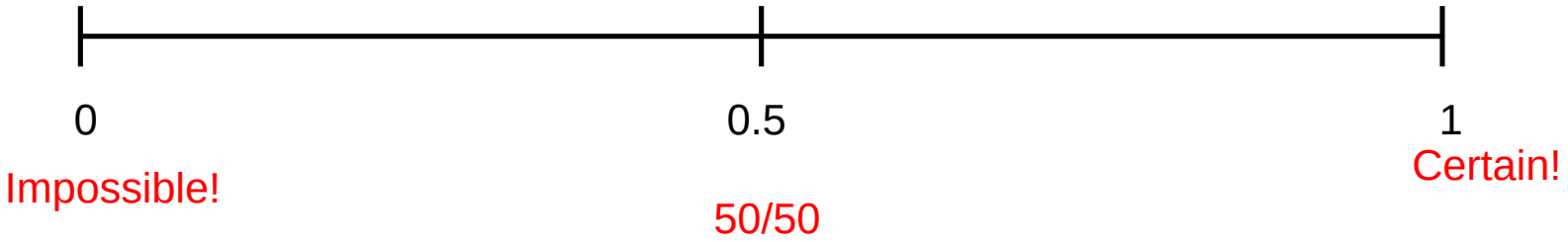

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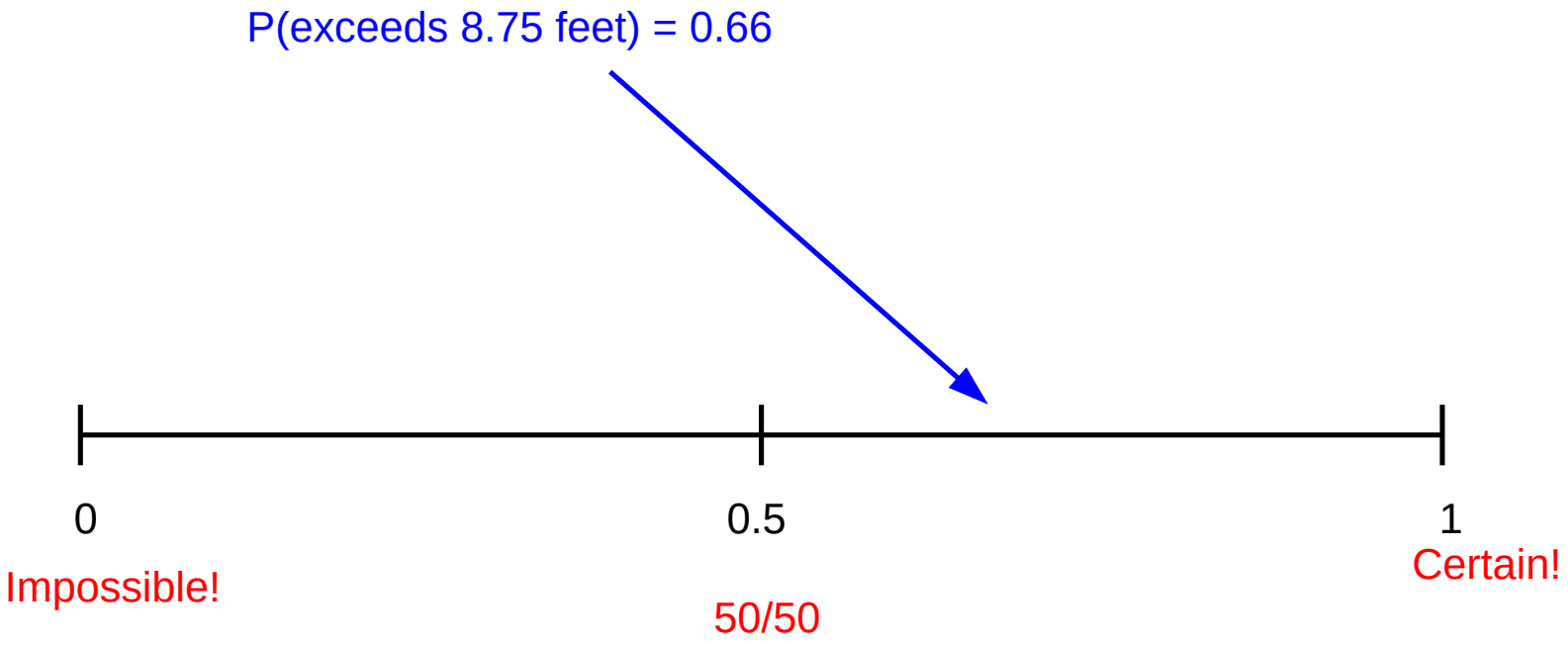

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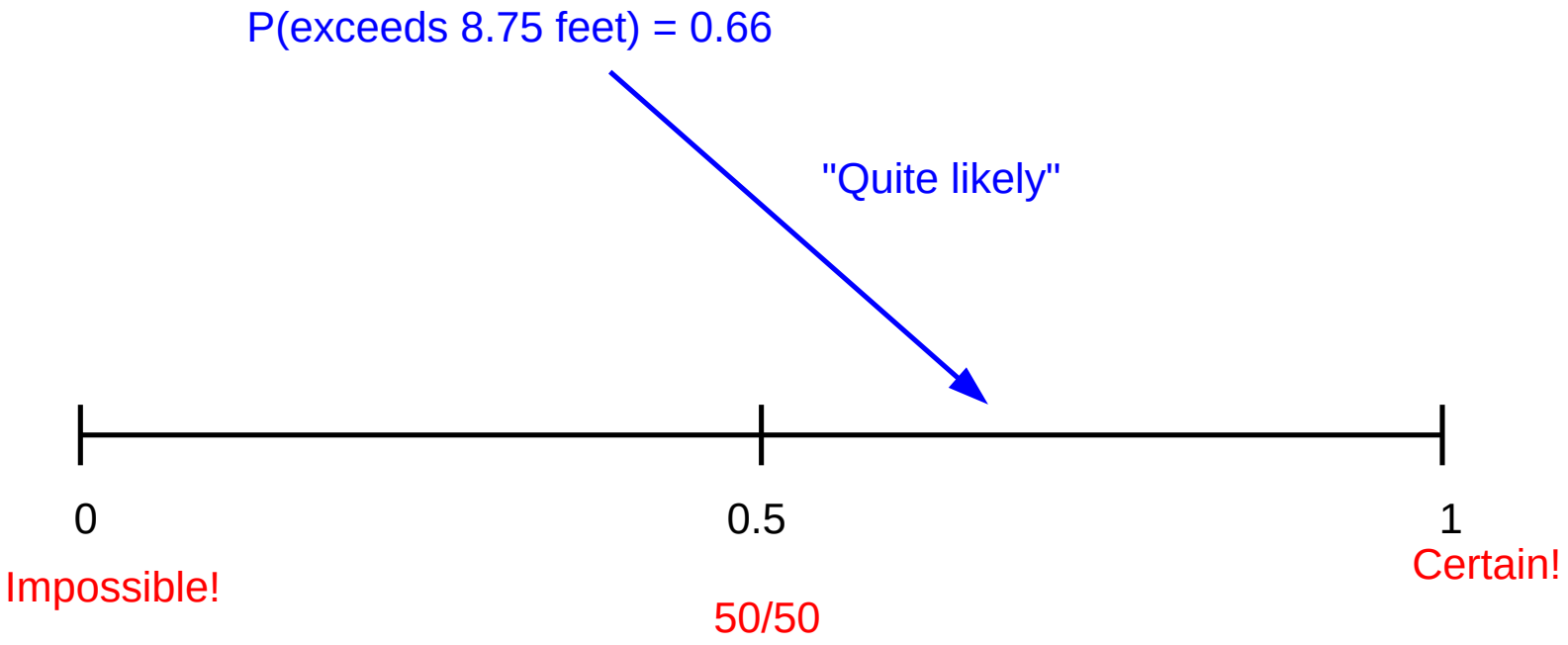

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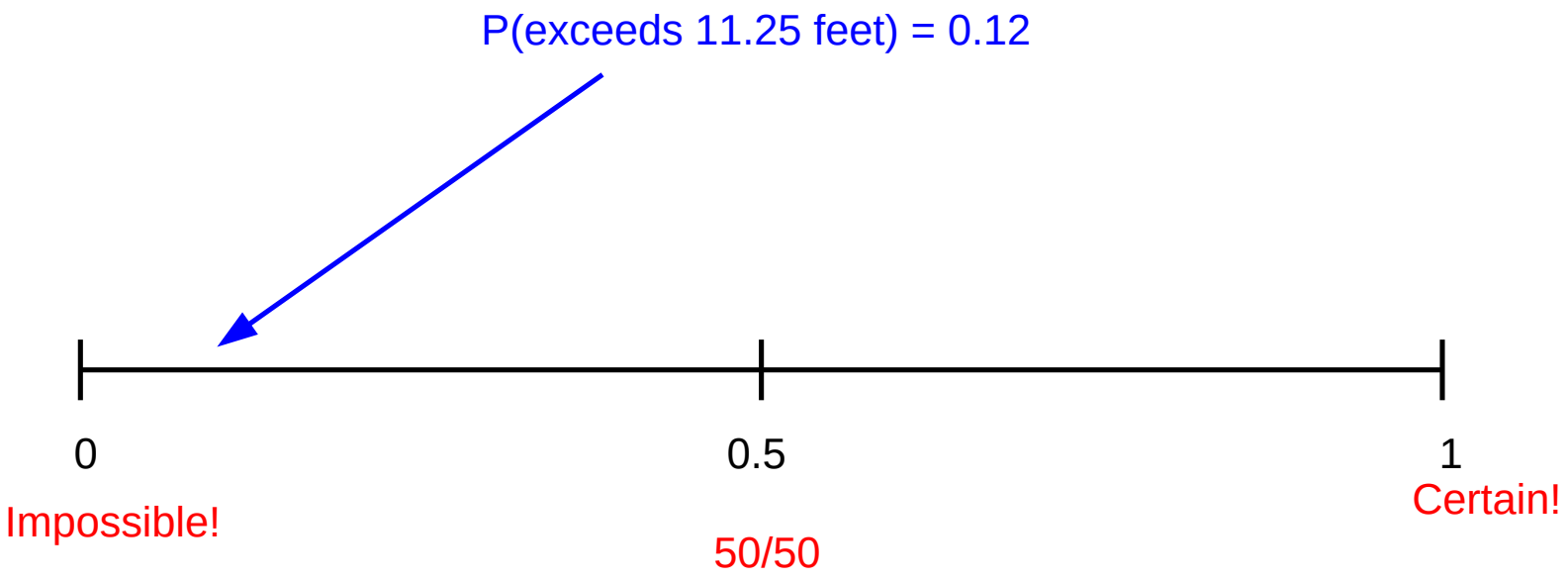

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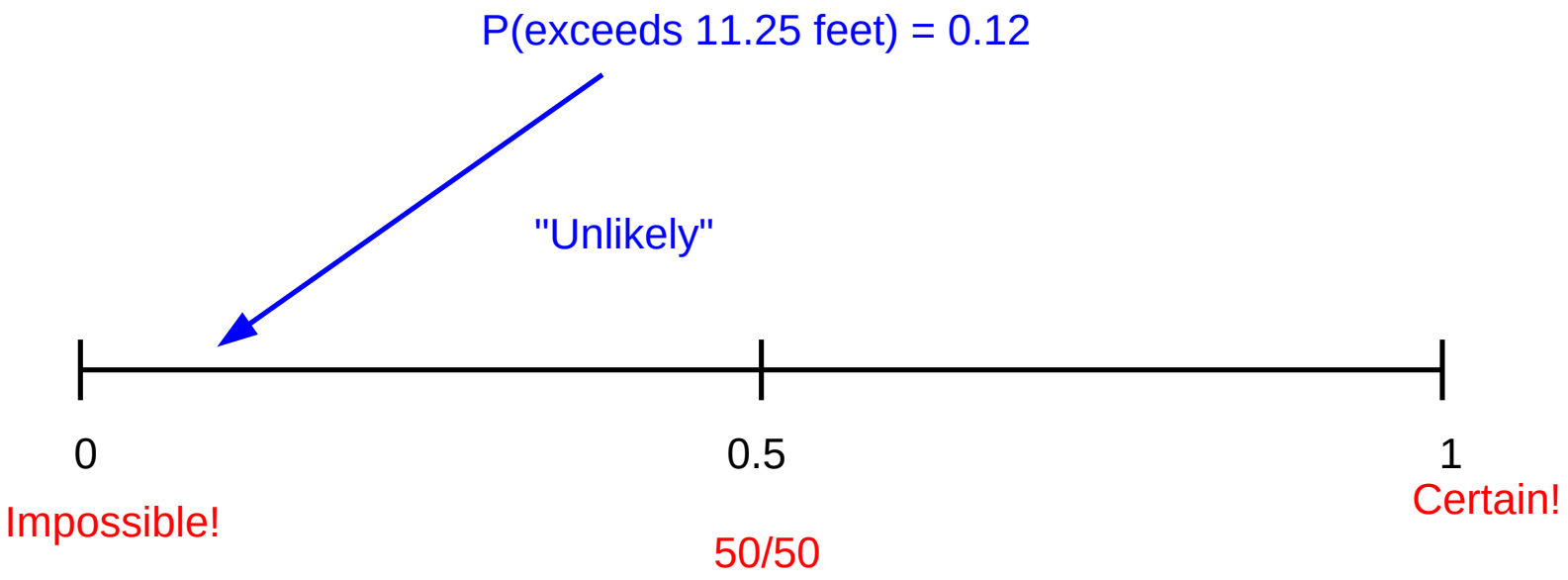

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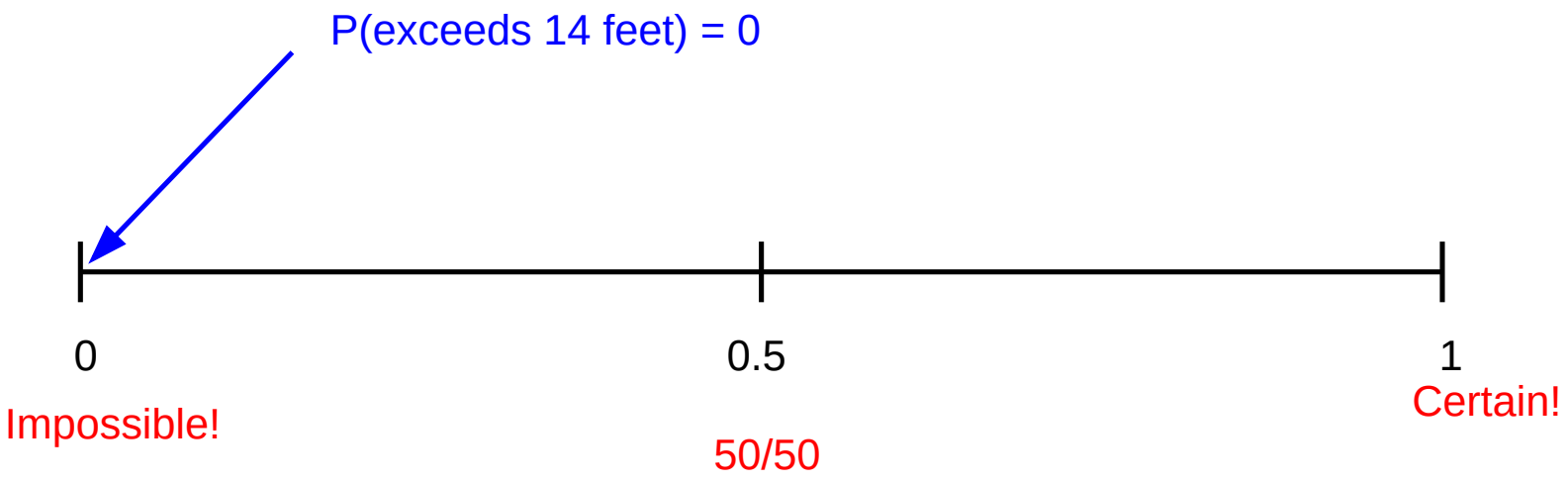

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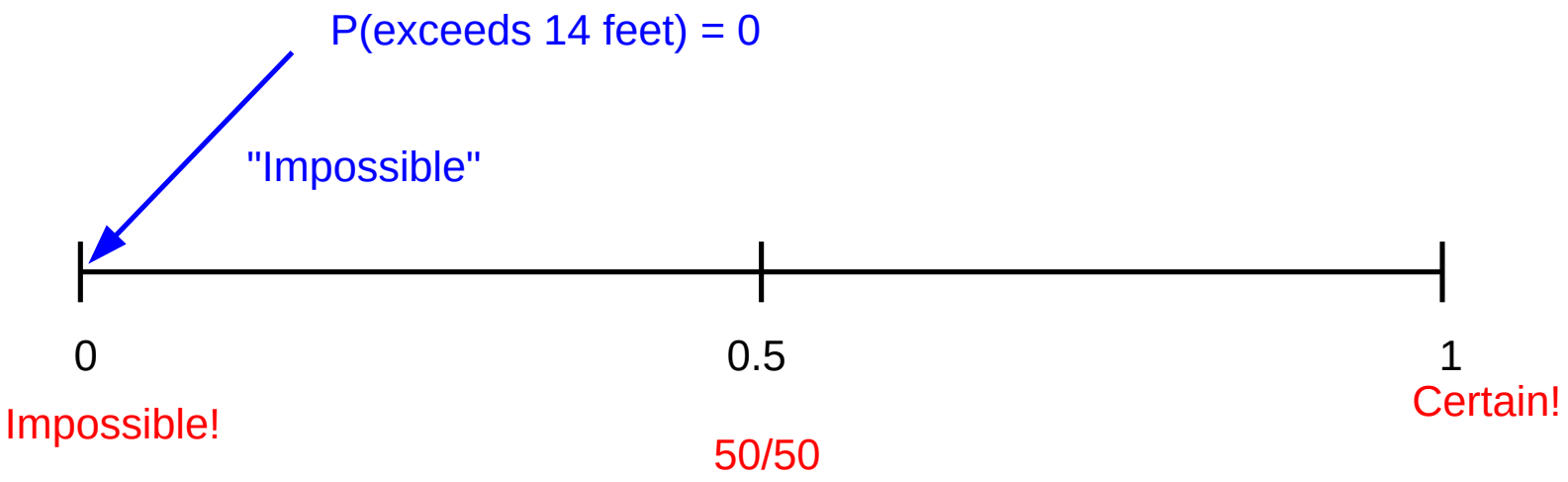

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What is wrong with the exceedance probability associated with a sea-surge of 14 feet?

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Probability models provide a way forward here.

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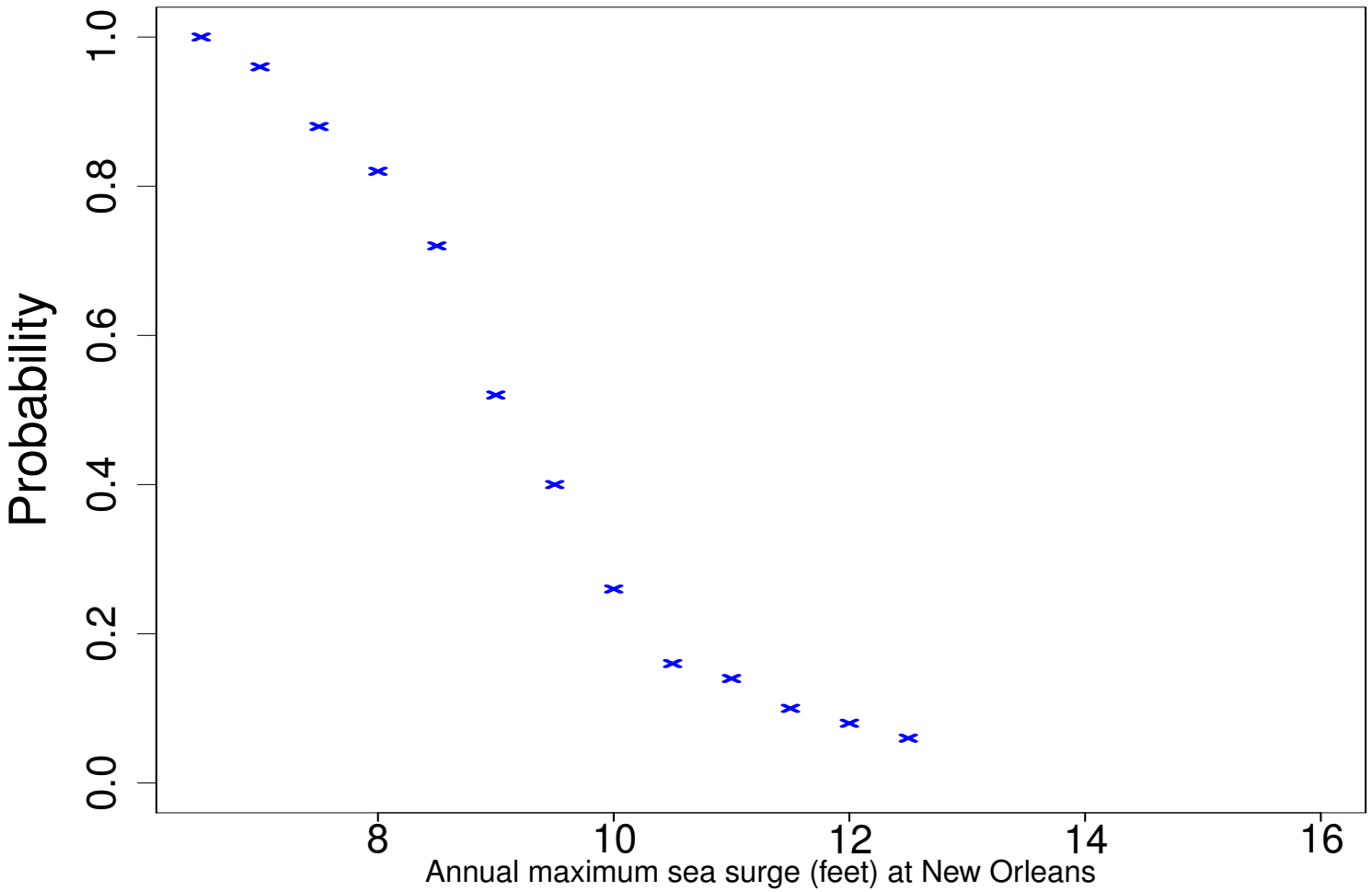
A probability model for extremes

Before we think about **probability models**, first of all let's consider exceedance probabilities for a range of values:

Sea–surge	No. of exceedances	Exceedance probability
6.5	50	$50/50 =$ **1.00**
7.0	48	$48/50 =$ **0.96**
7.5	44	$44/50 =$ **0.88**
8.0	41	$41/50 =$ **0.82**
8.5	36	$36/50 =$ **0.72**
9.0	26	$26/50 =$ **0.52**
9.5	20	$20/50 =$ **0.40**
10.0	13	$13/50 =$ **0.26**
10.5	8	$8/50 =$ **0.16**
11.0	7	$7/50 =$ **0.14**
11.5	5	$5/50 =$ **0.10**
12.0	4	$4/50 =$ **0.08**
12.5	3	$3/50 =$ **0.06**

A probability model for extremes



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Between 1920 and the mid–1950s, some eminent mathematicians developed probability models for extremes.

The aim was to provide a **mathematical formula** that could "predict" the exceedance probabilities of real–life extremes, as in the table we've just drawn up for the New Orleans data.

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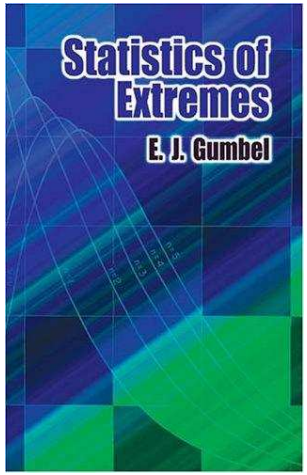

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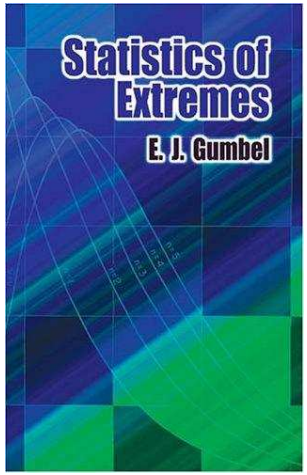

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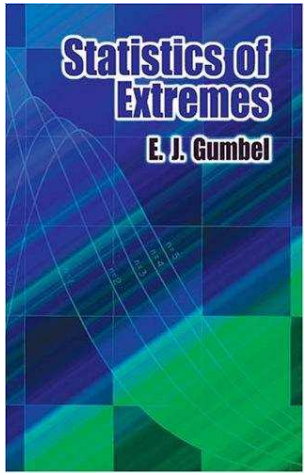

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The so–called **Gumbel model** for the exceedance probabilities of extremes is given by the following formula:

$$\boldsymbol{P}(X > x) = 1 - \exp\left[-\exp\left\{-\left(\frac{x-\mu}{\sigma}\right)\right\}\right],$$

where

- μ is the "**location**" parameter
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Using techniques students learn about at University, we can use the data to estimate the values of the location and scale parameters.

Doing so, gives

$$\mu = 8.536 \qquad \text{and} \qquad \sigma = 1.2$$

for our dataset. Plugging these into Gumbel's formula gives

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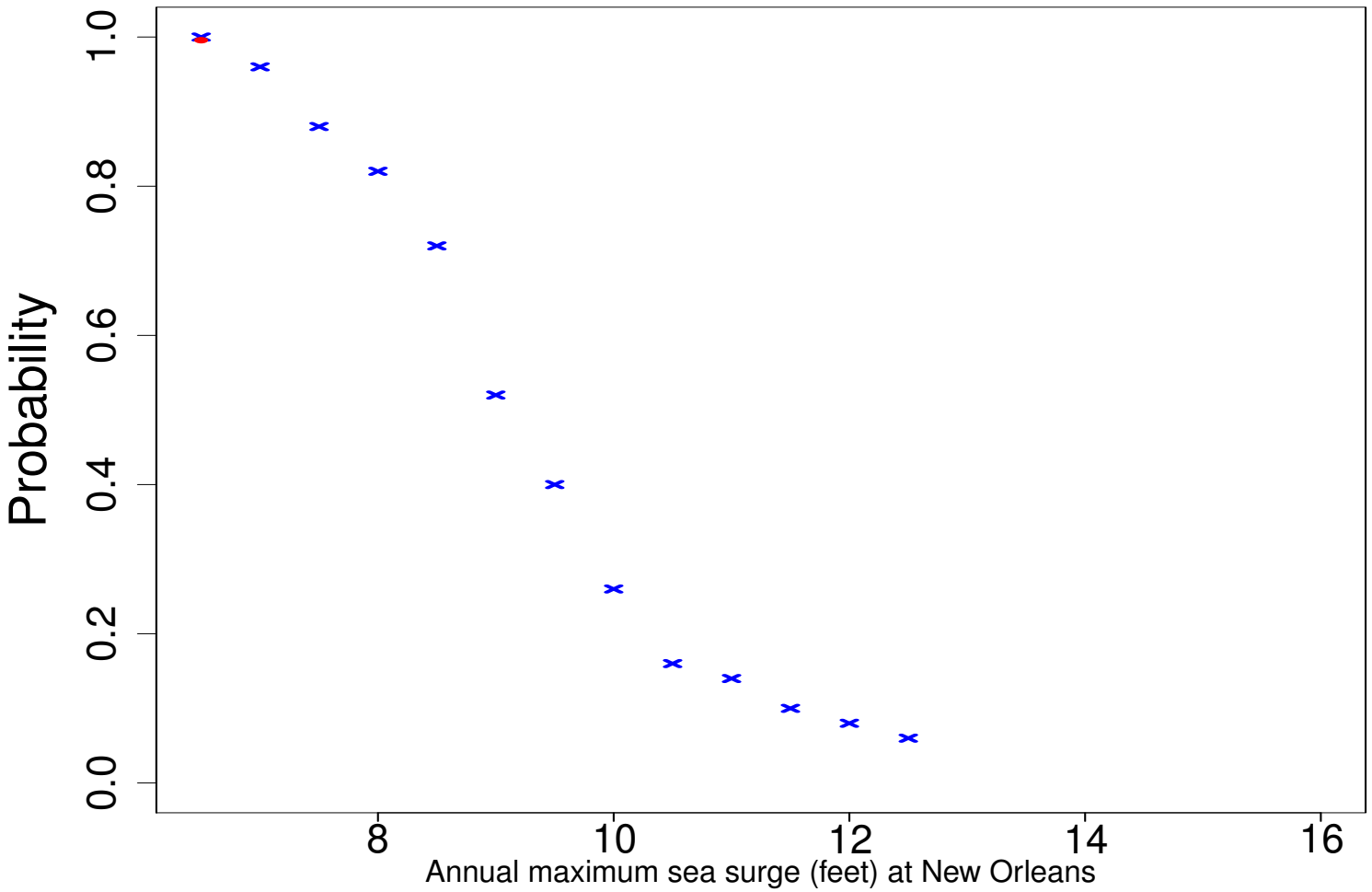

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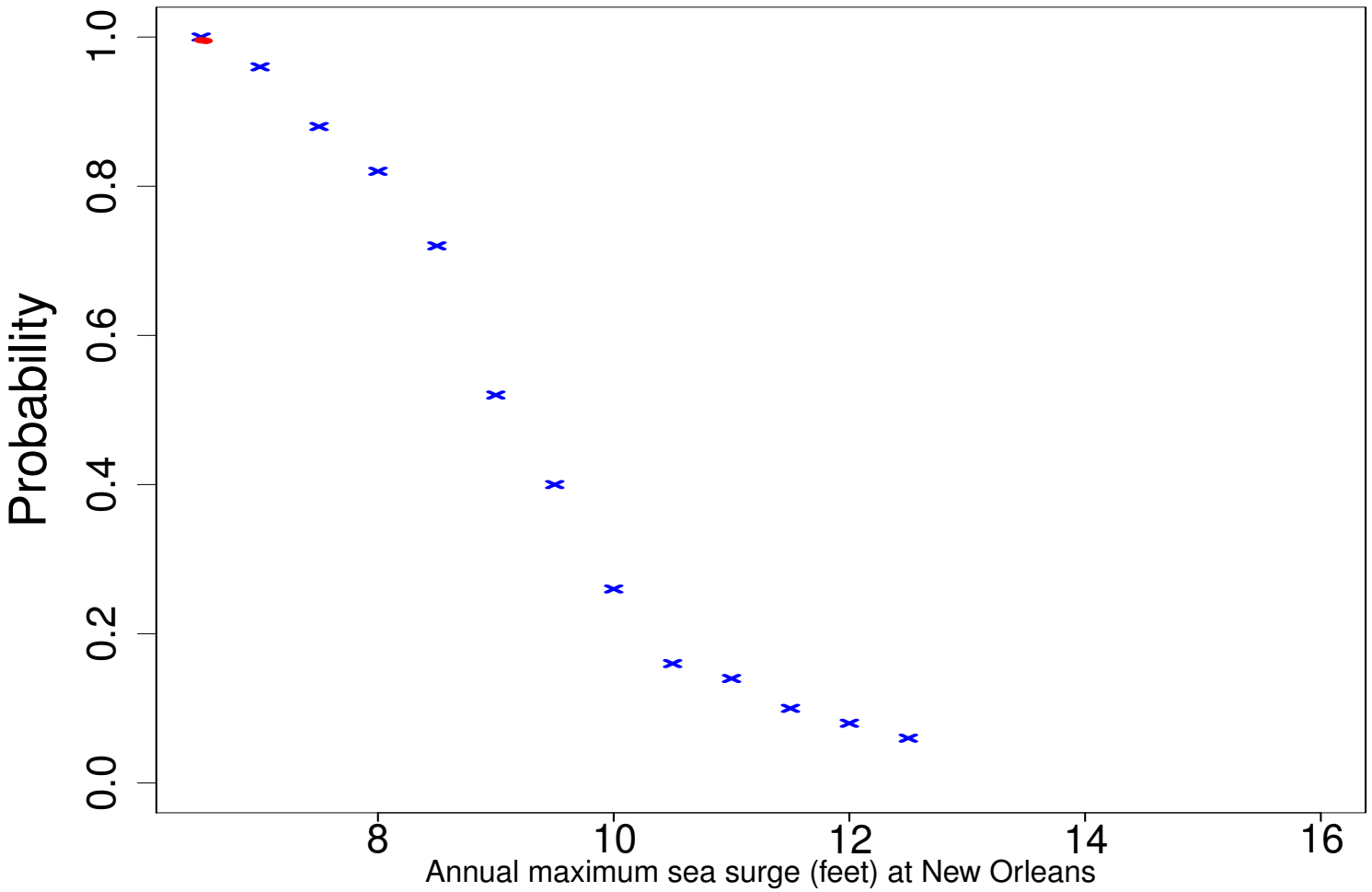

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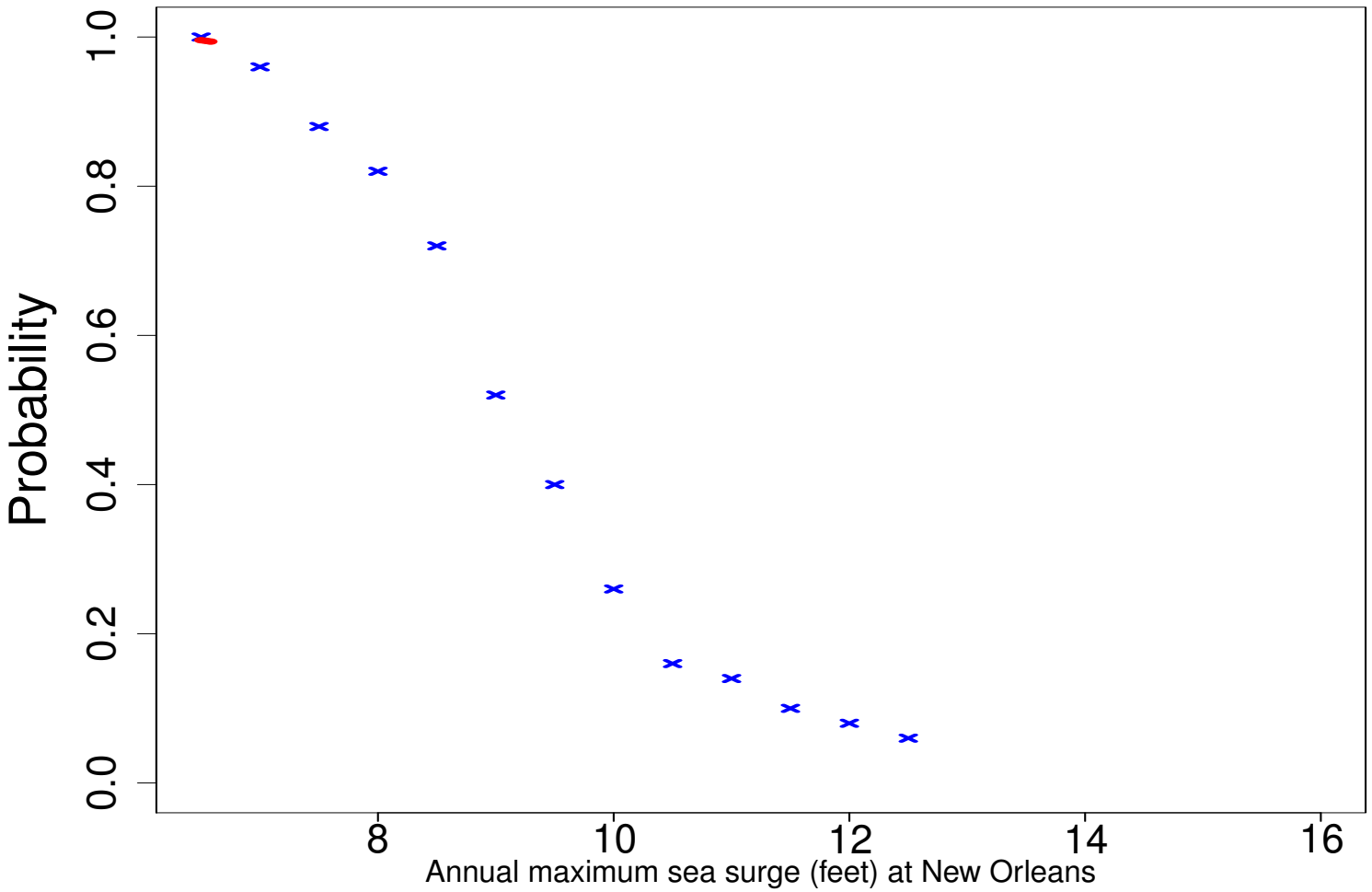

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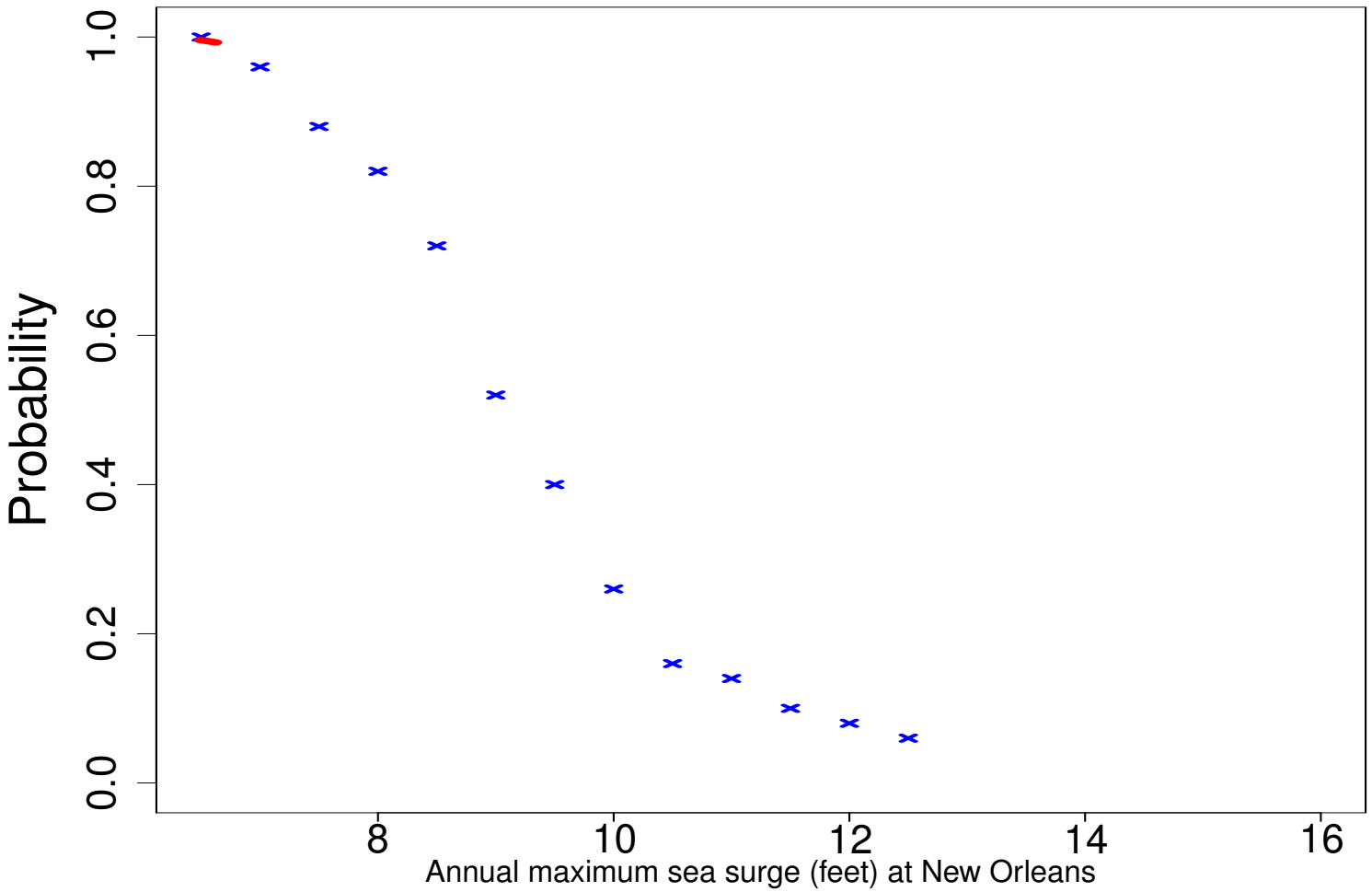

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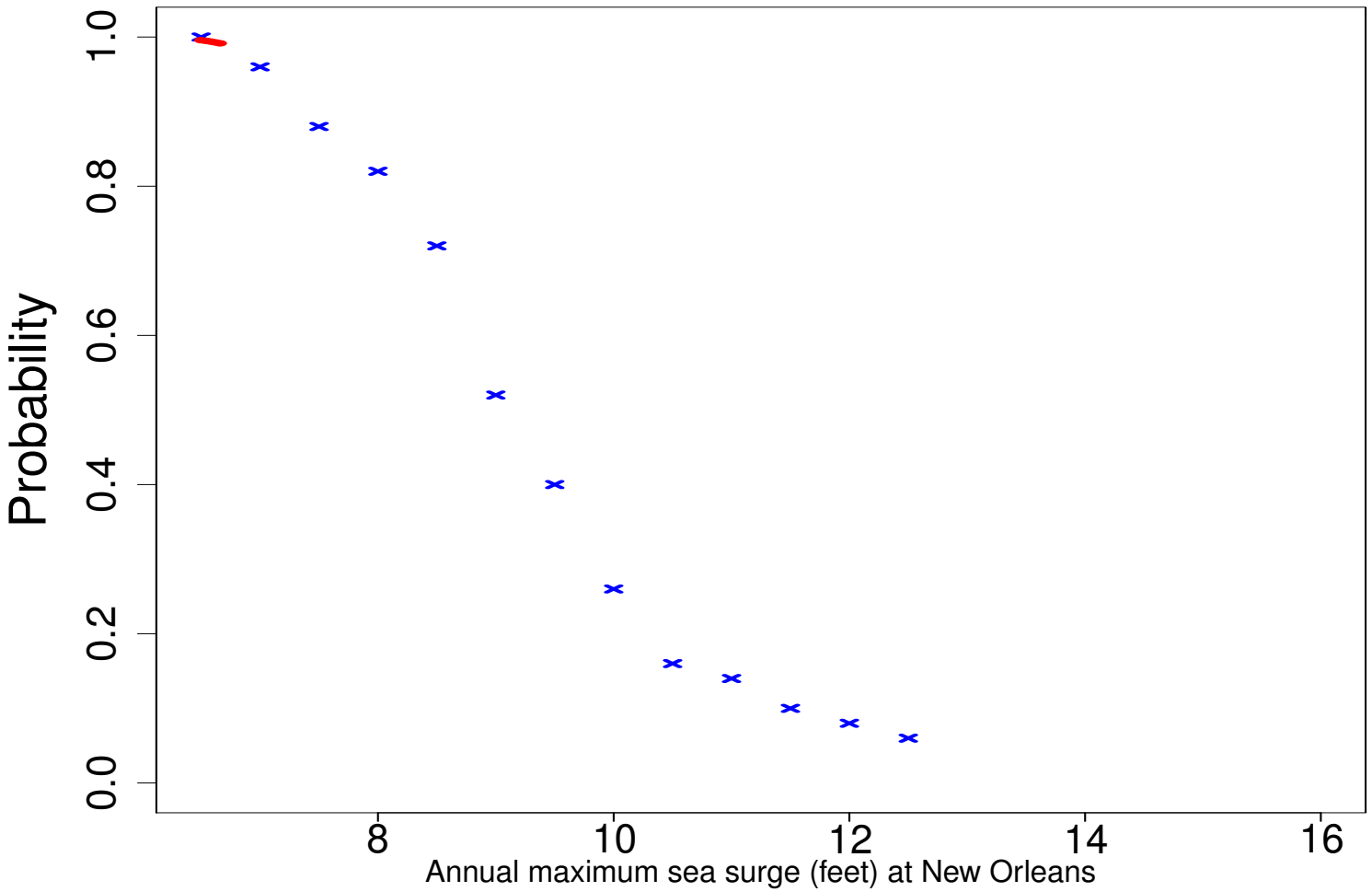

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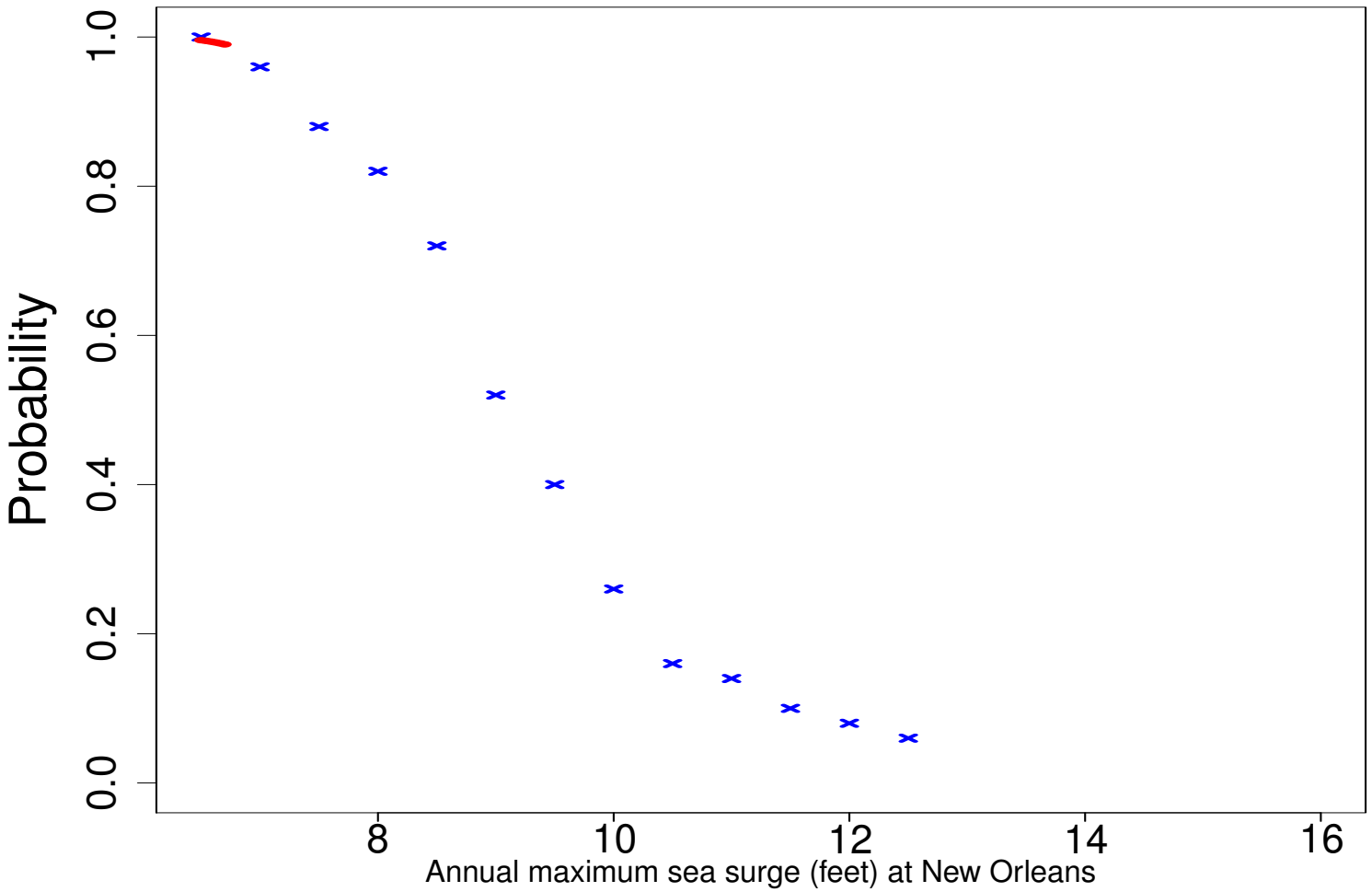

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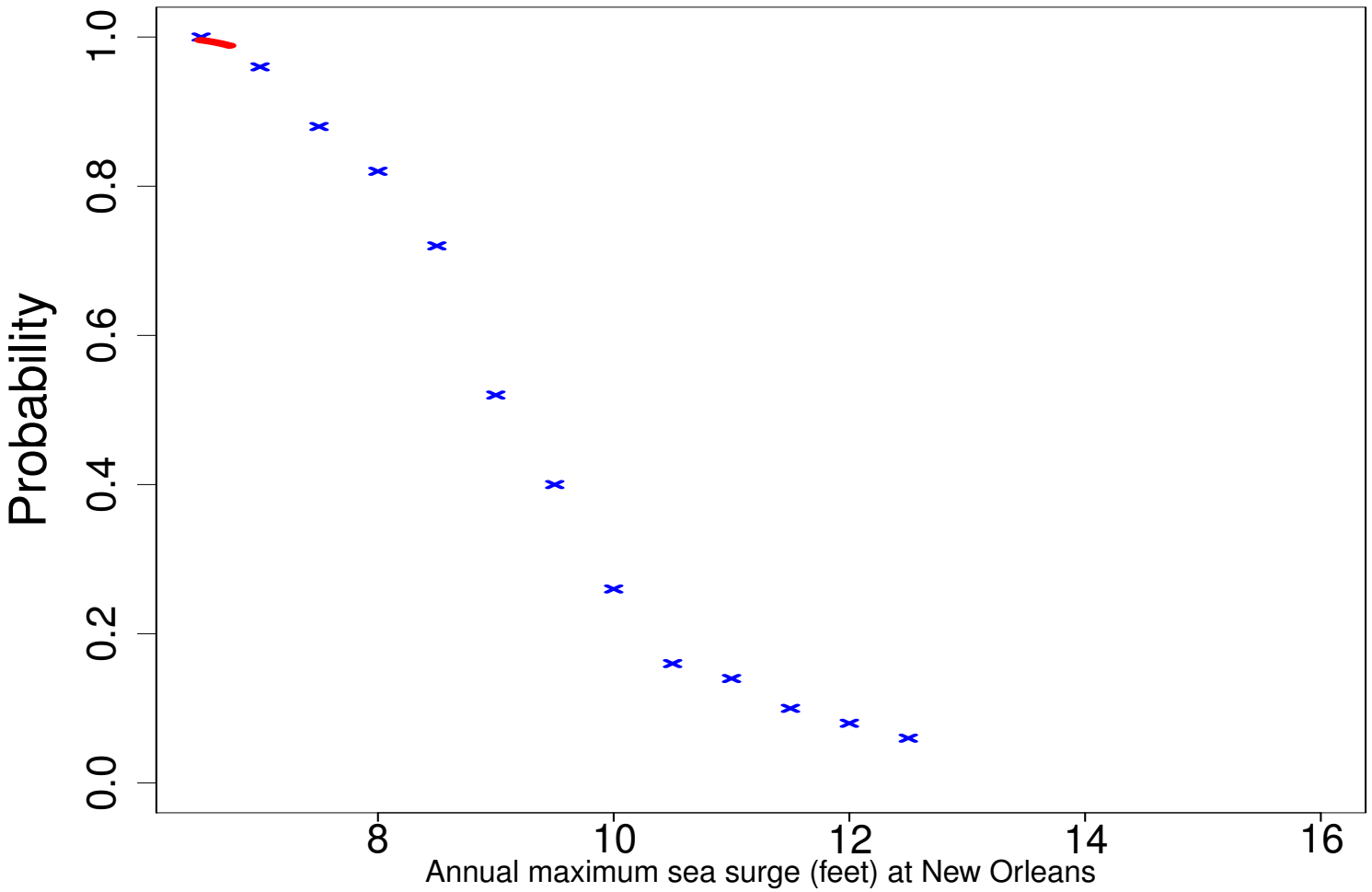

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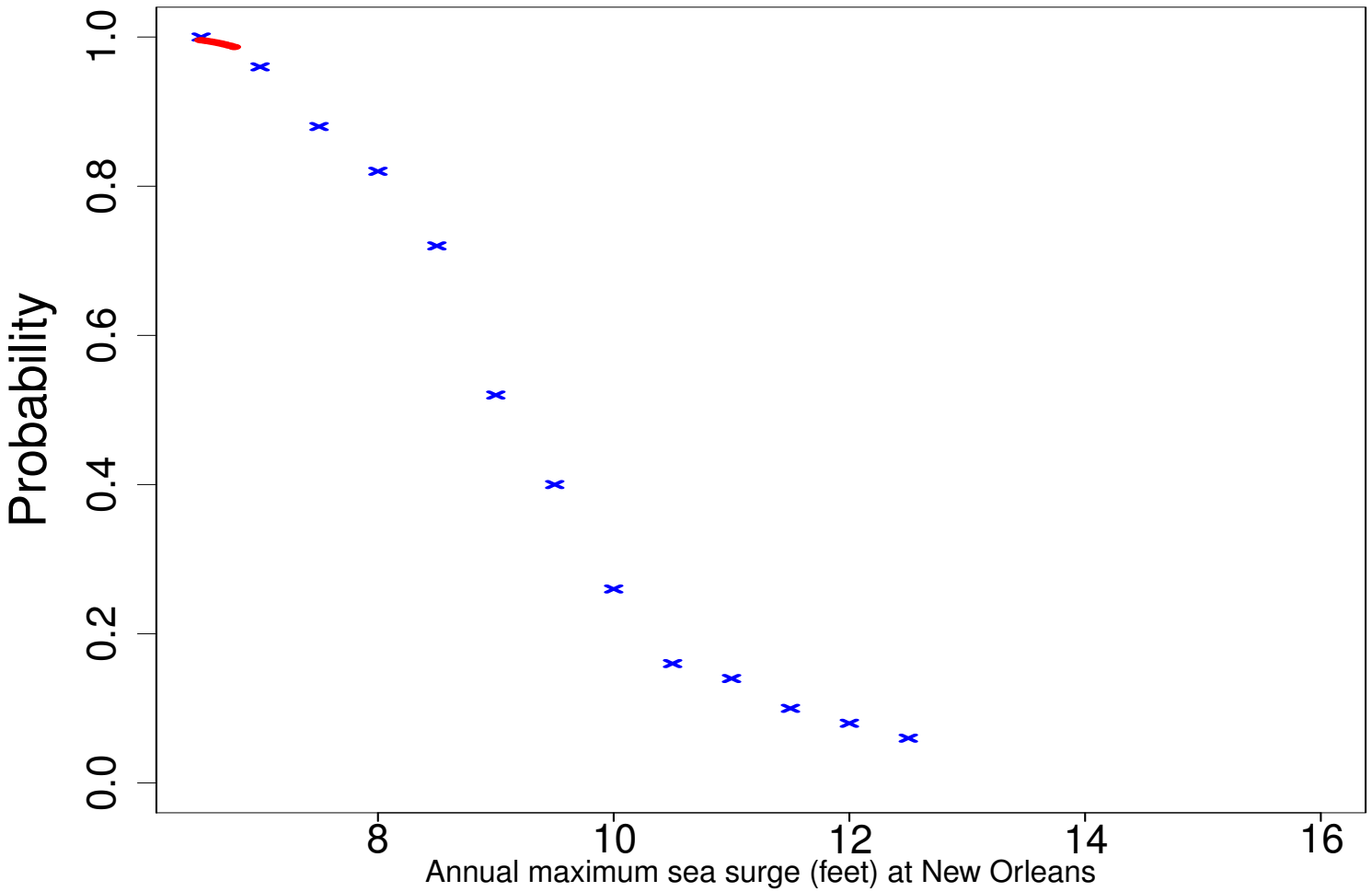

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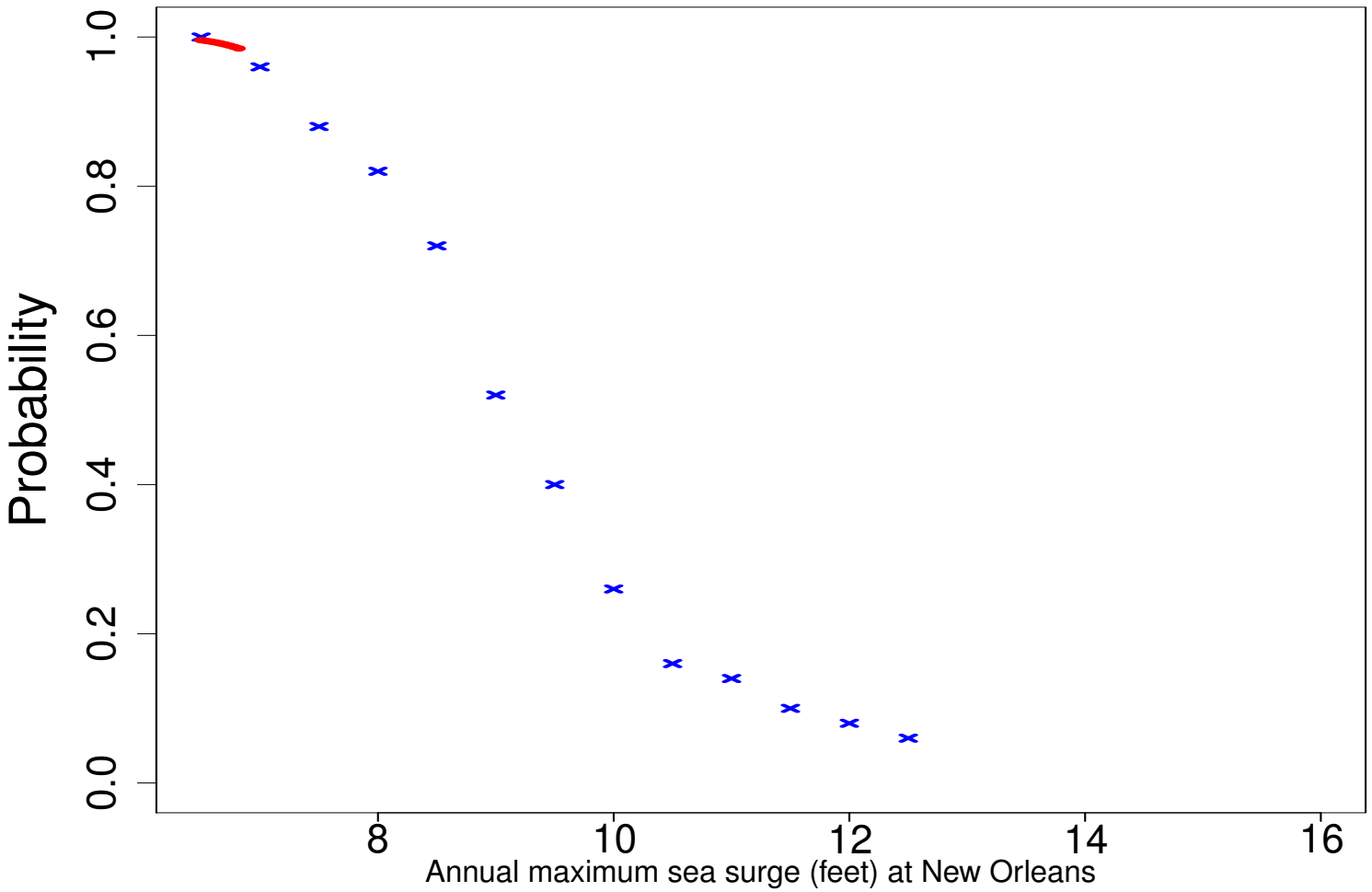

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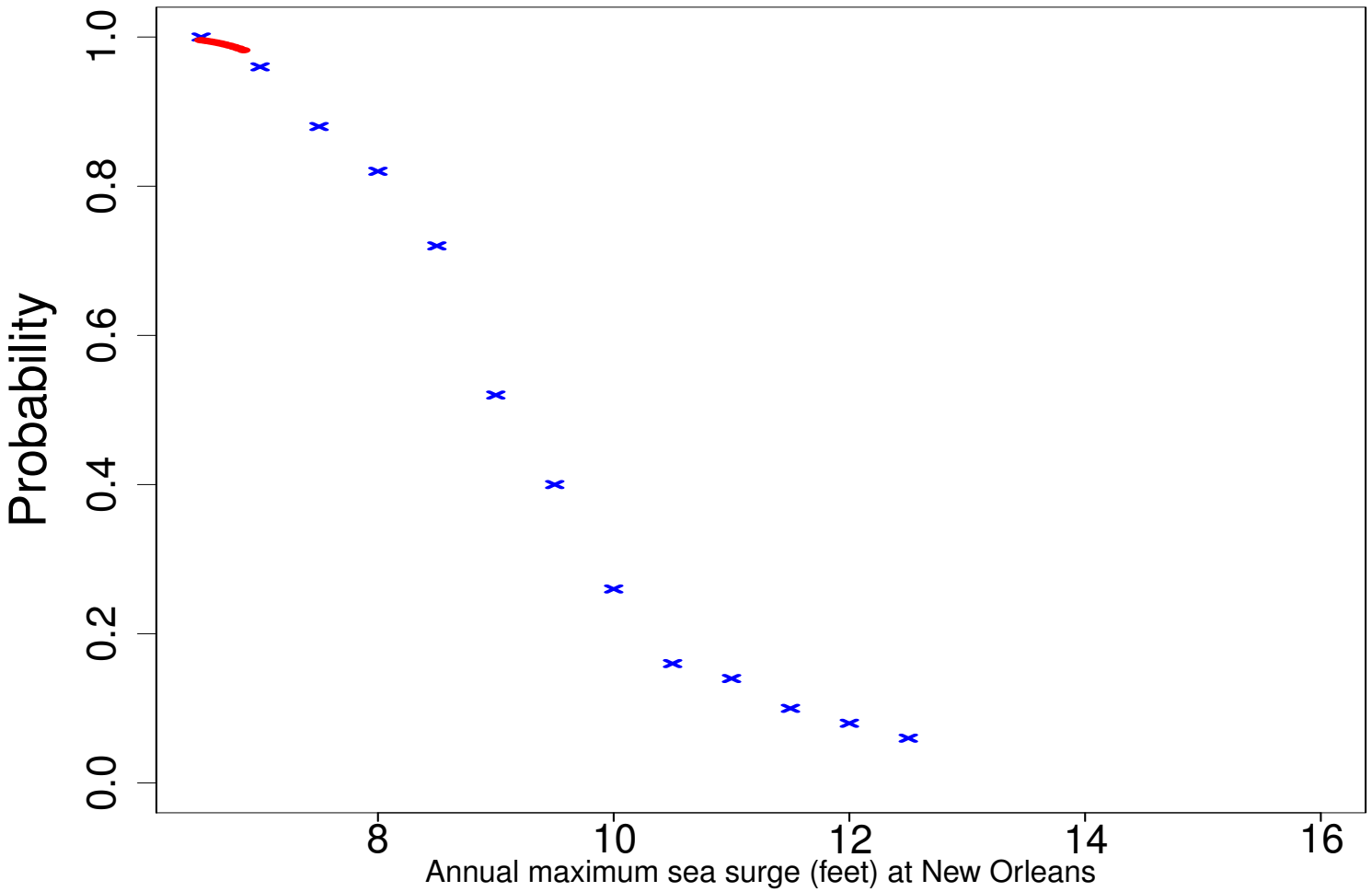

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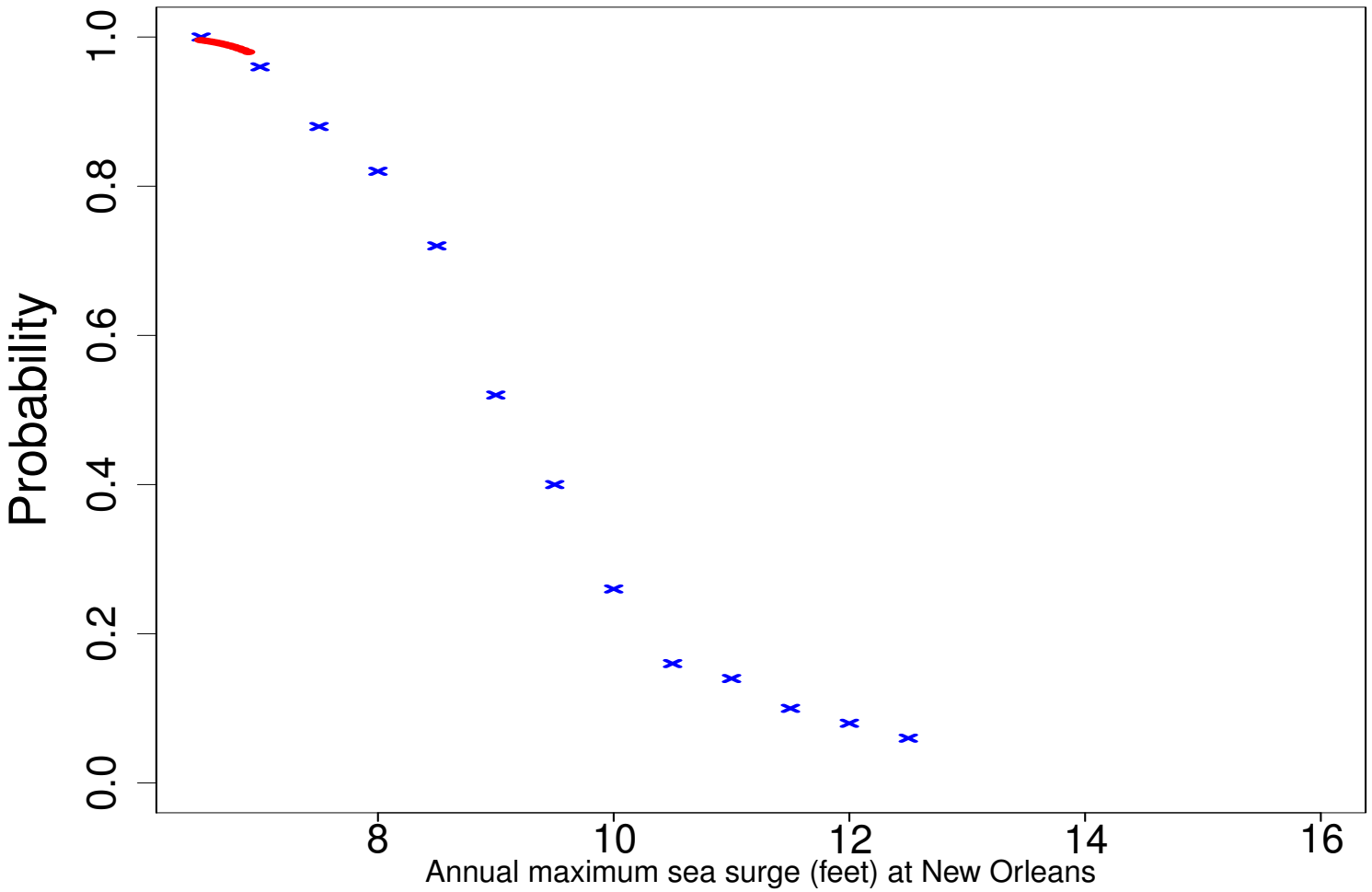

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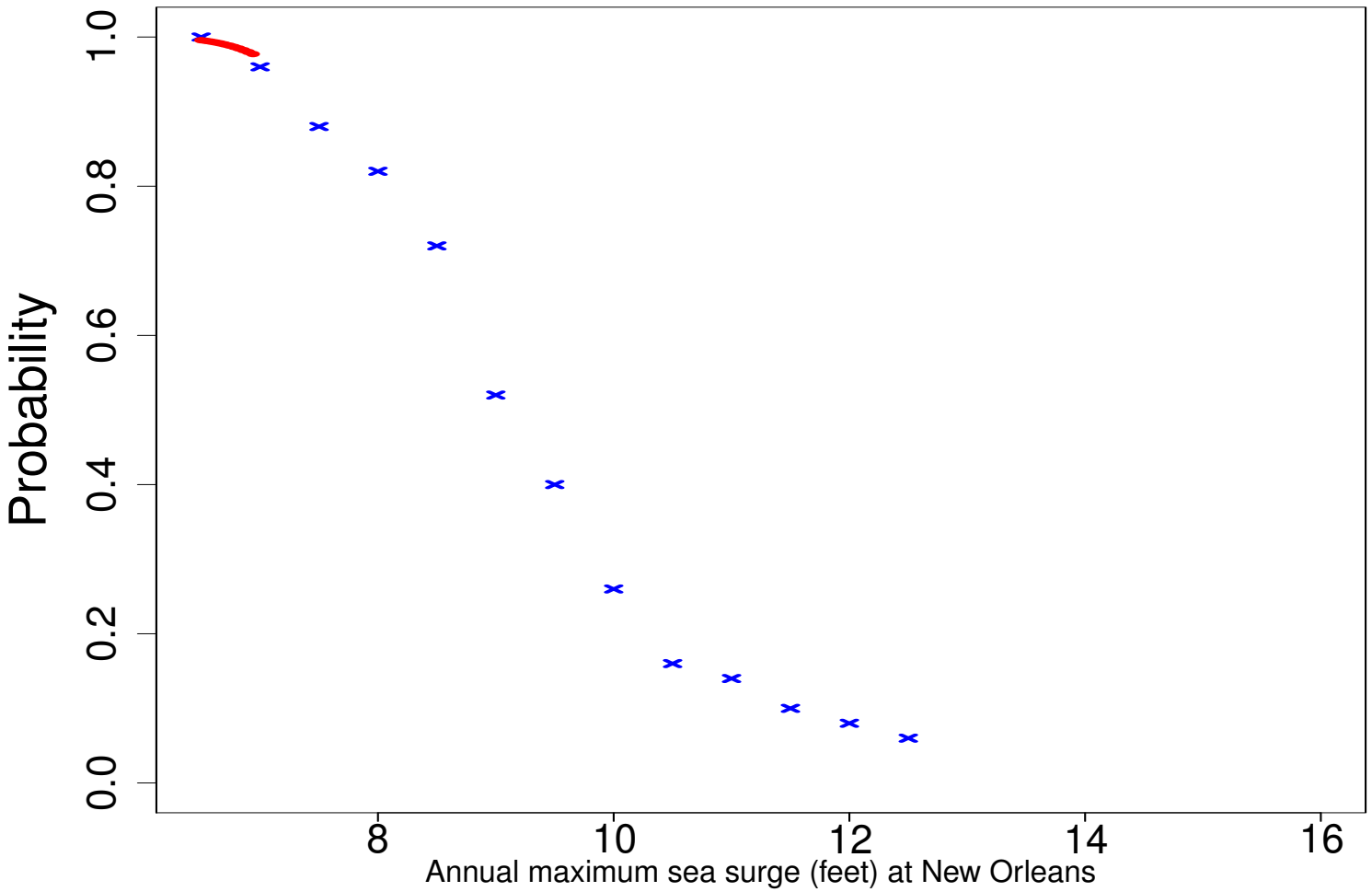

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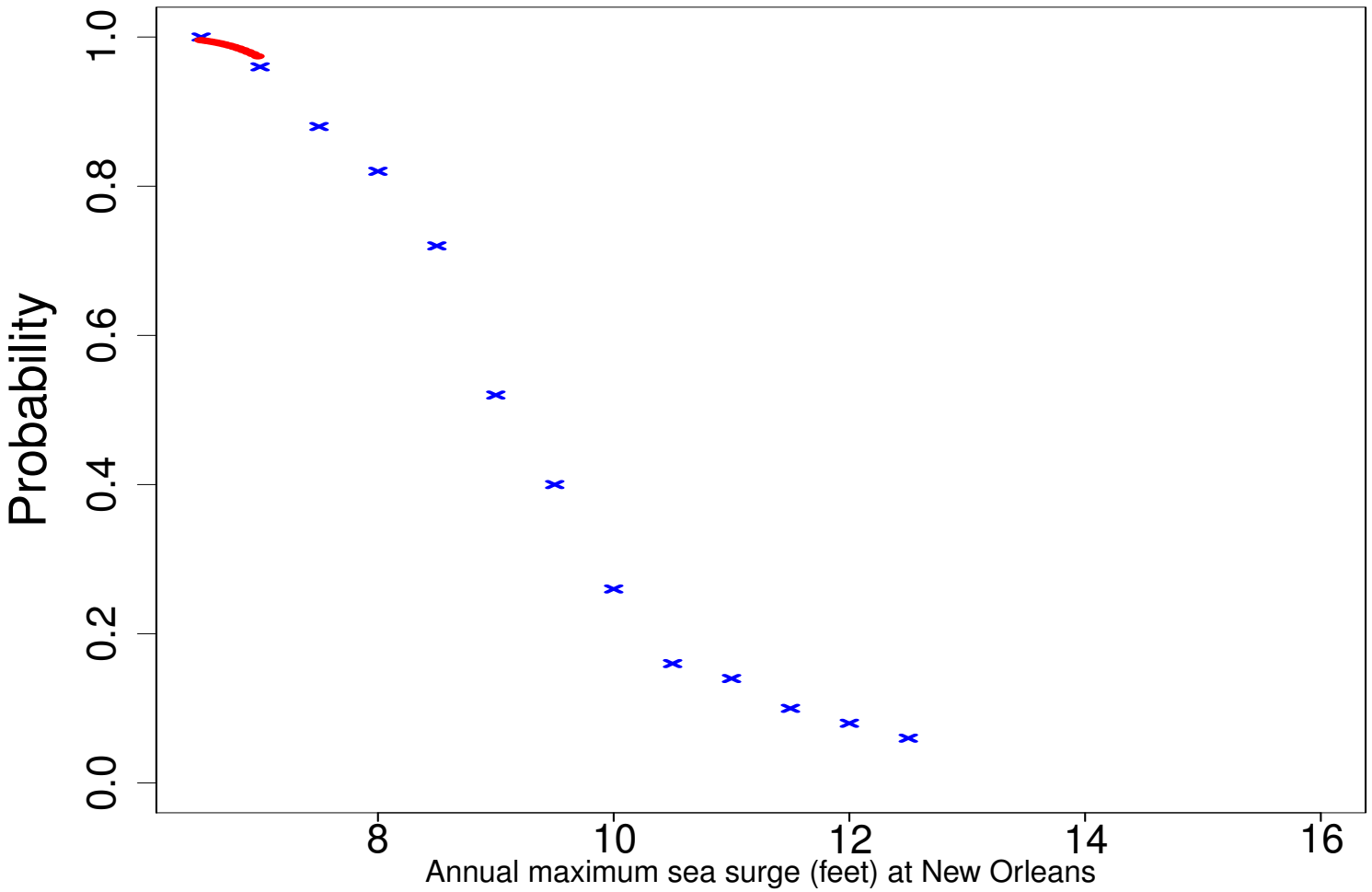

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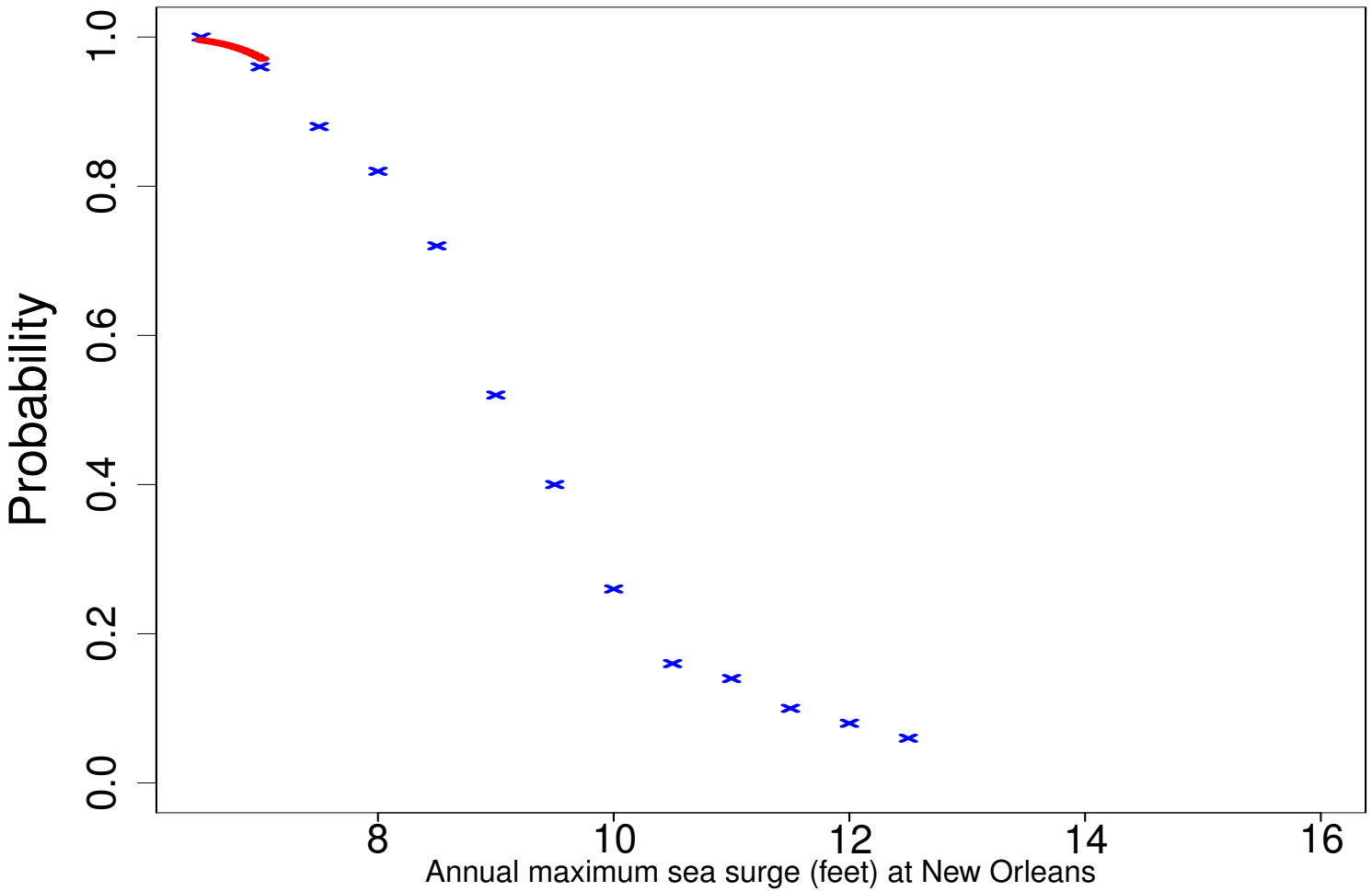

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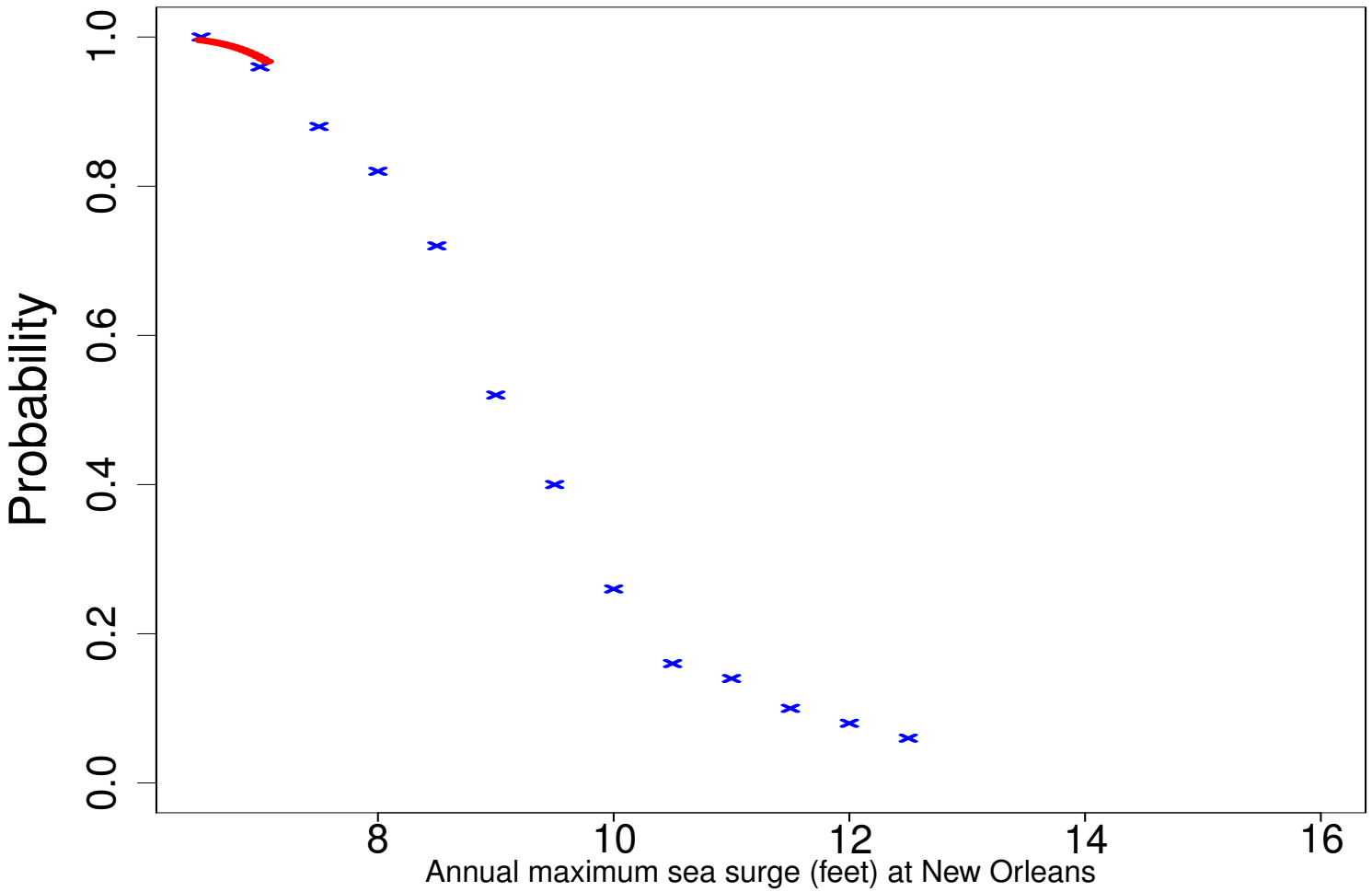

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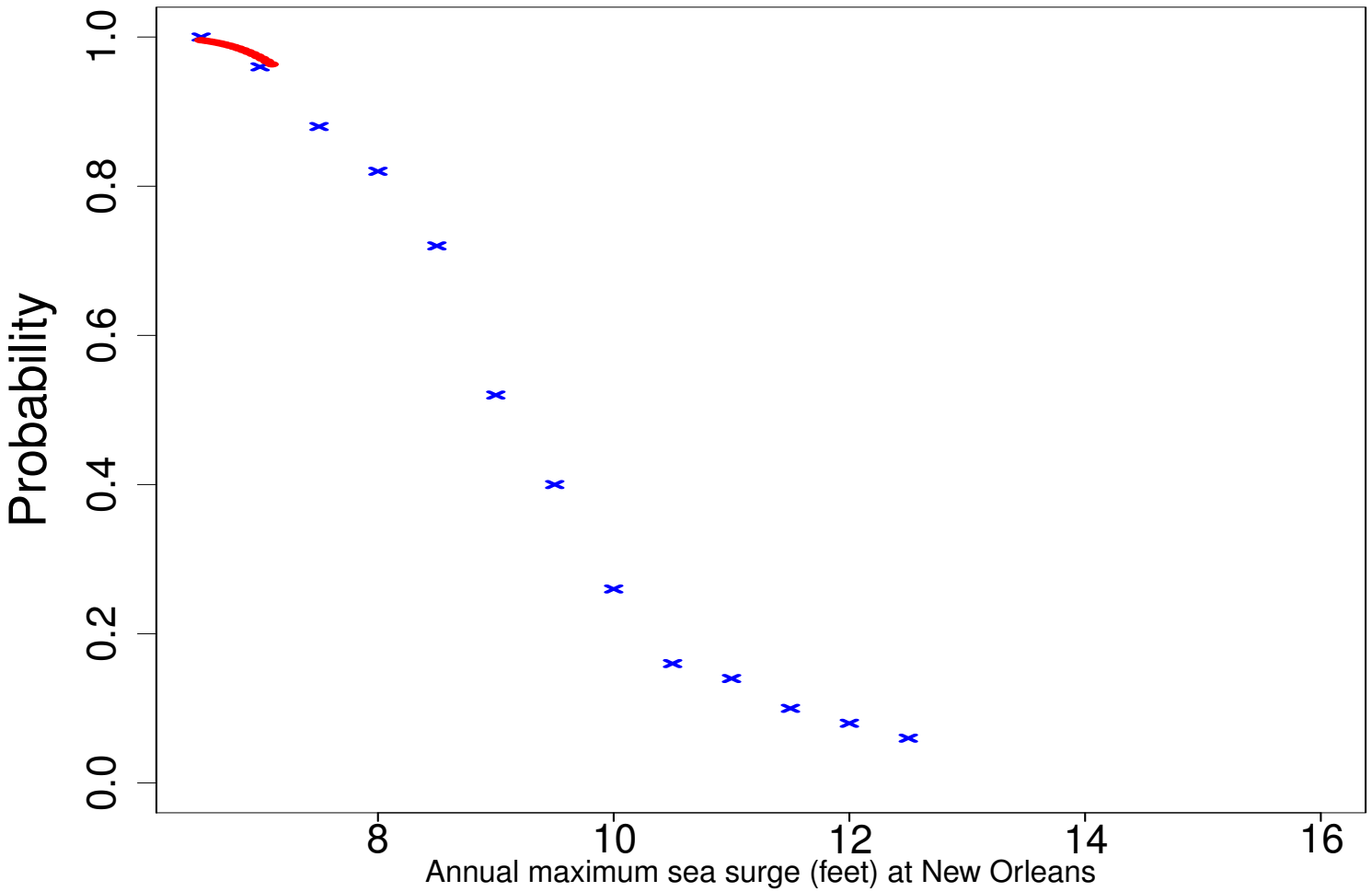

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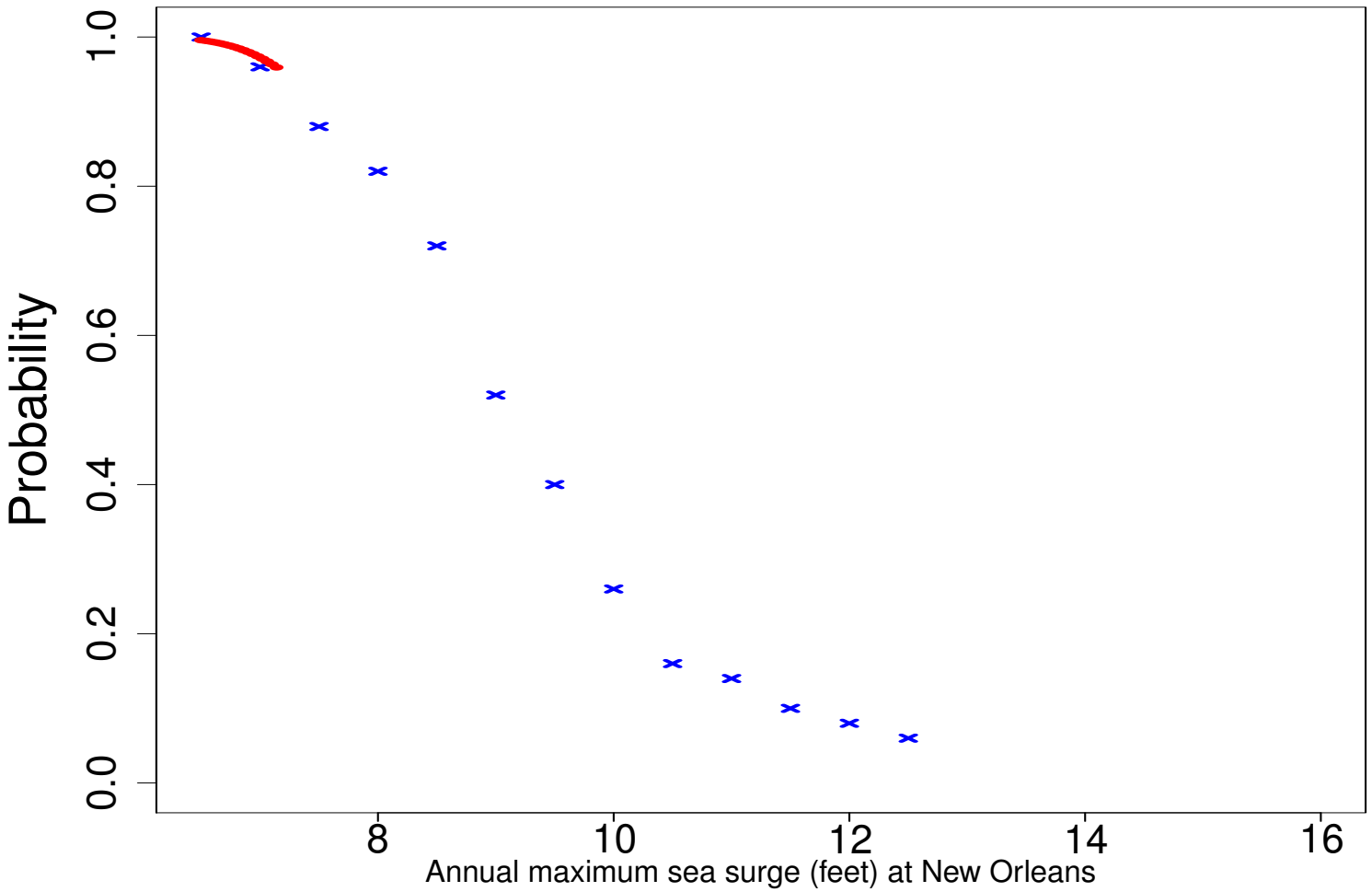

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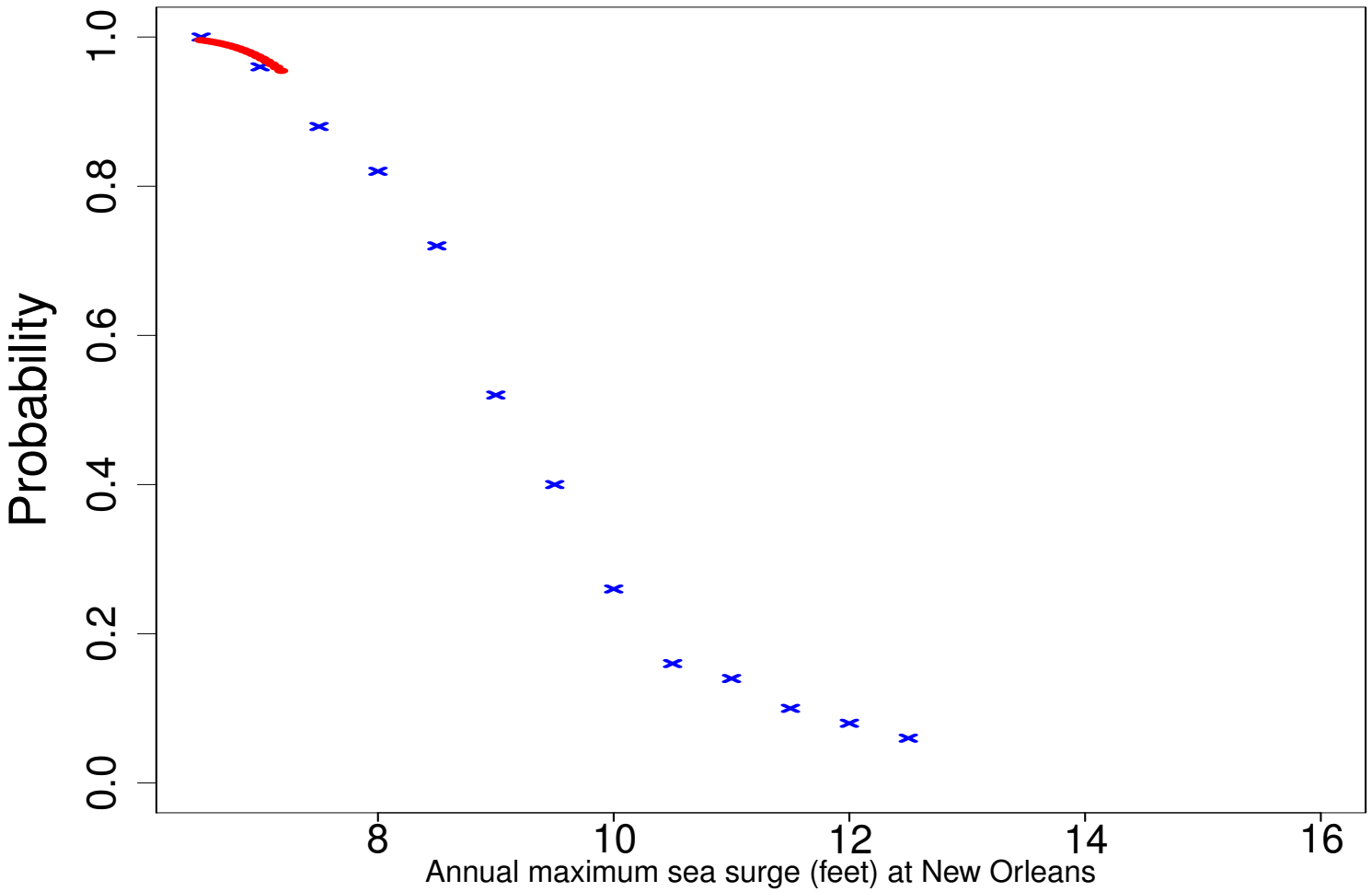

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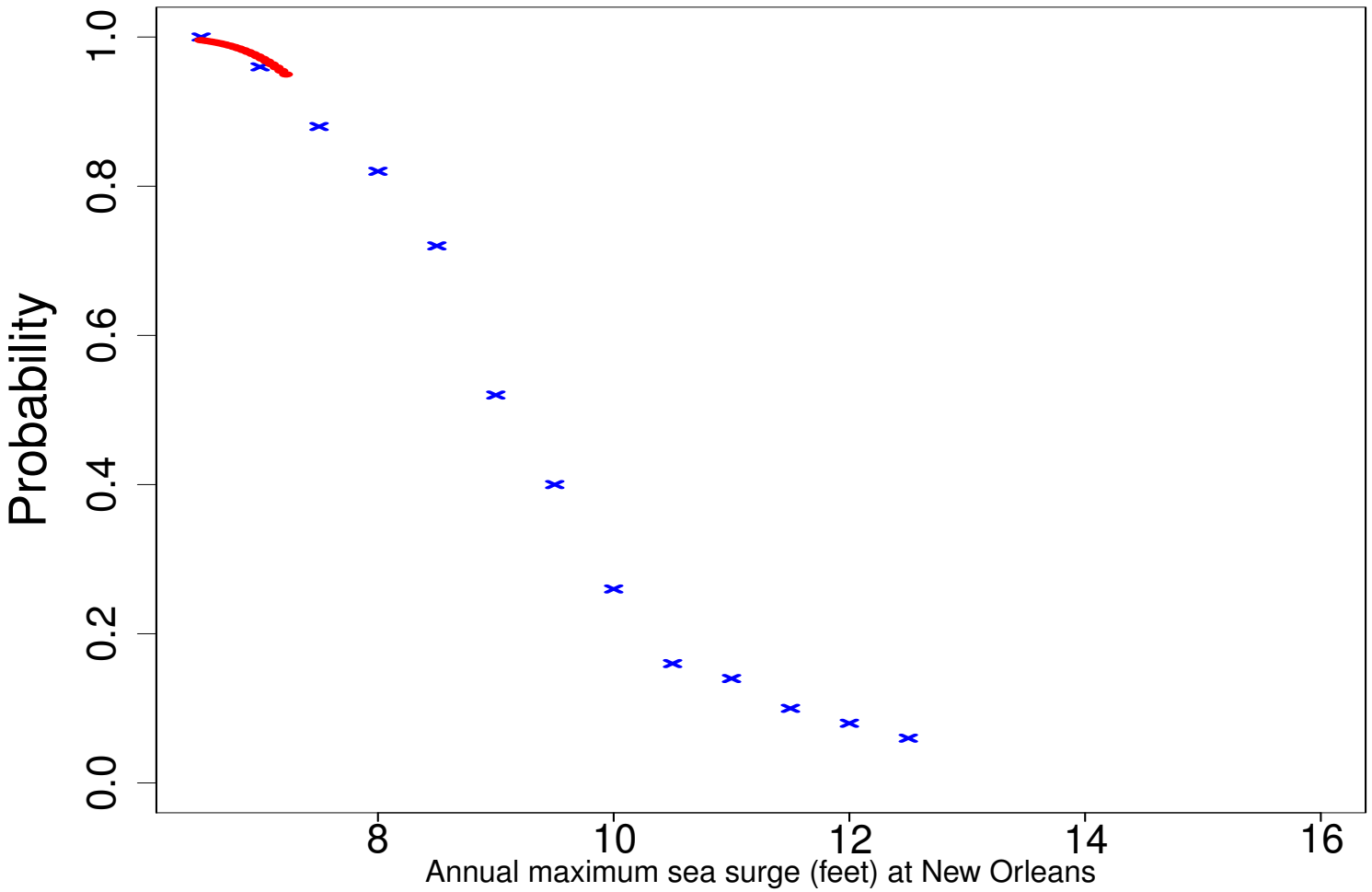

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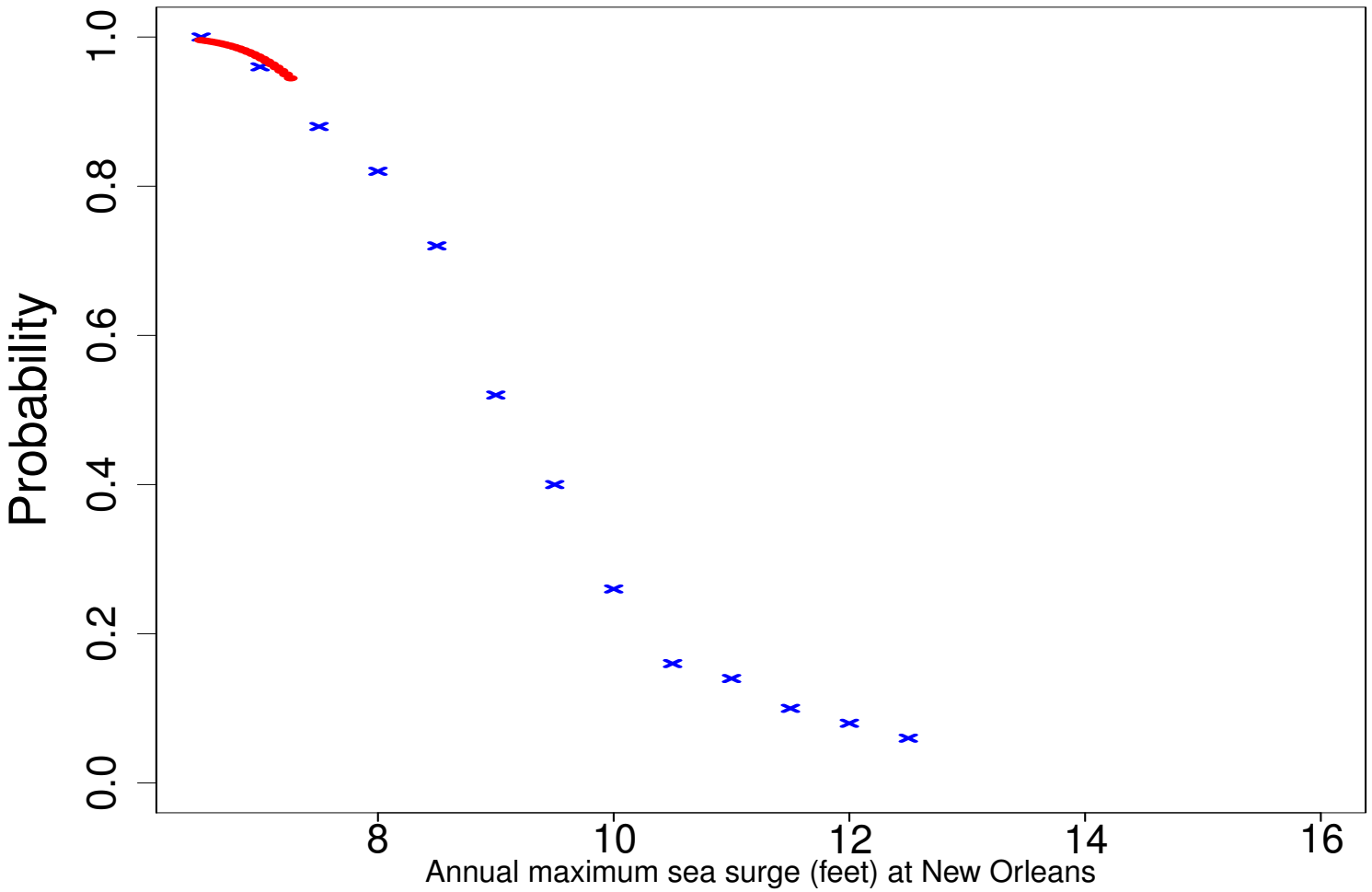

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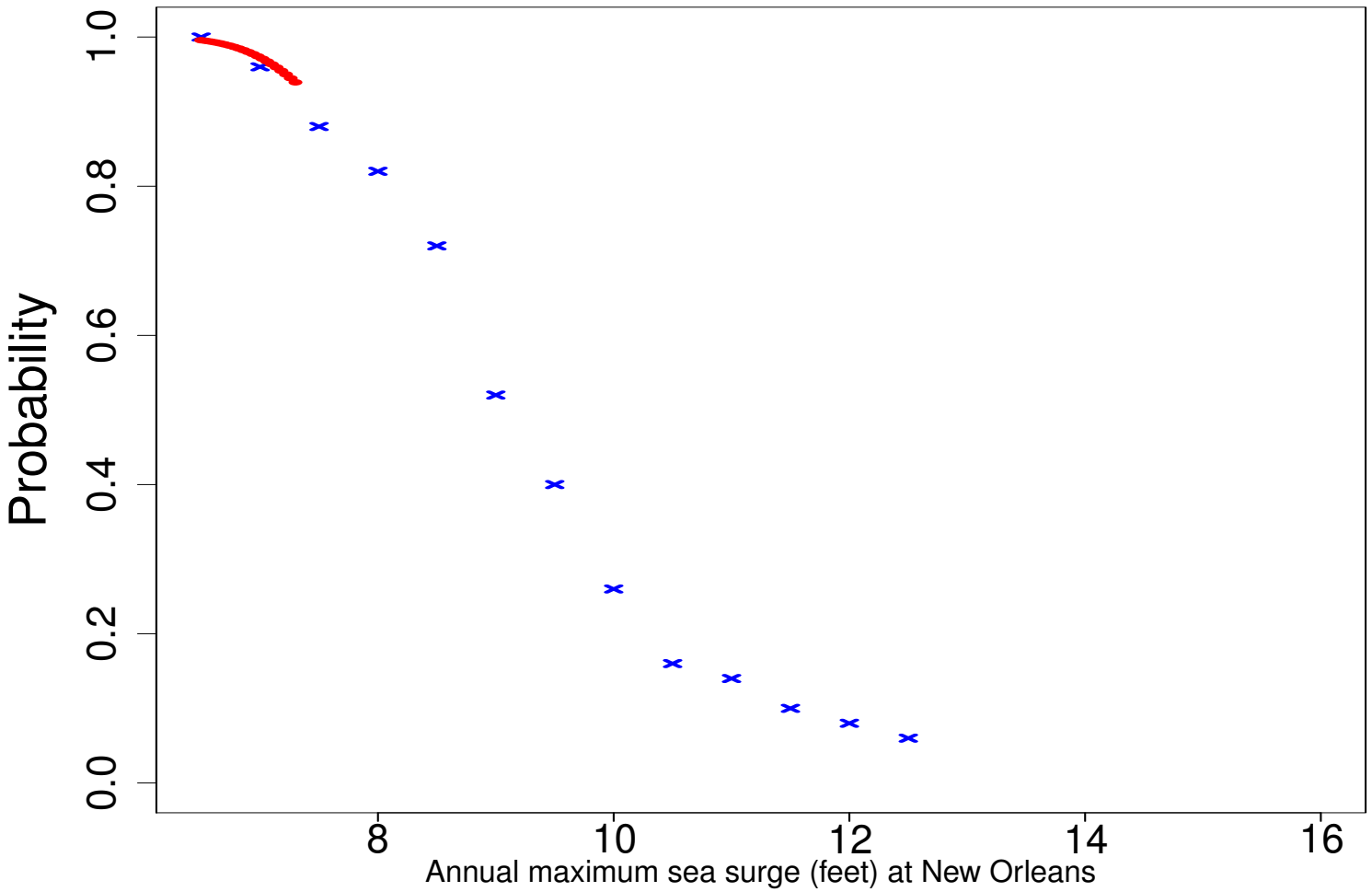

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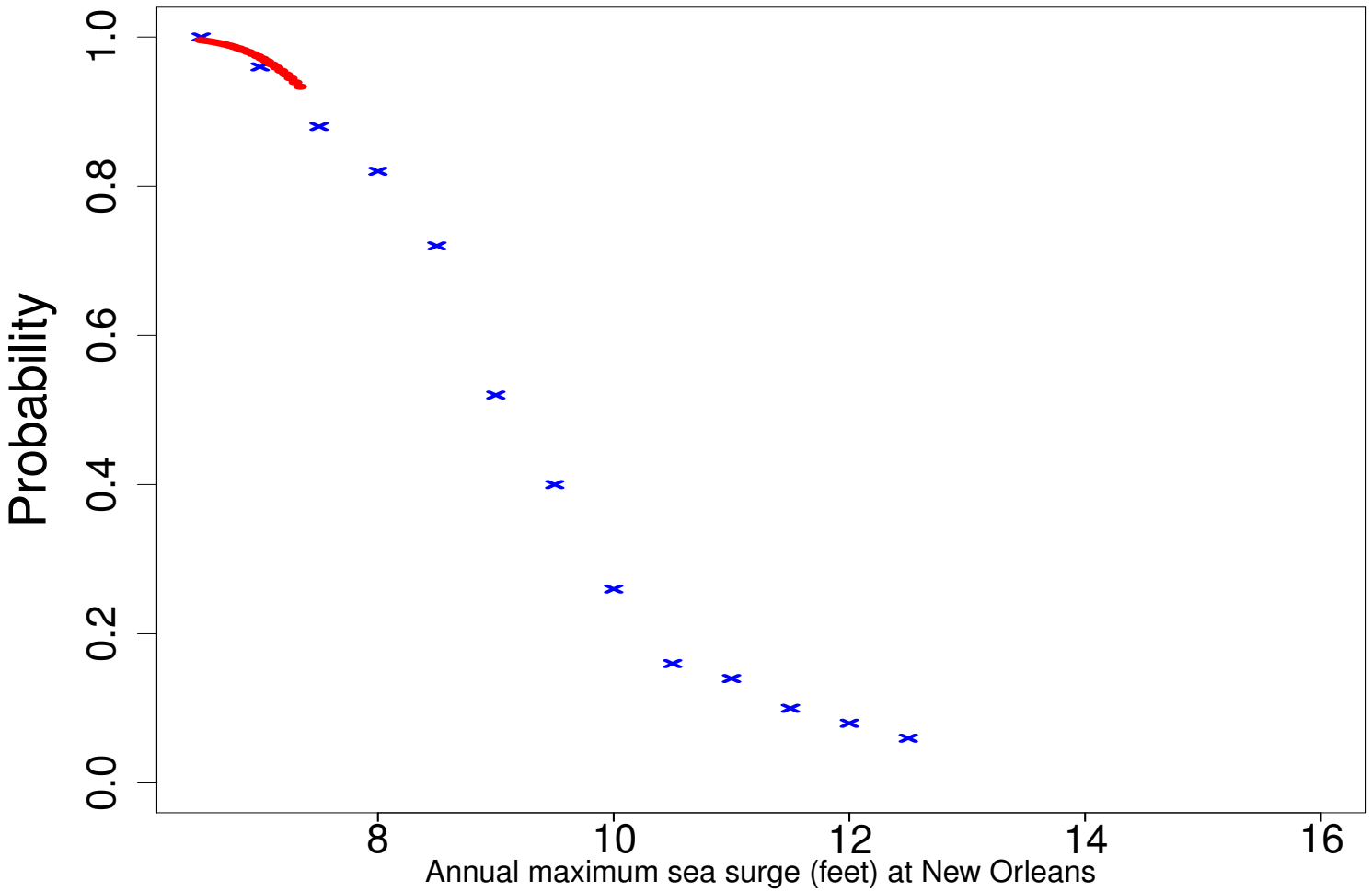

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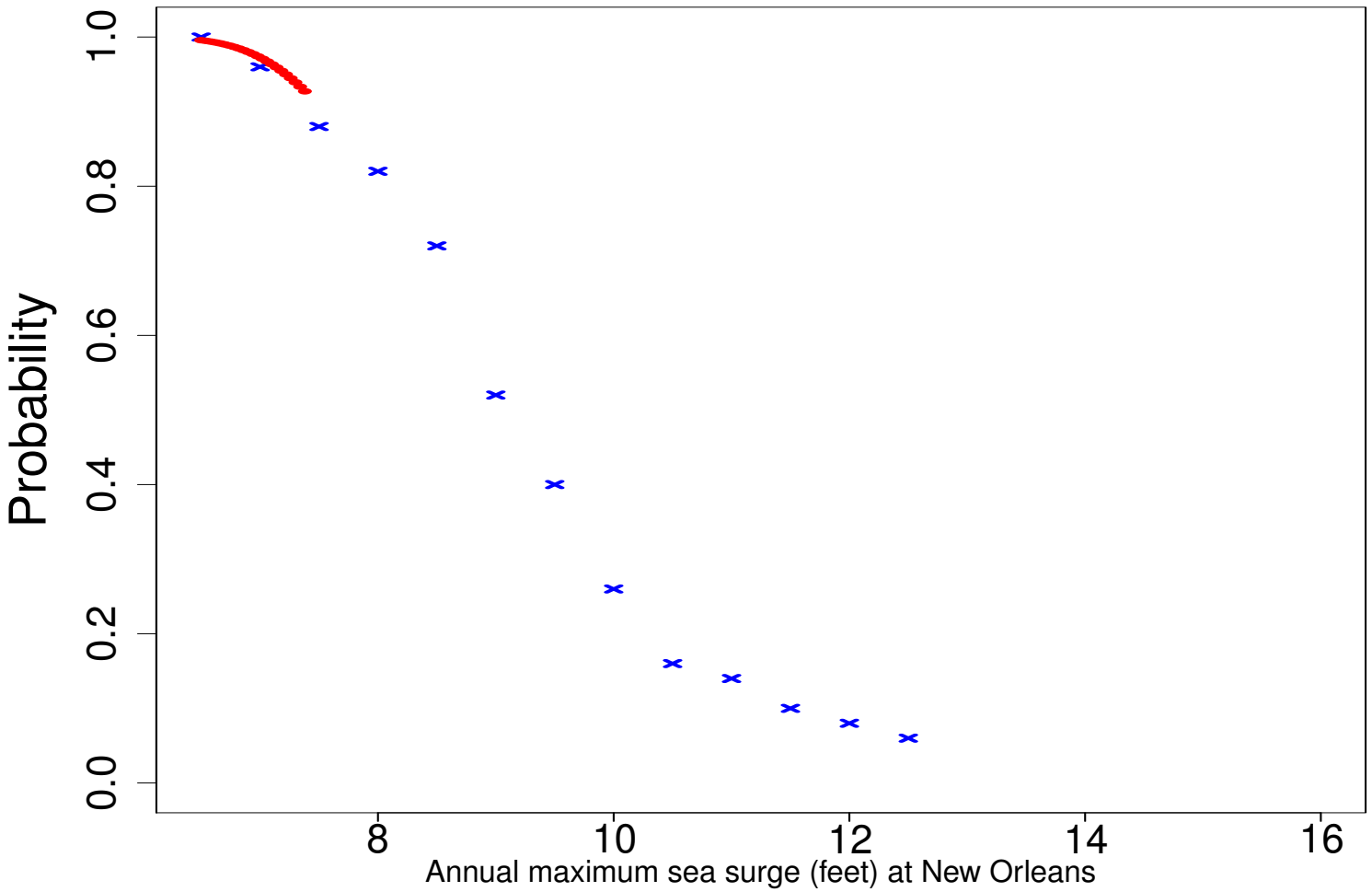

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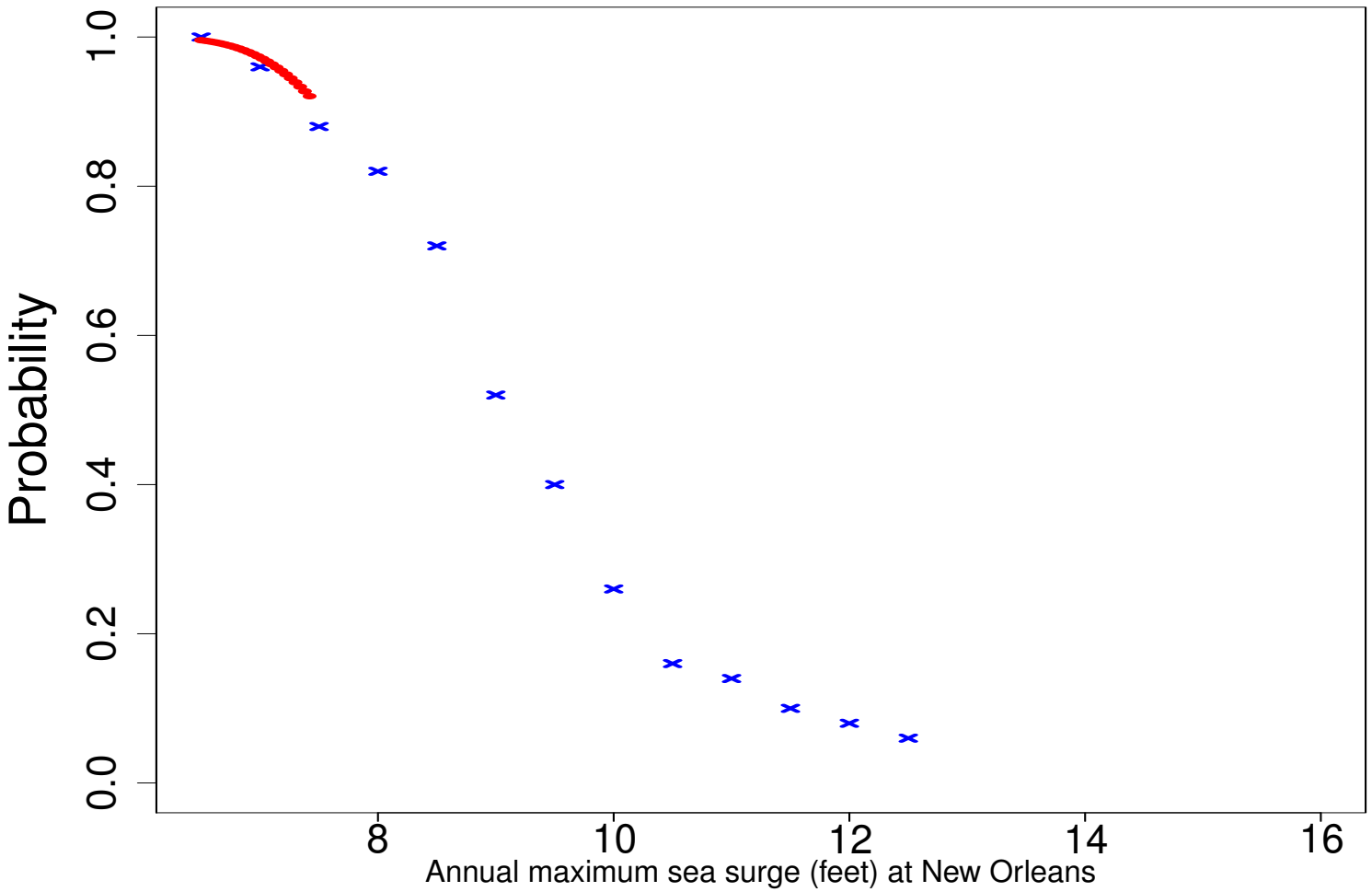

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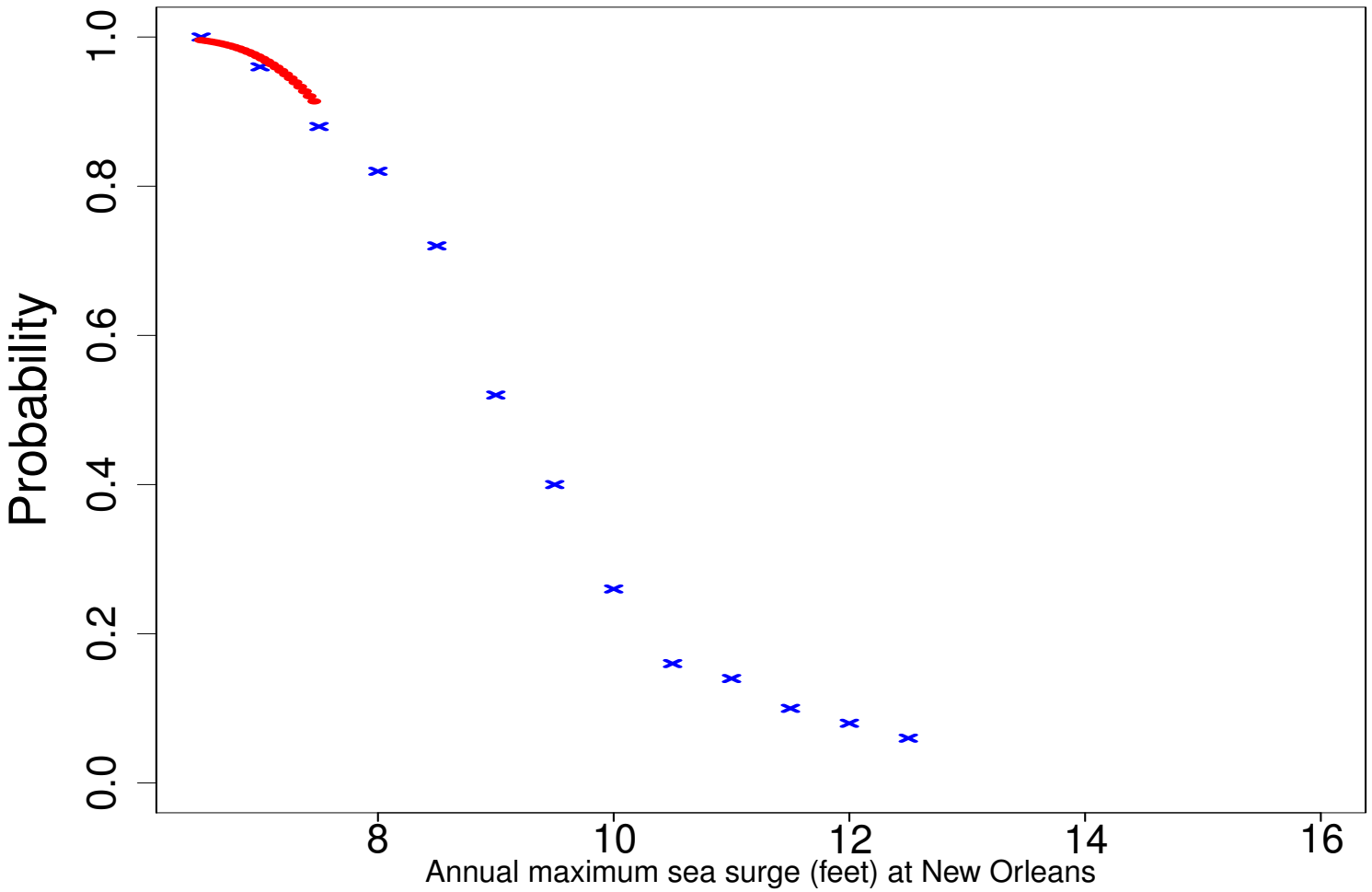

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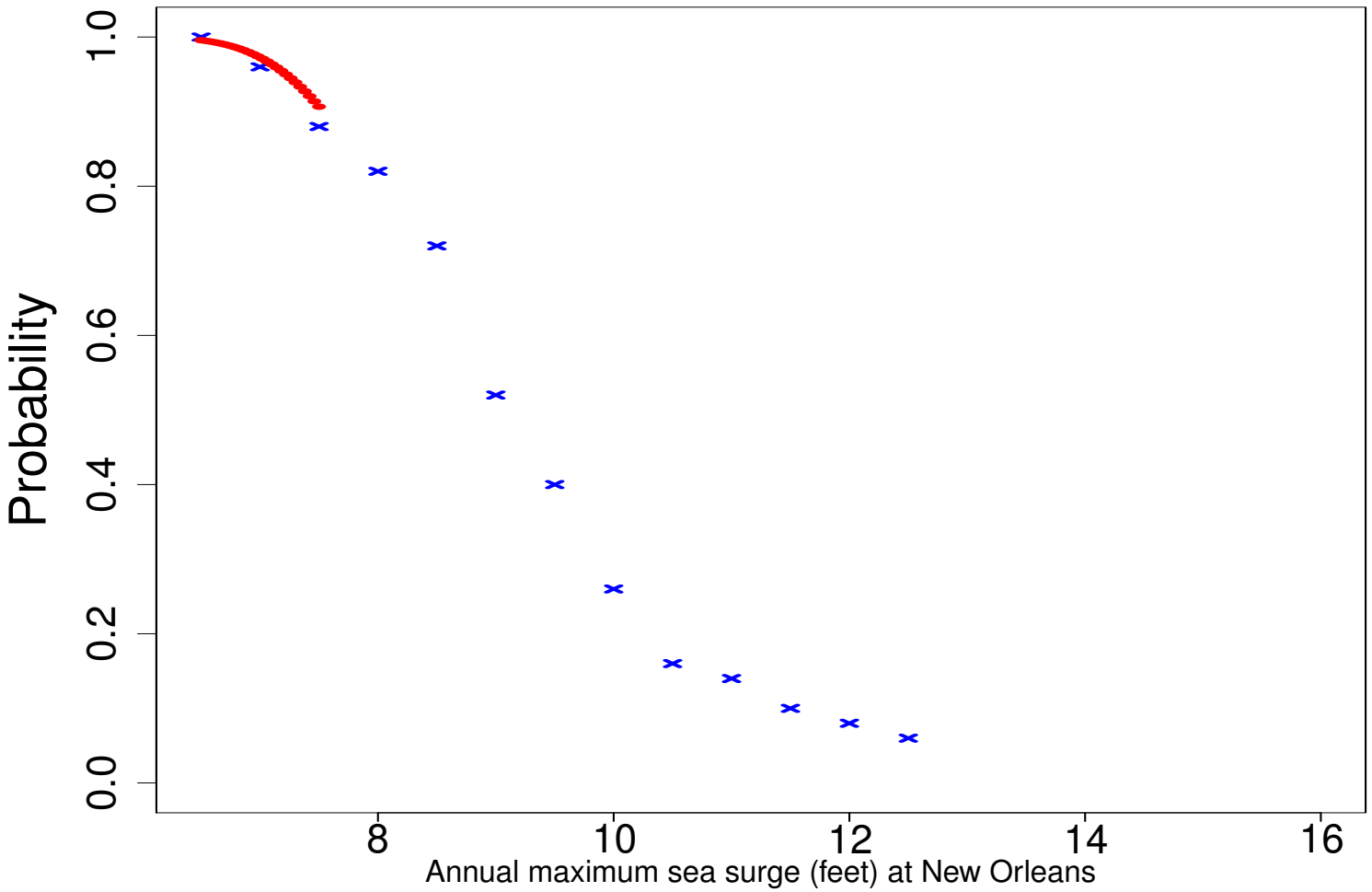

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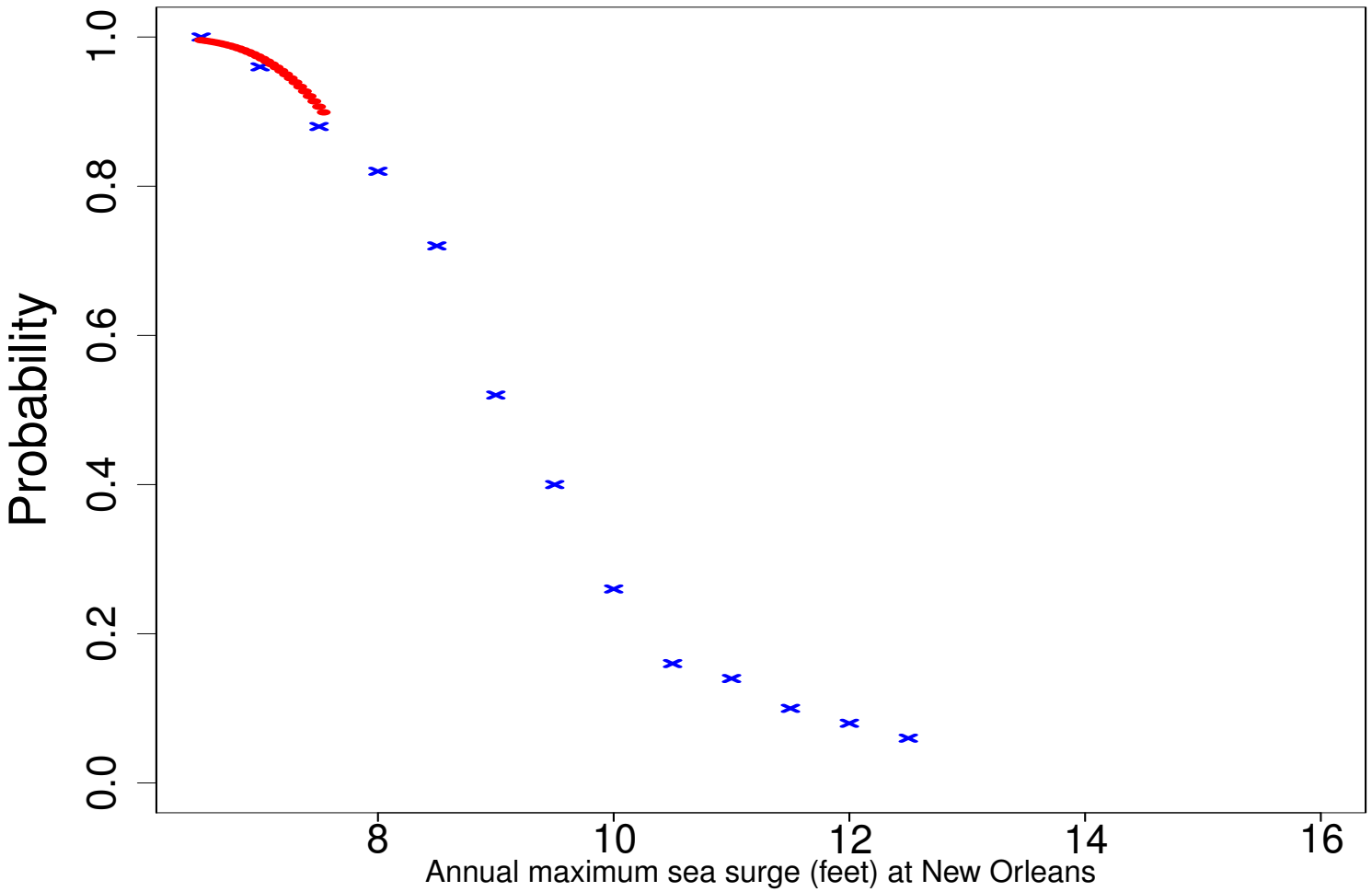

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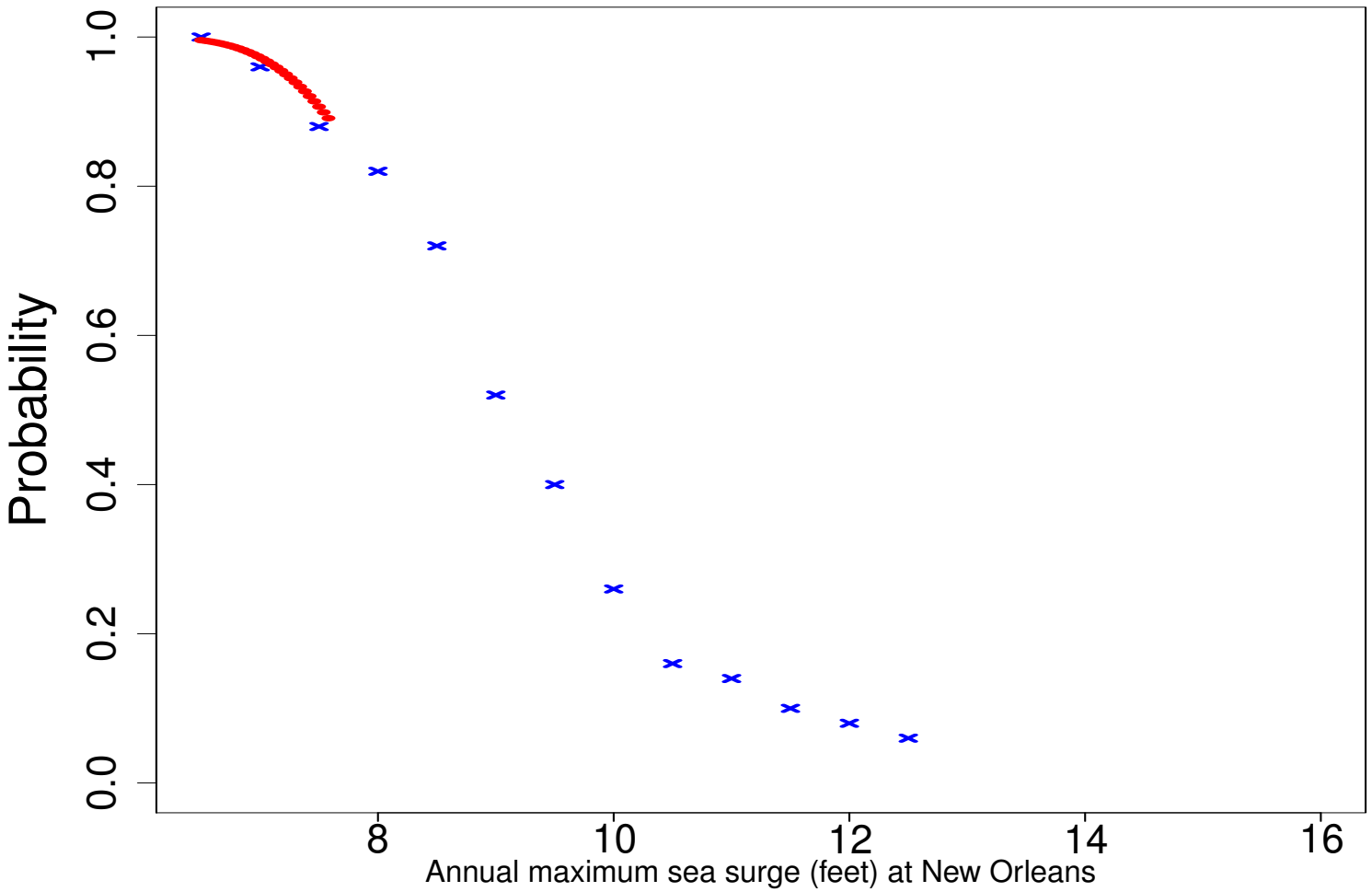

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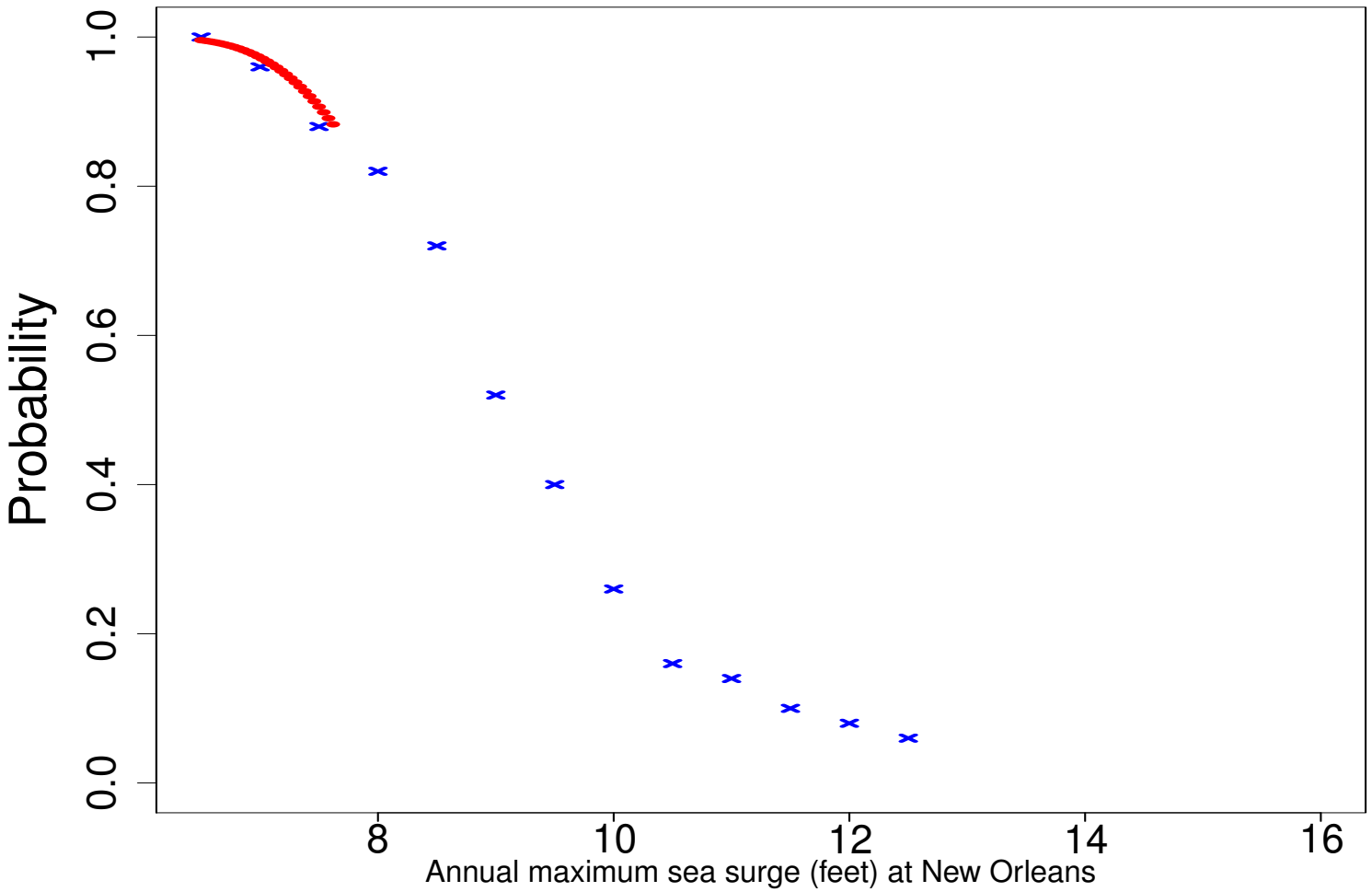

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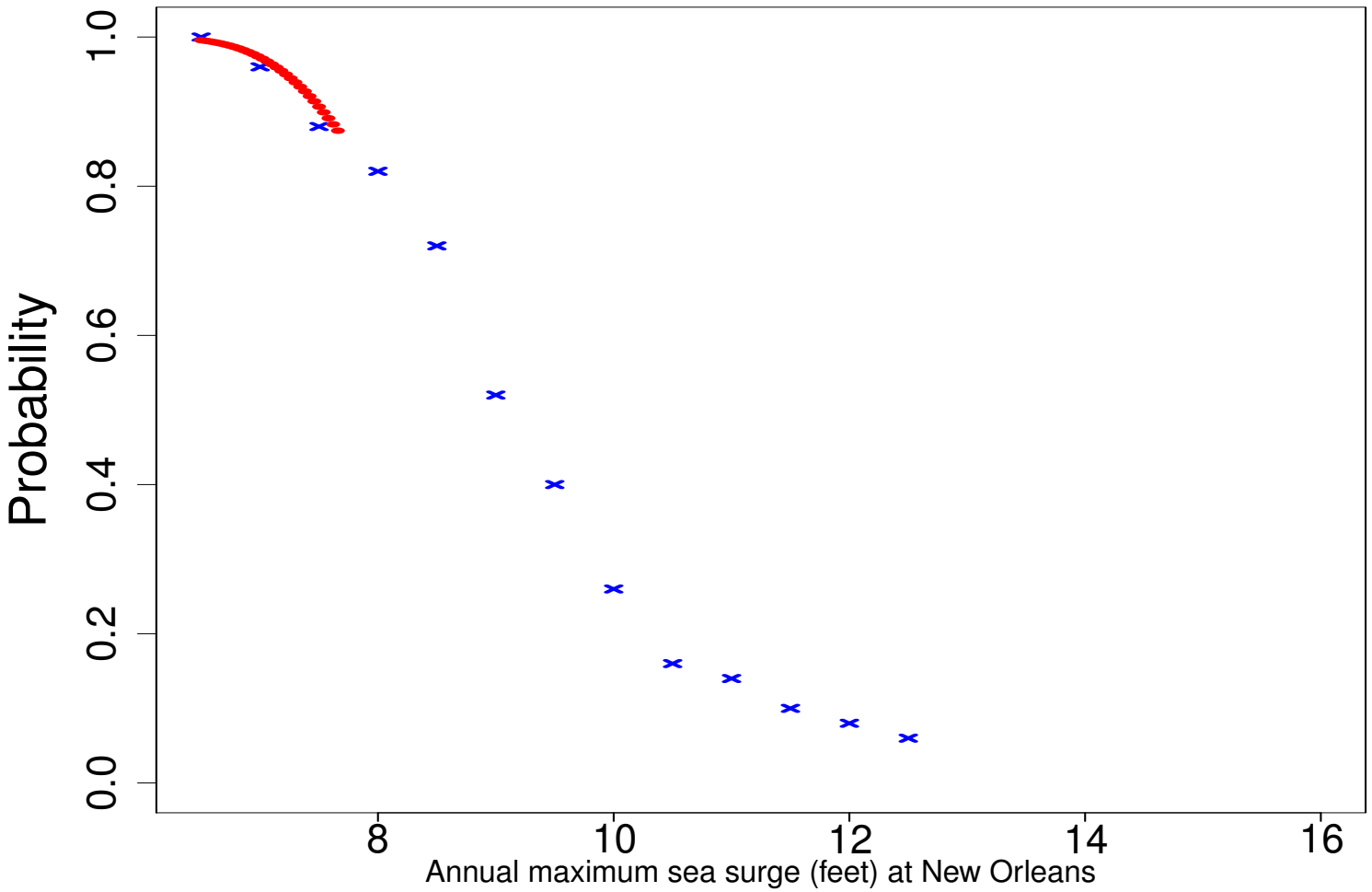

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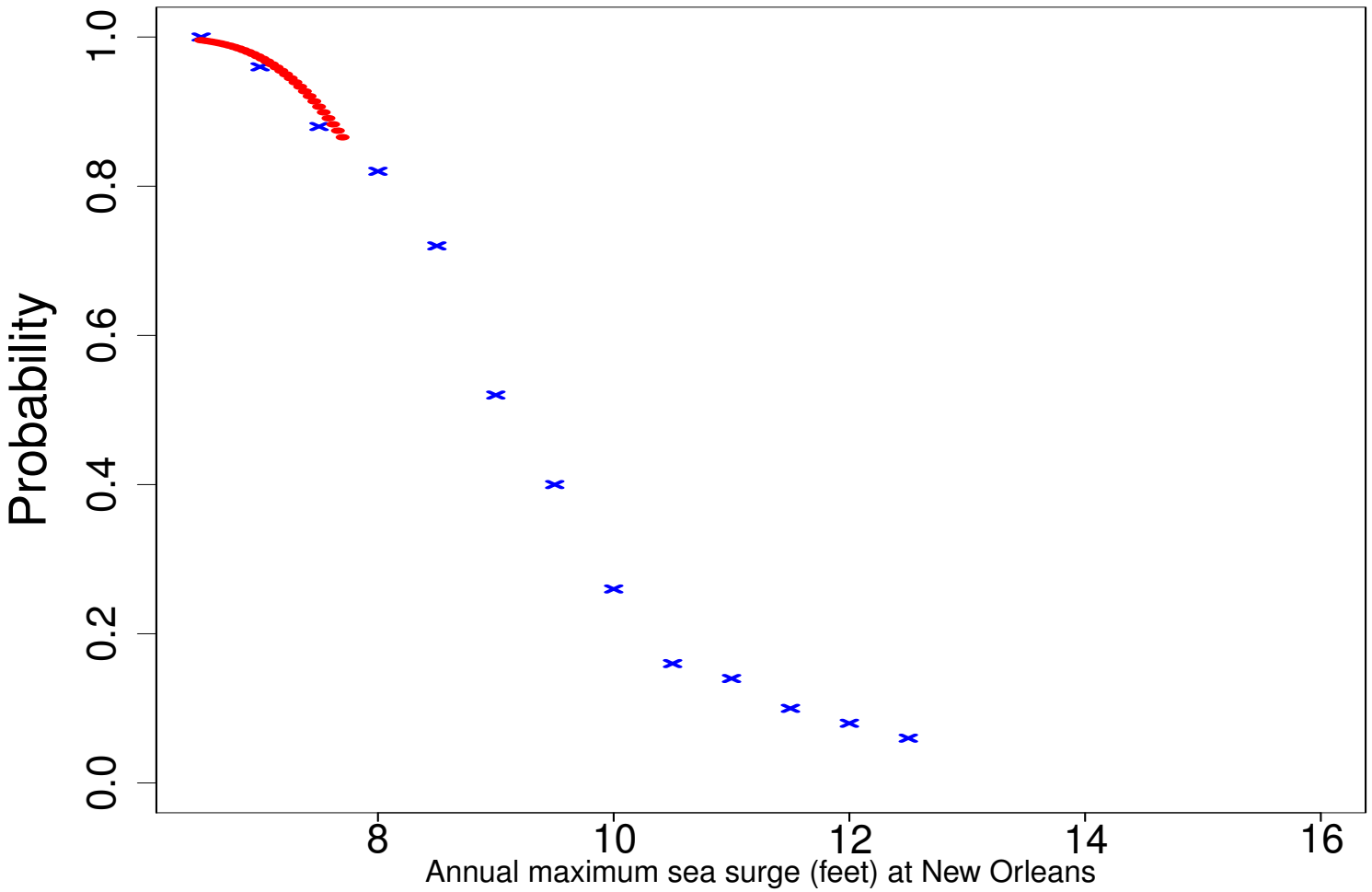

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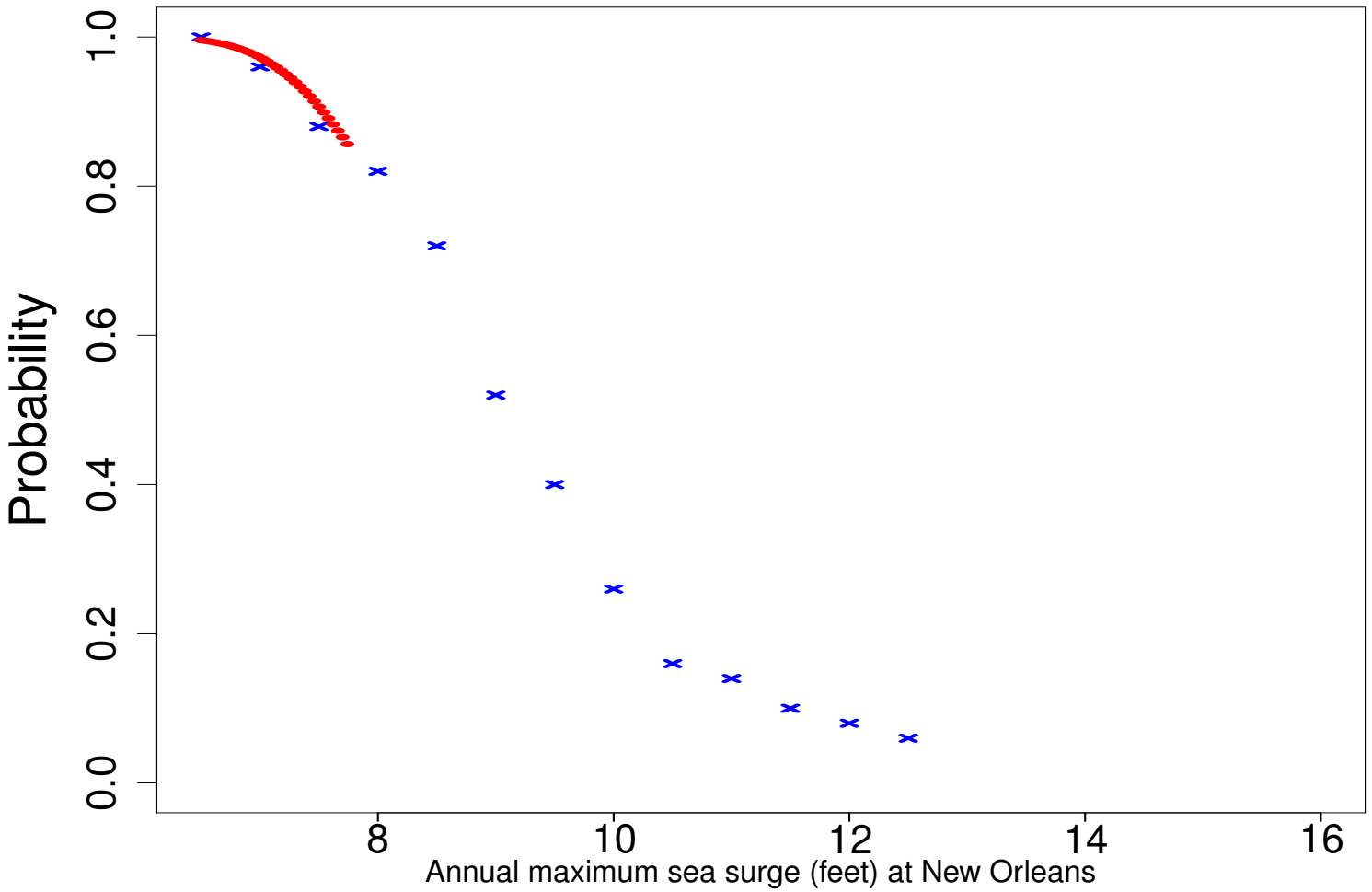

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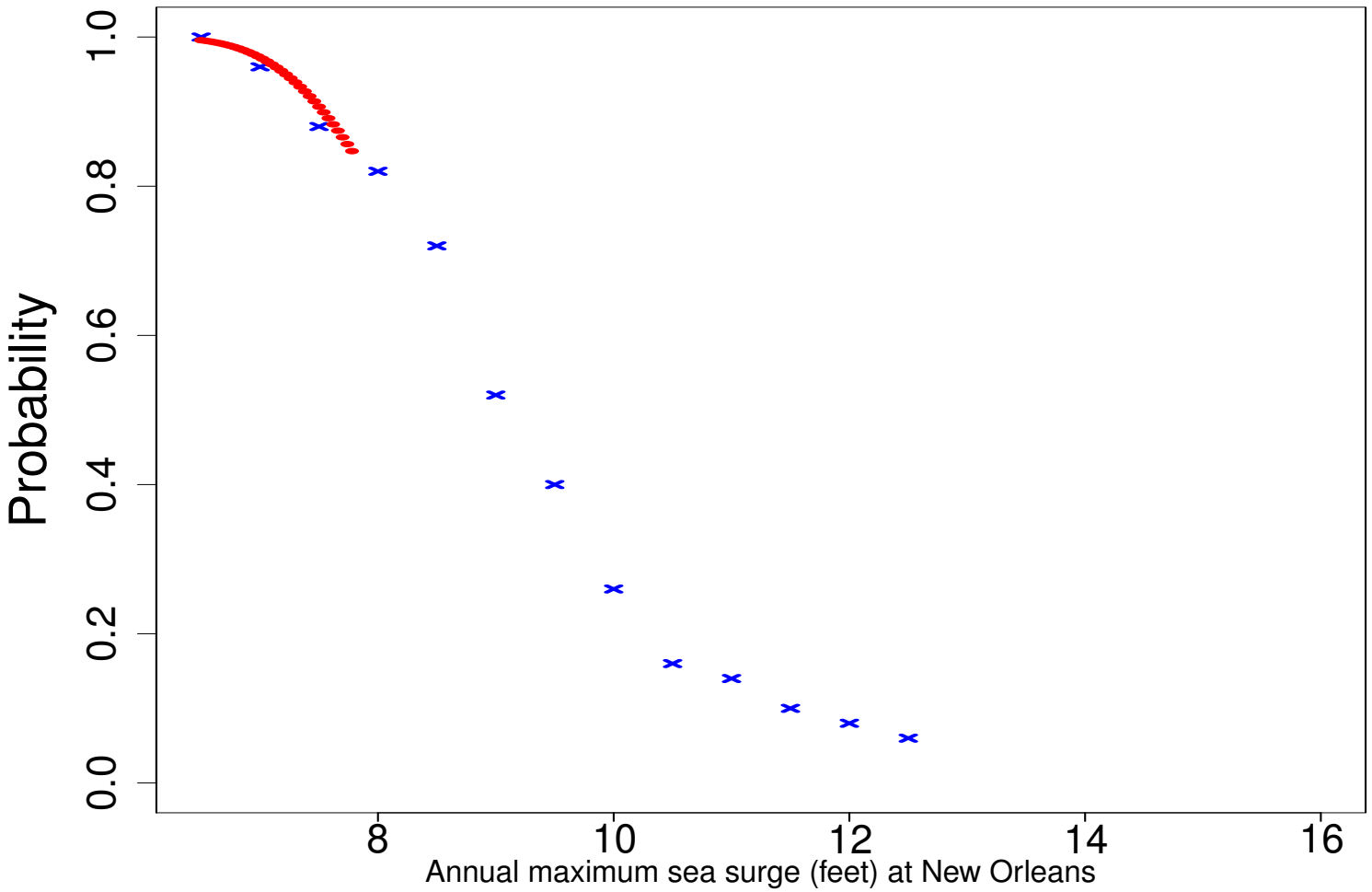

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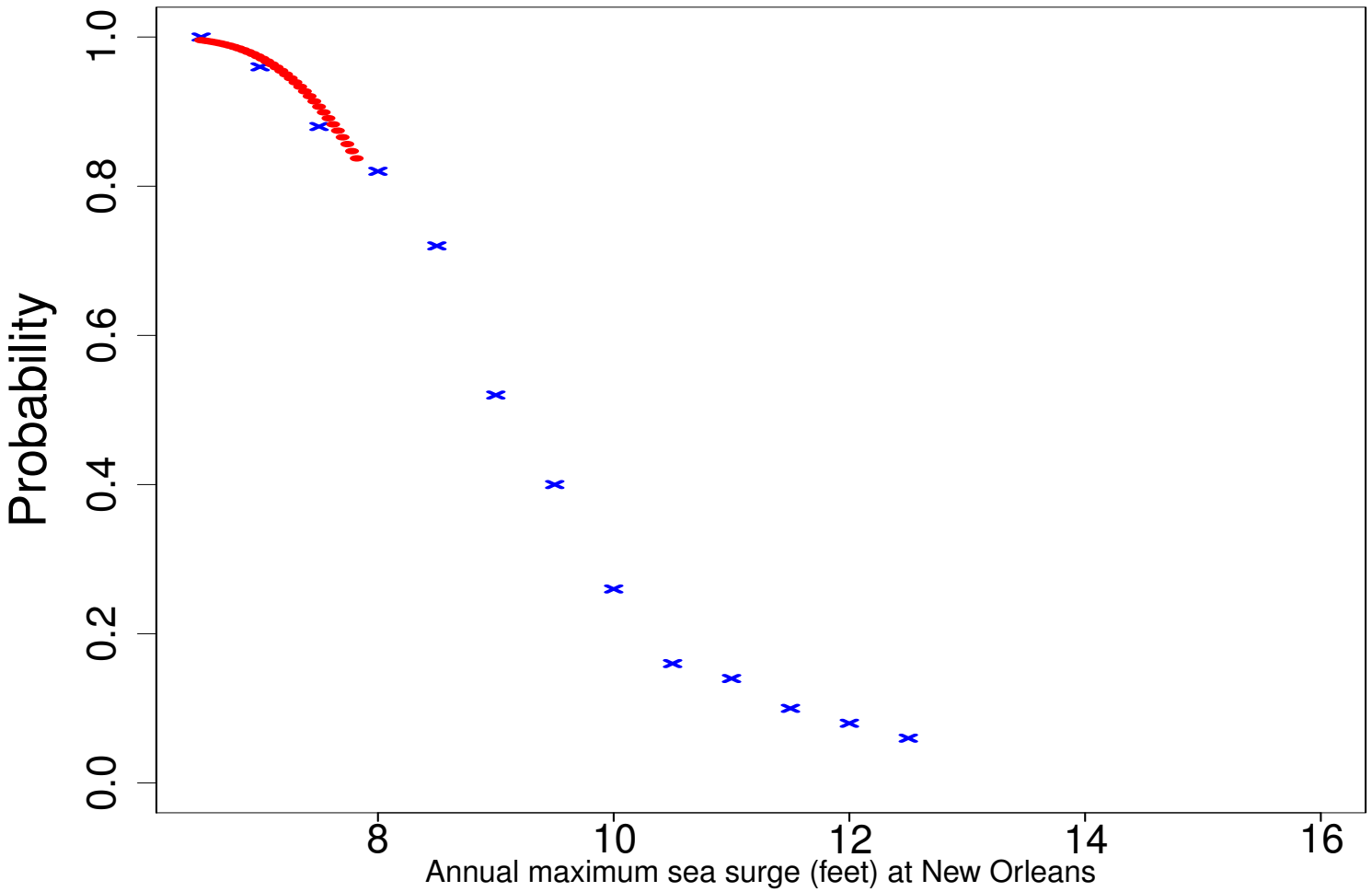

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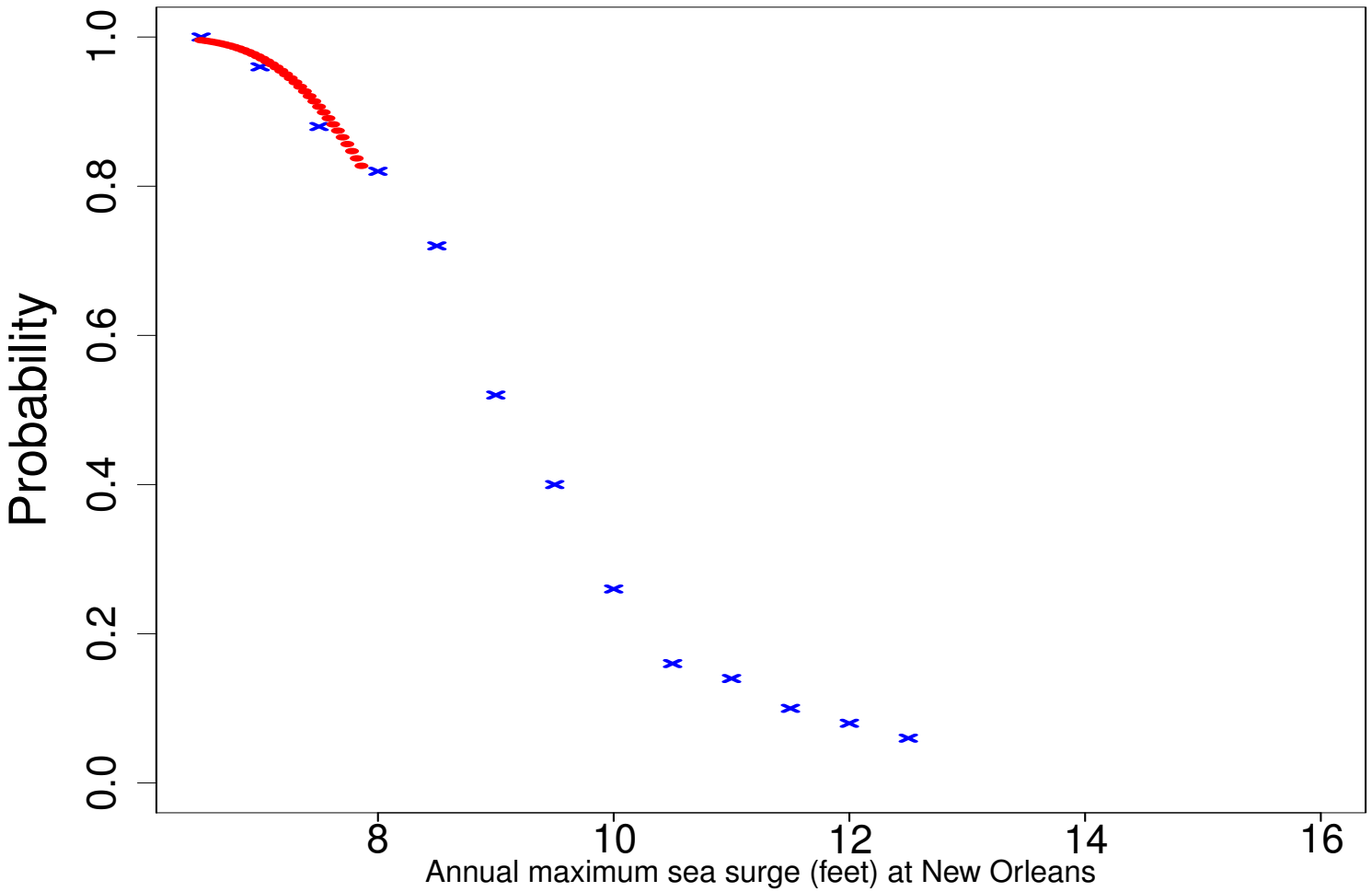

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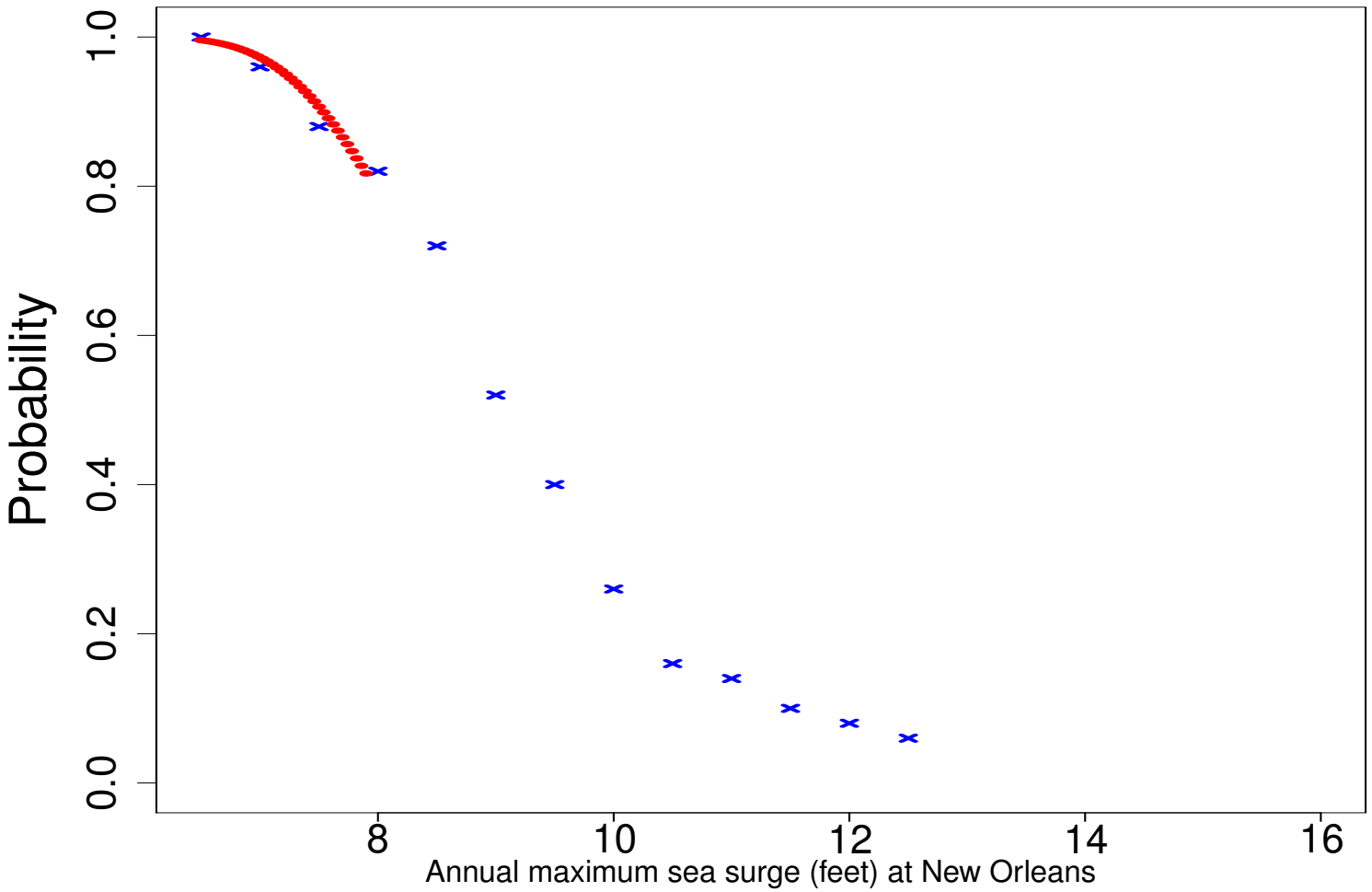

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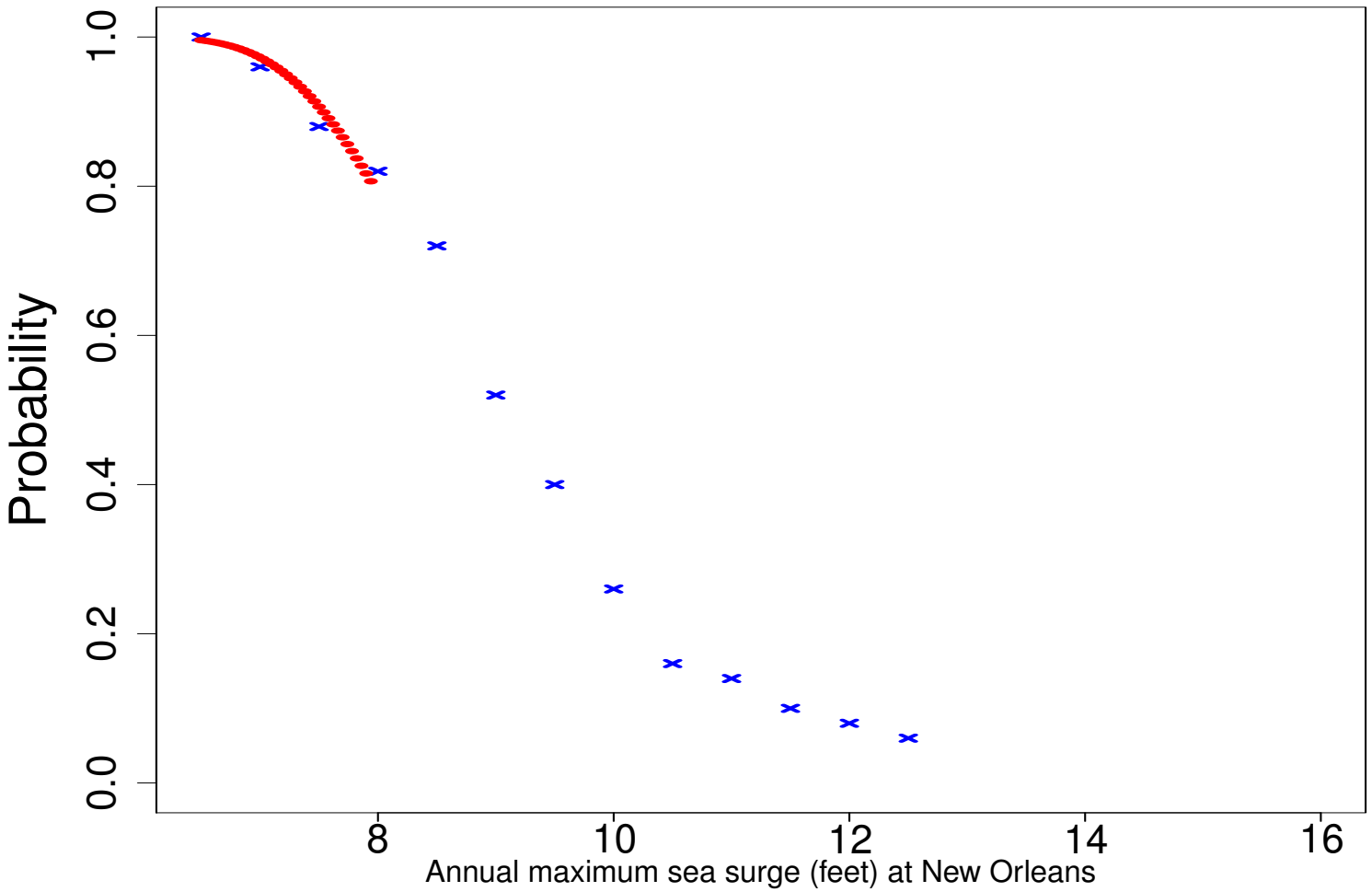

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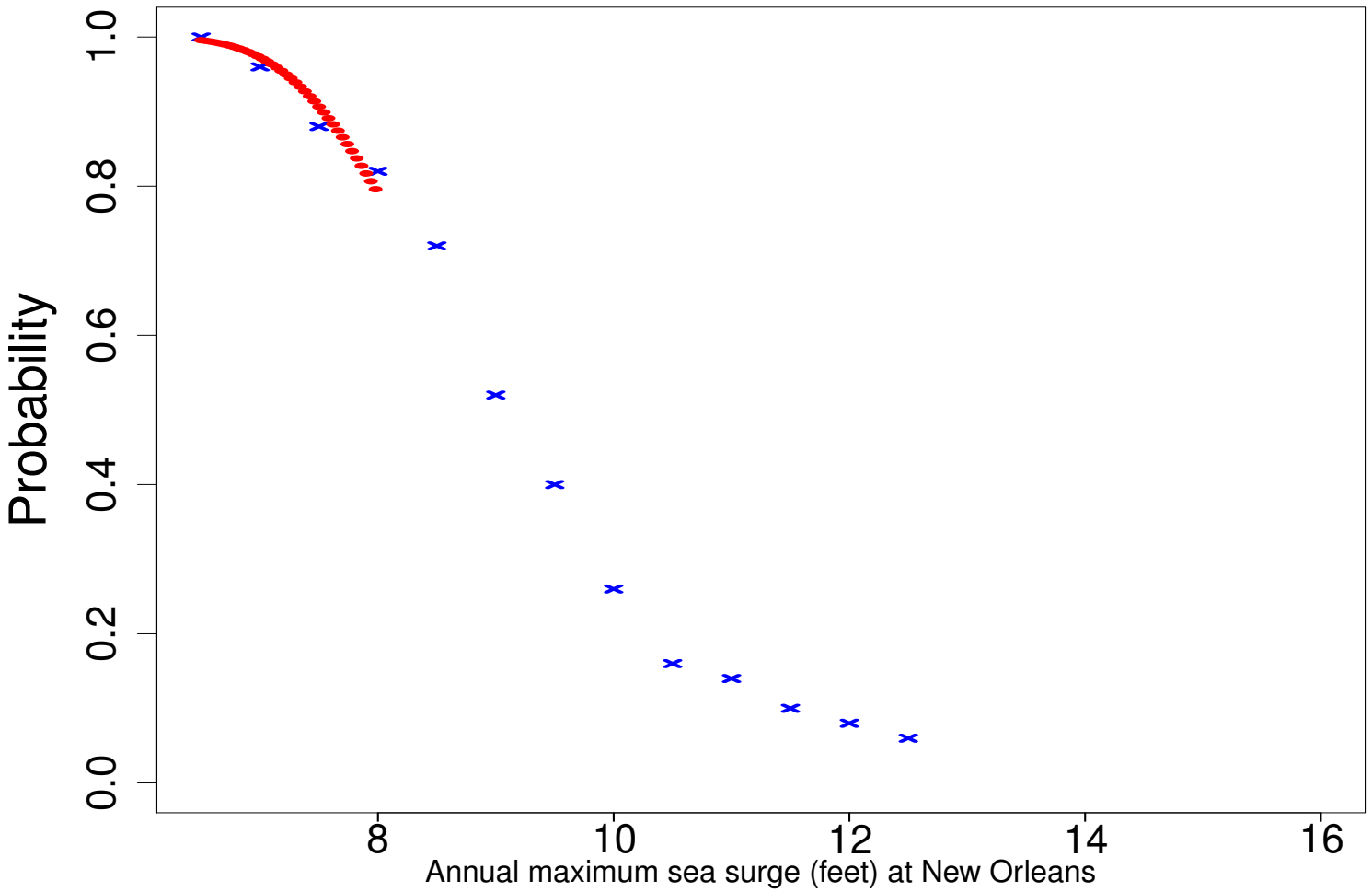

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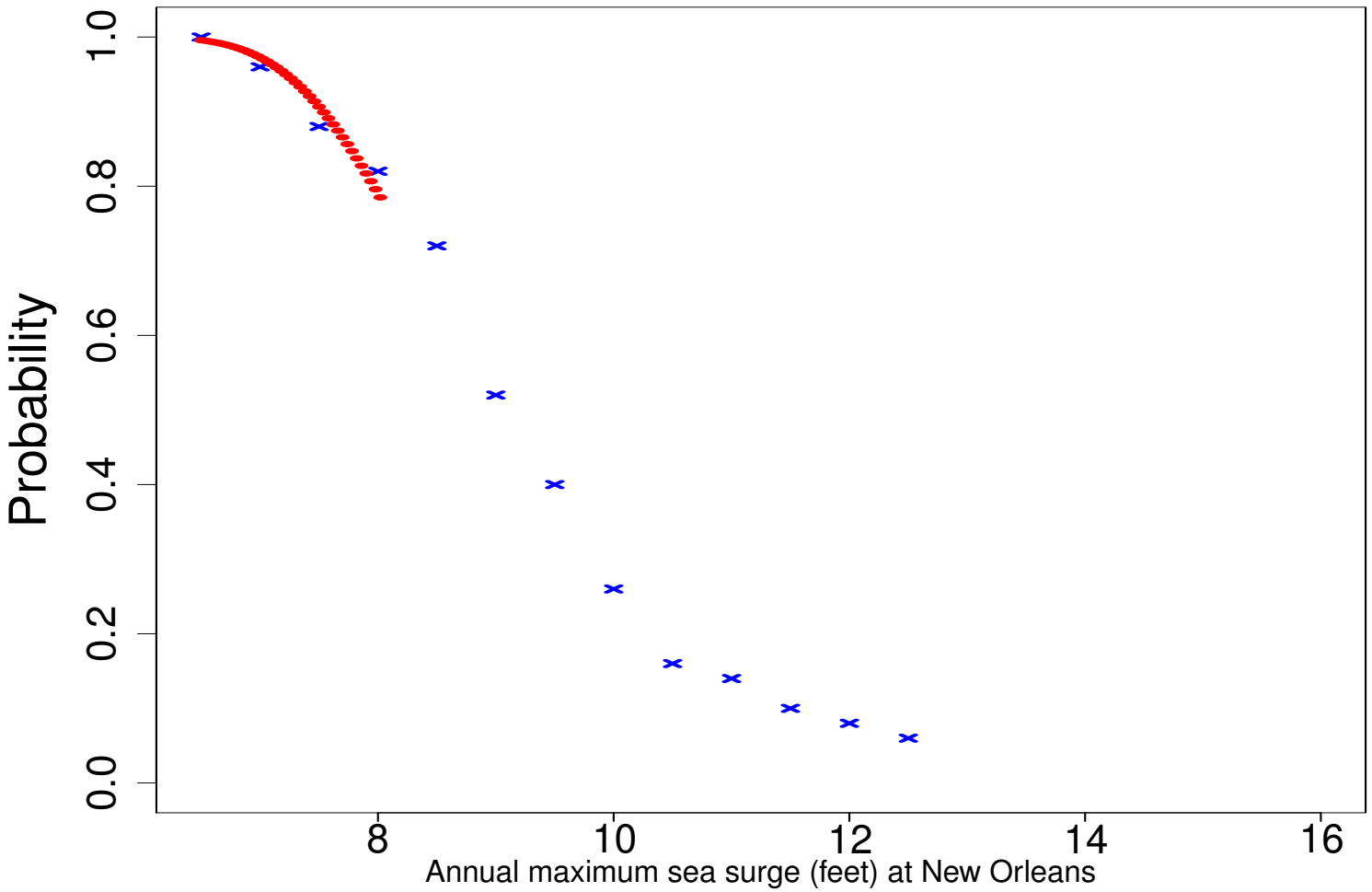

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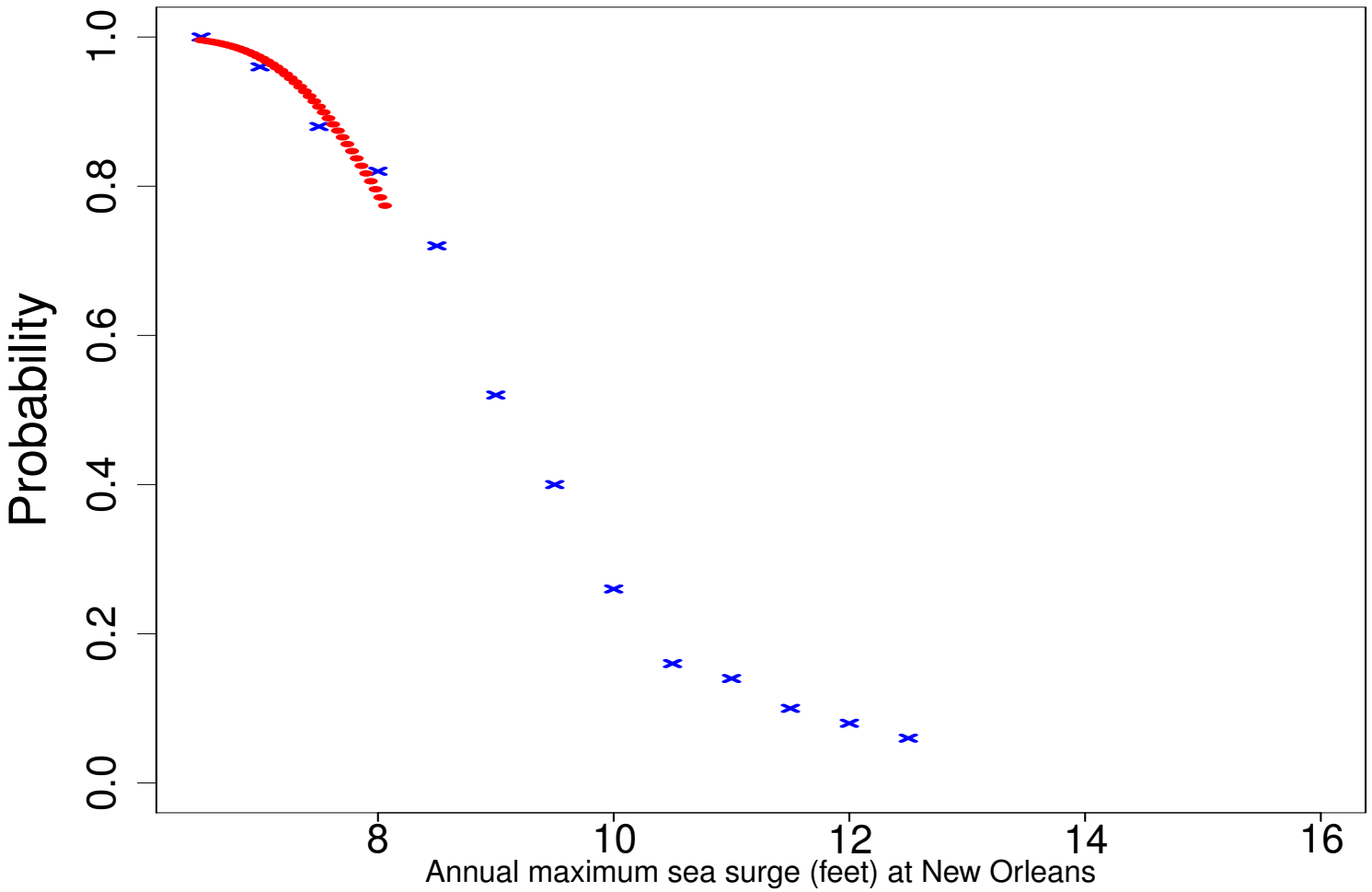

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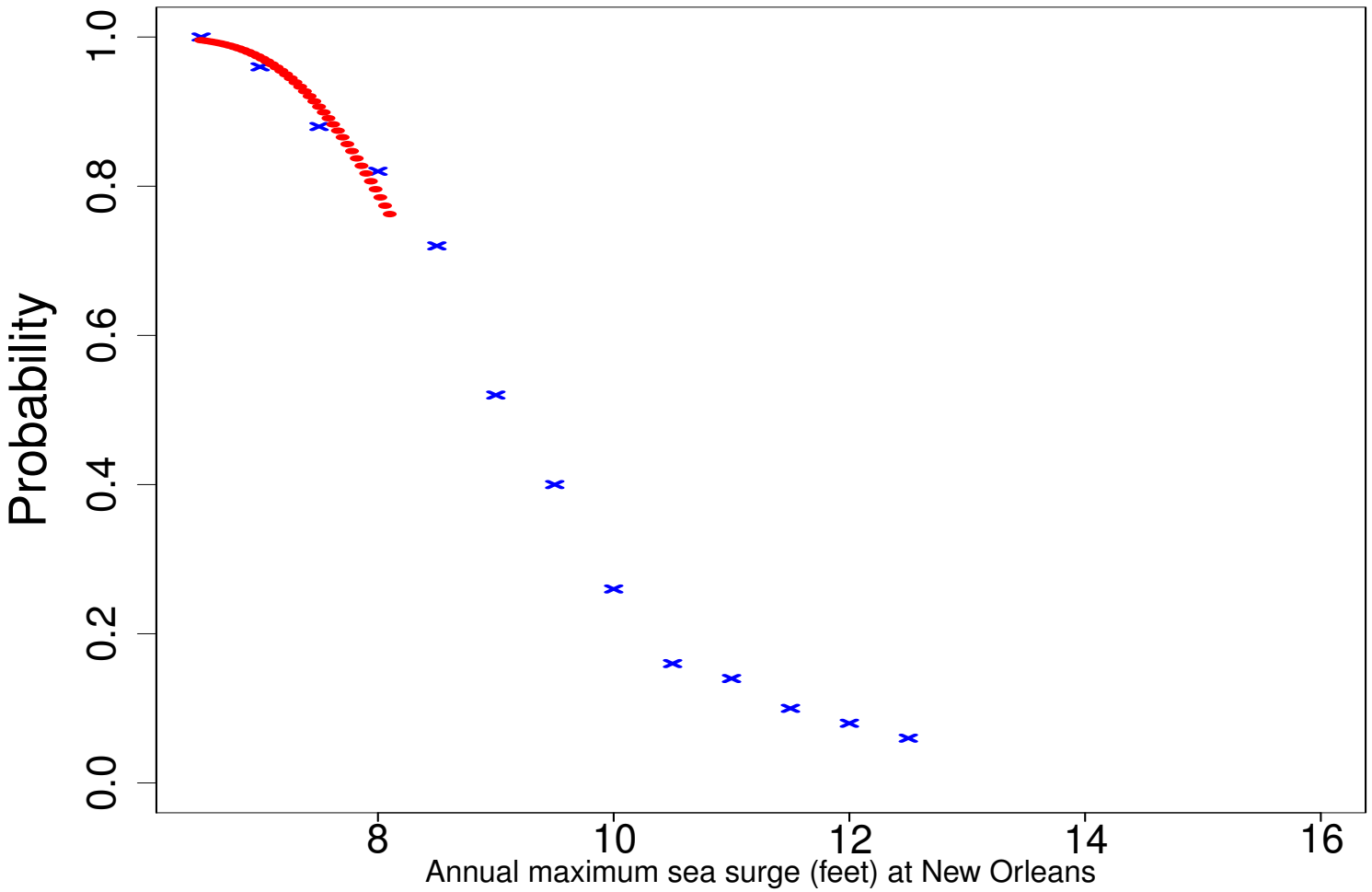

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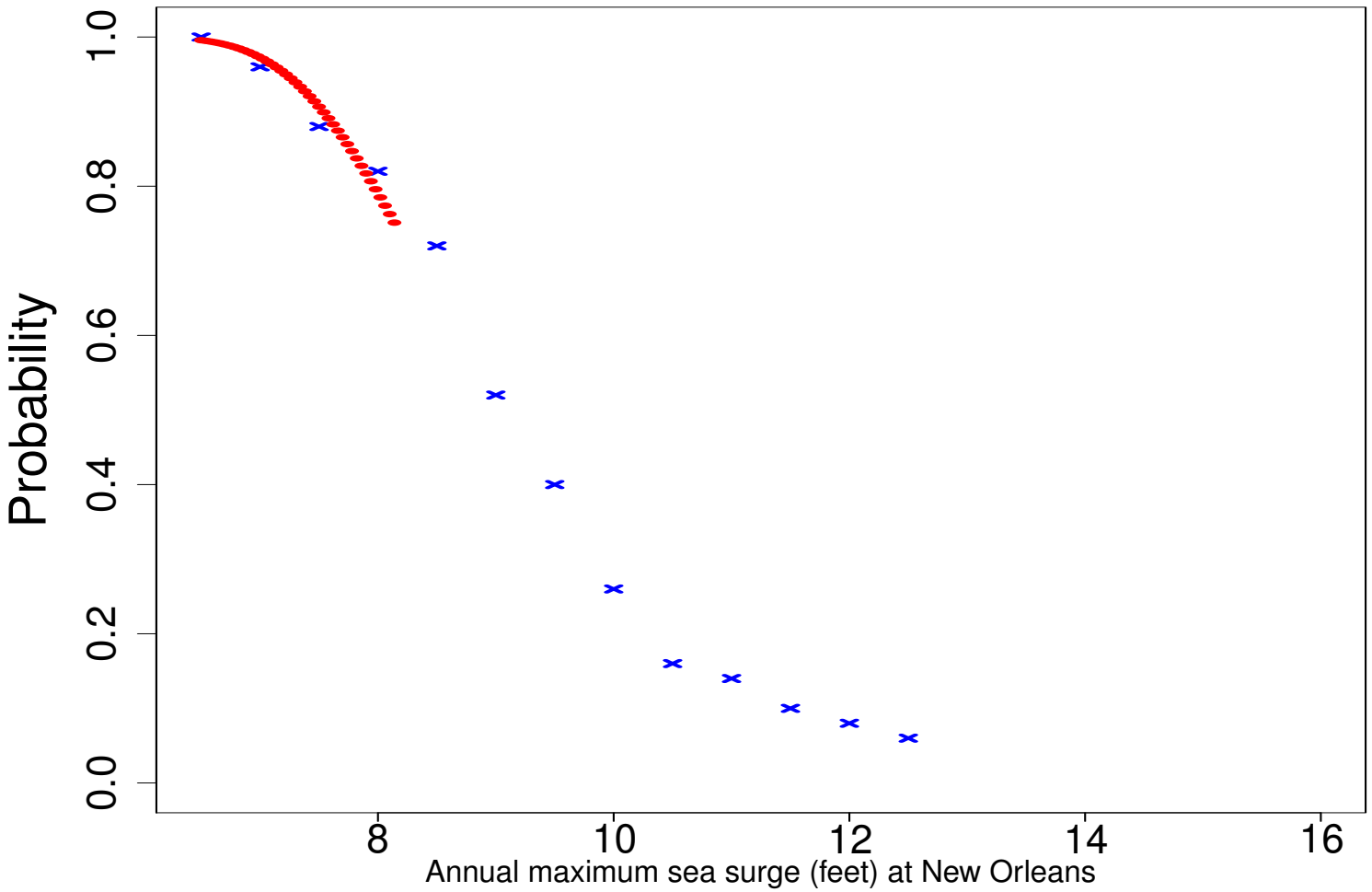

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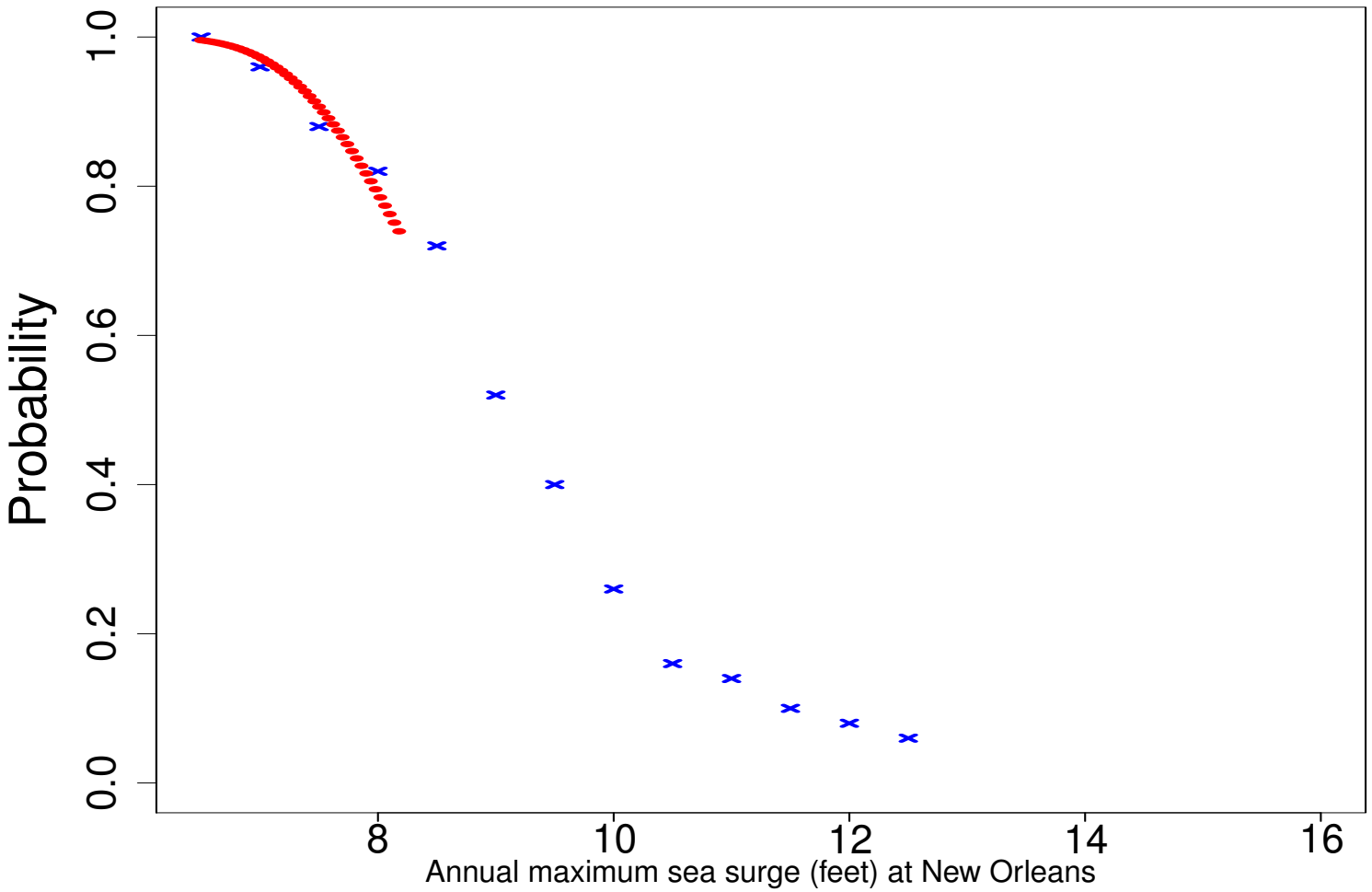

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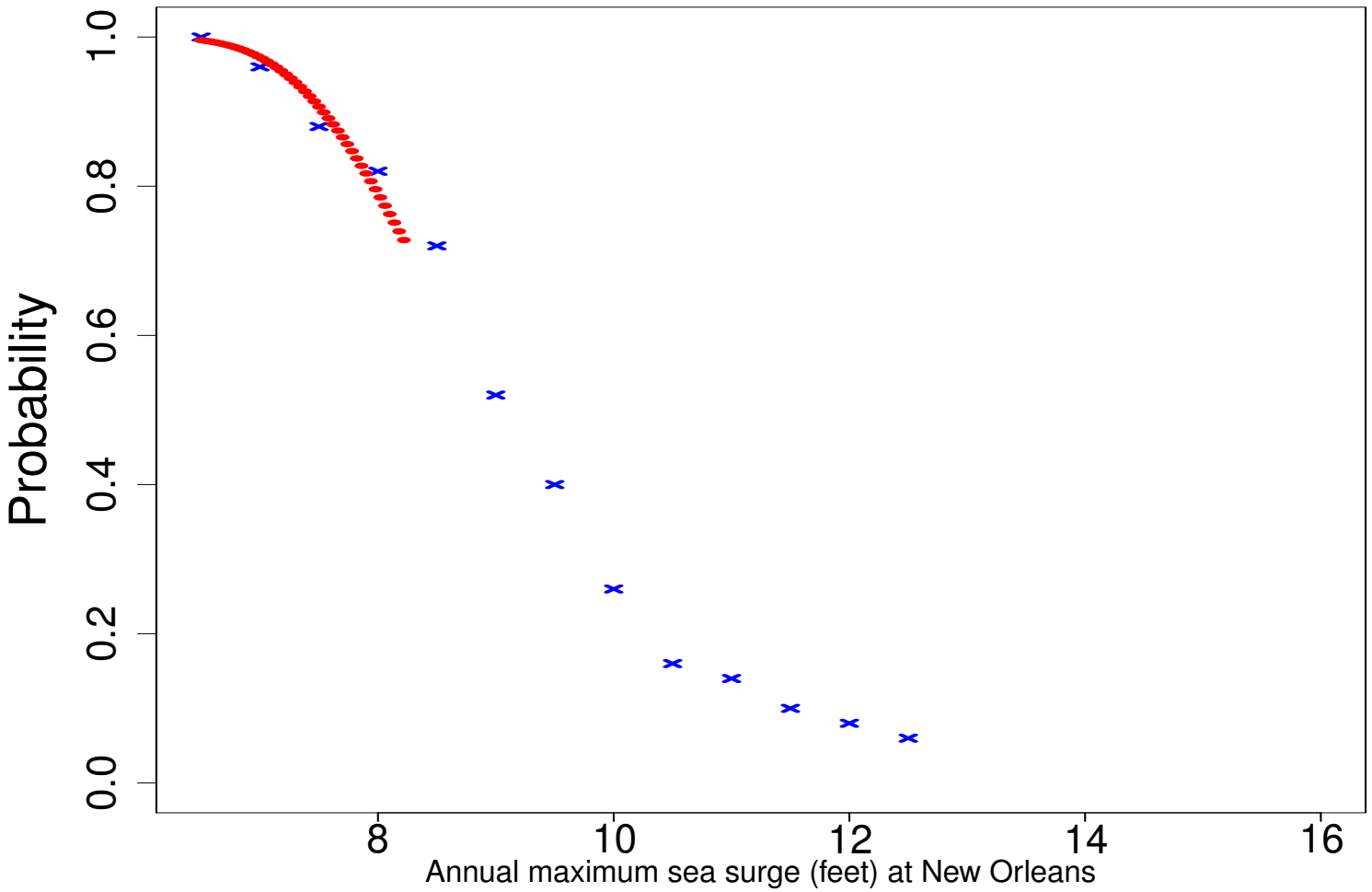

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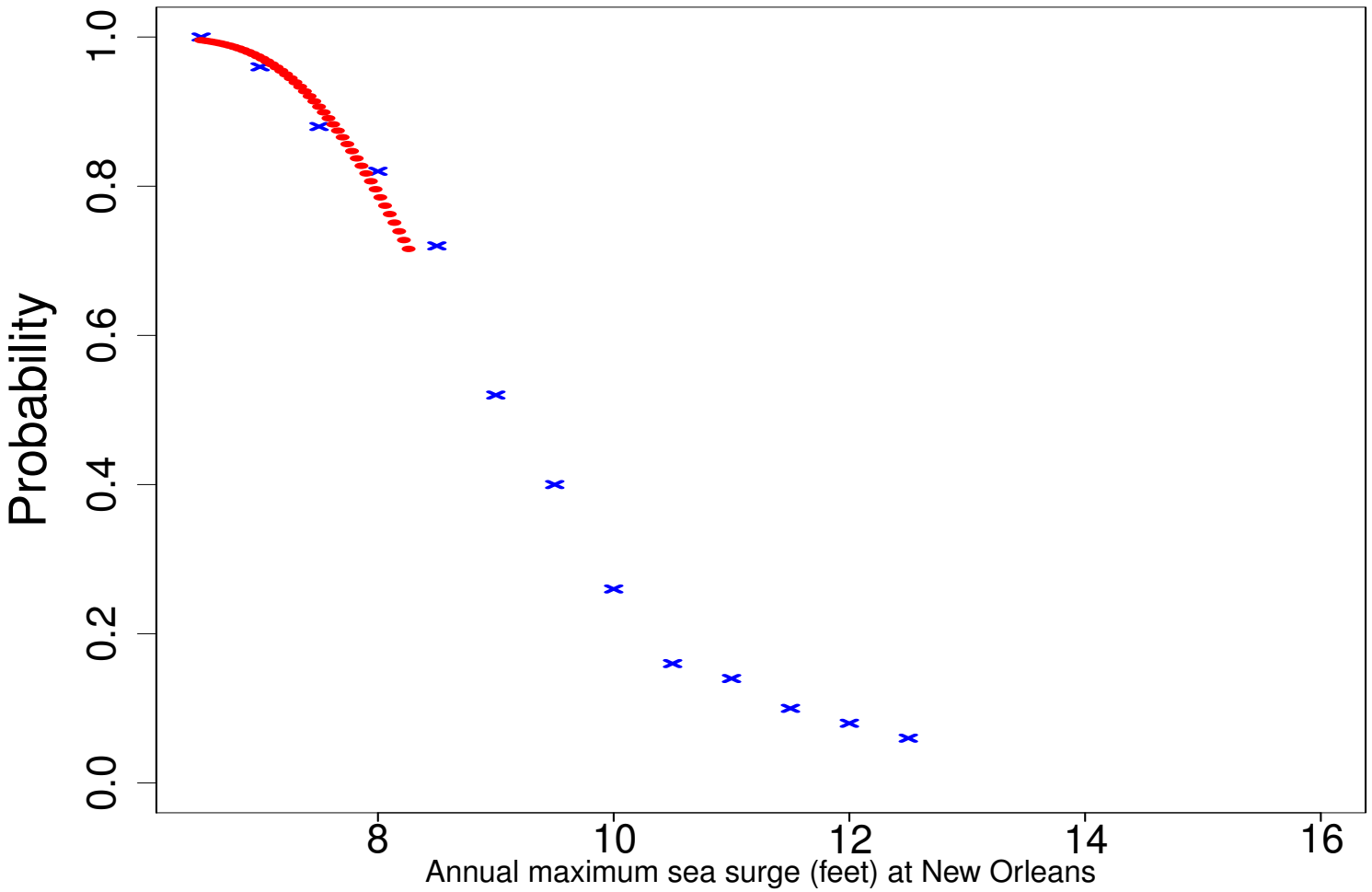

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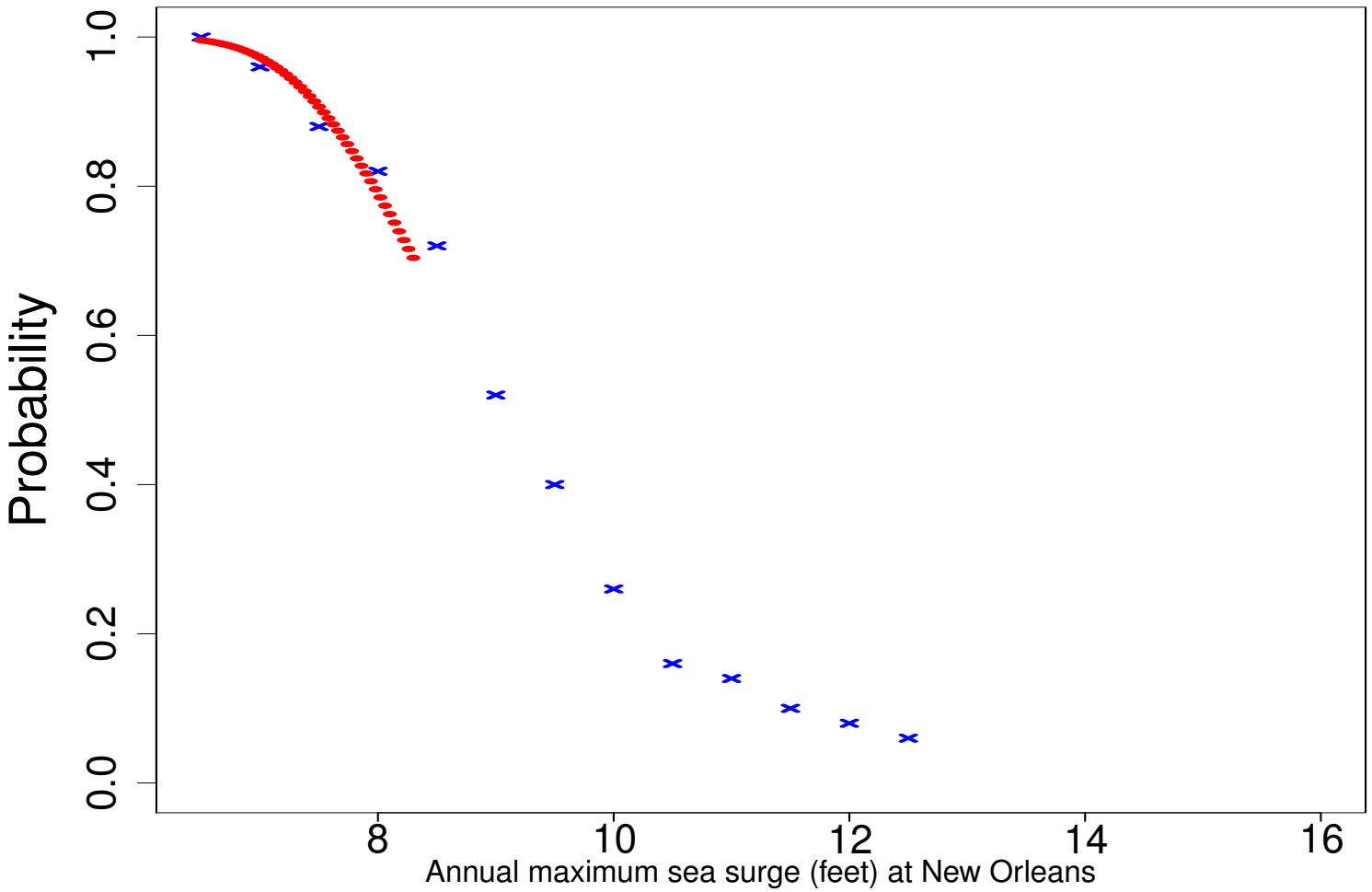

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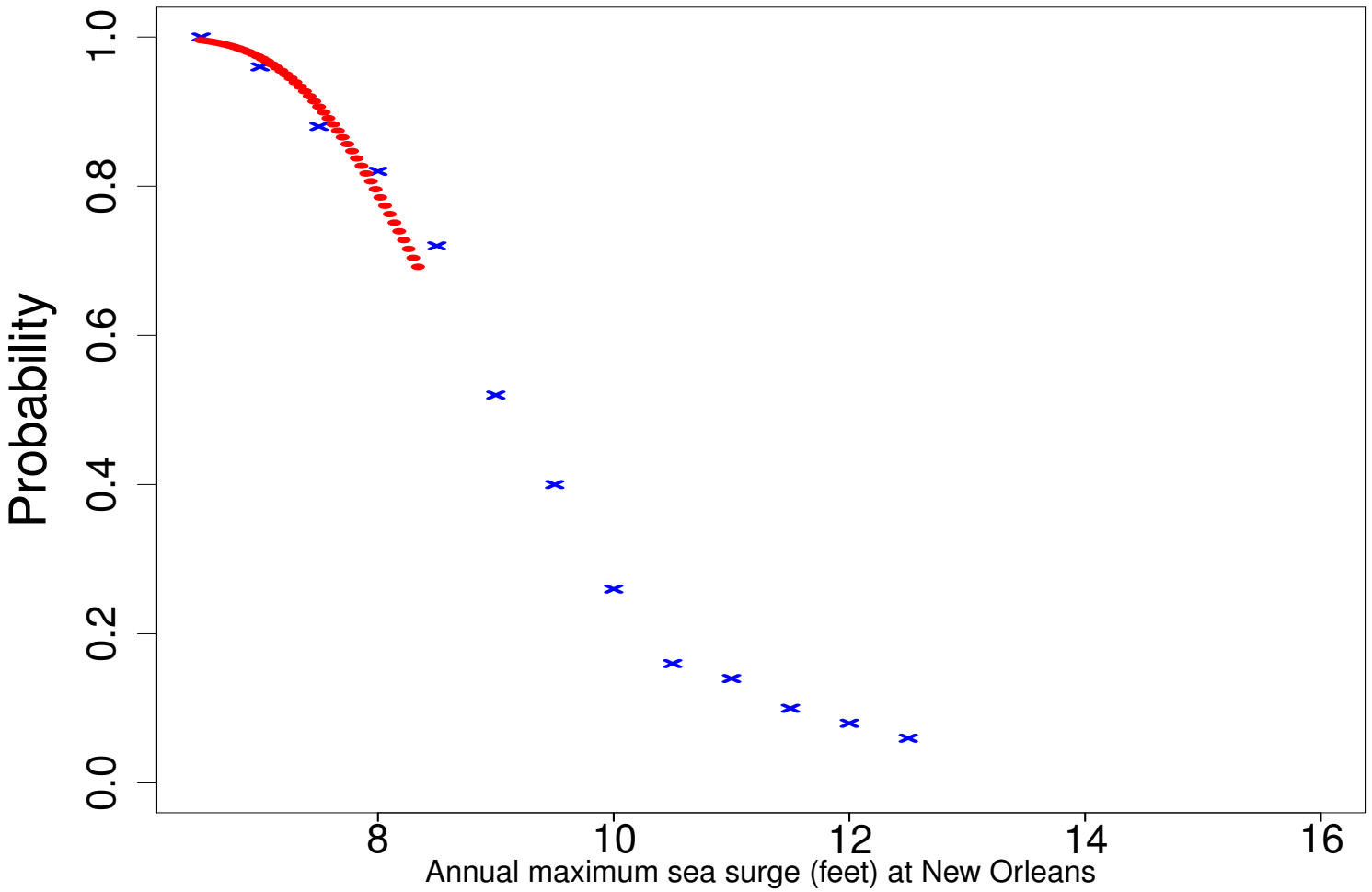

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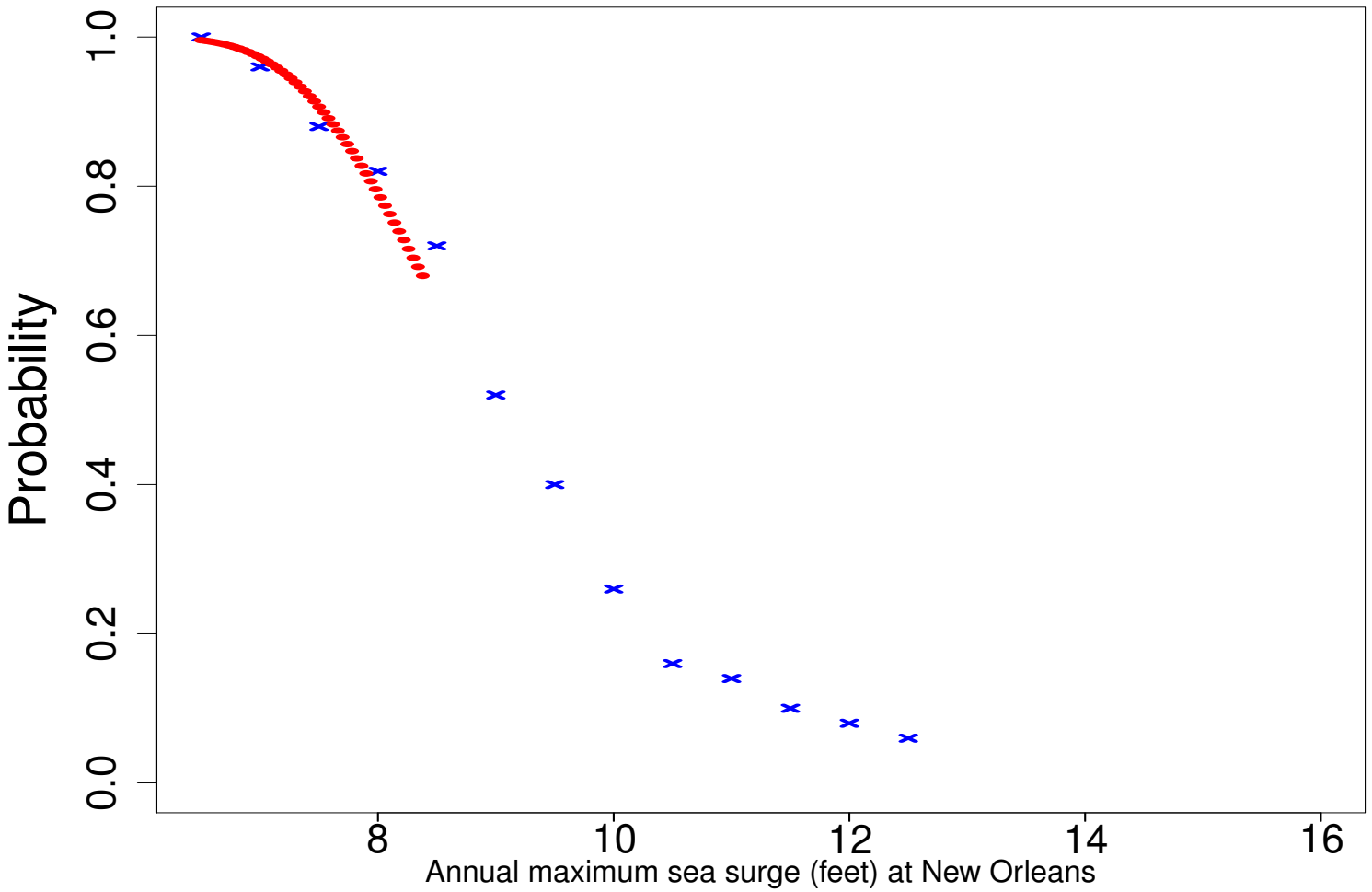

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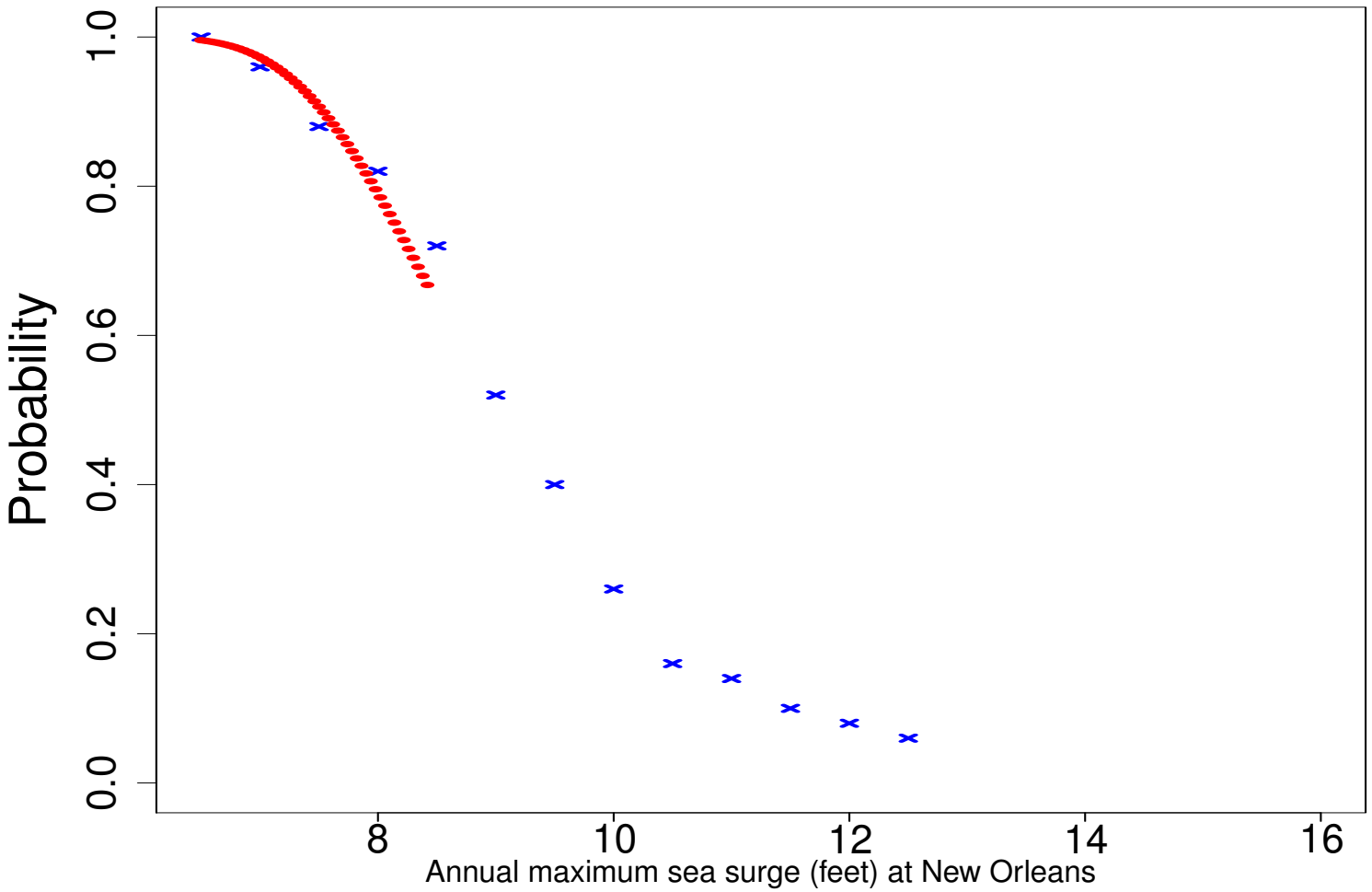

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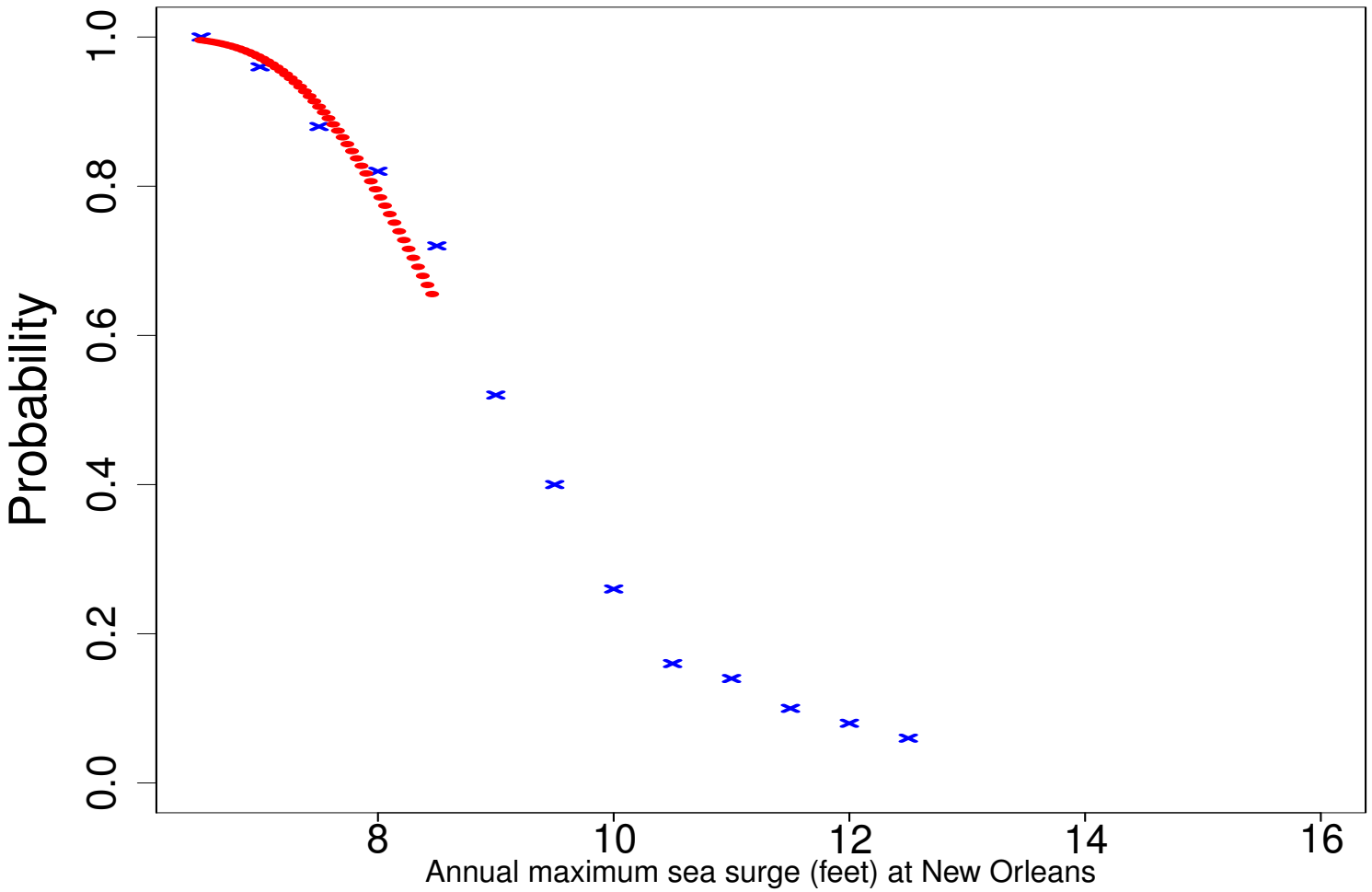

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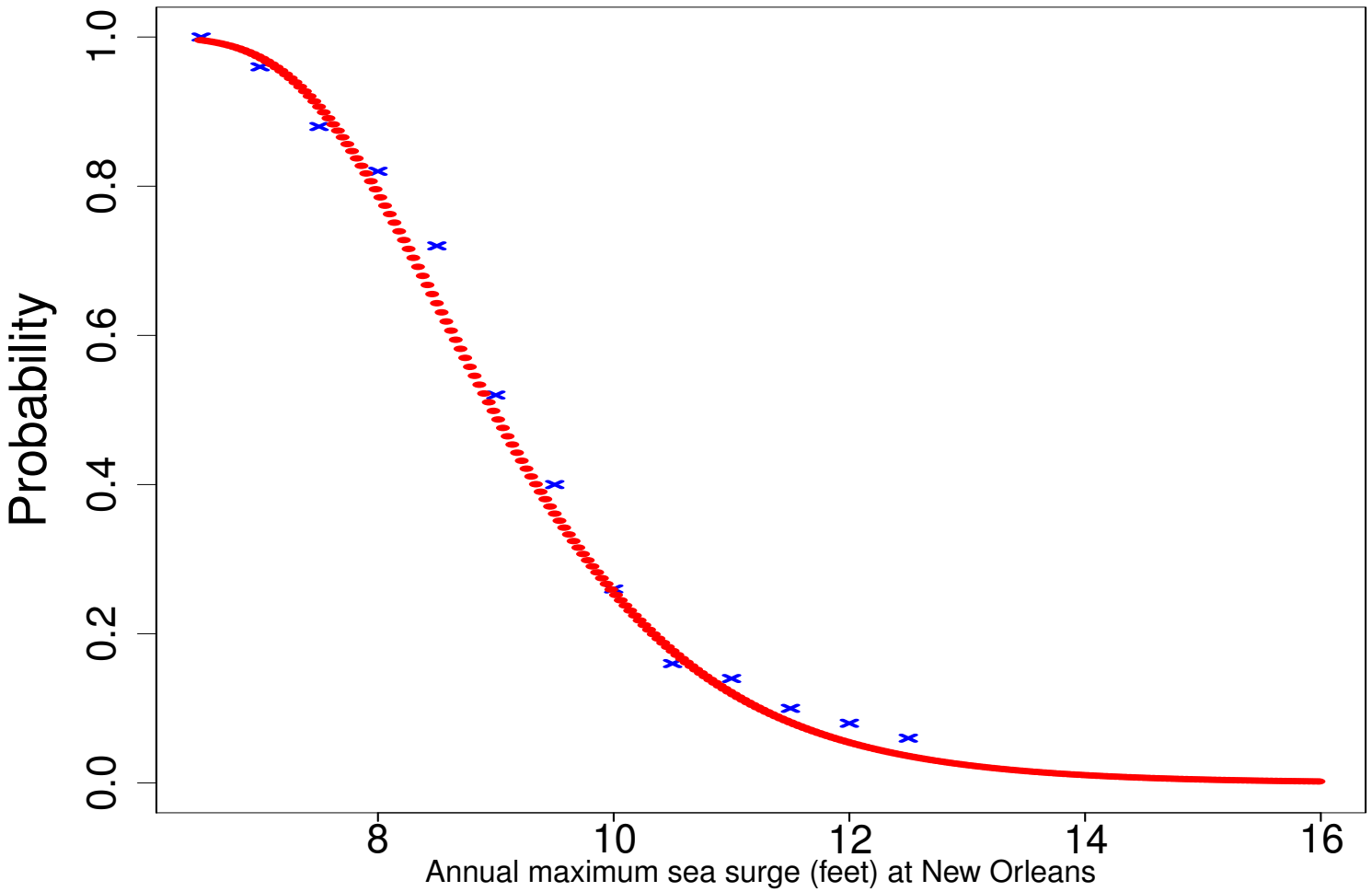

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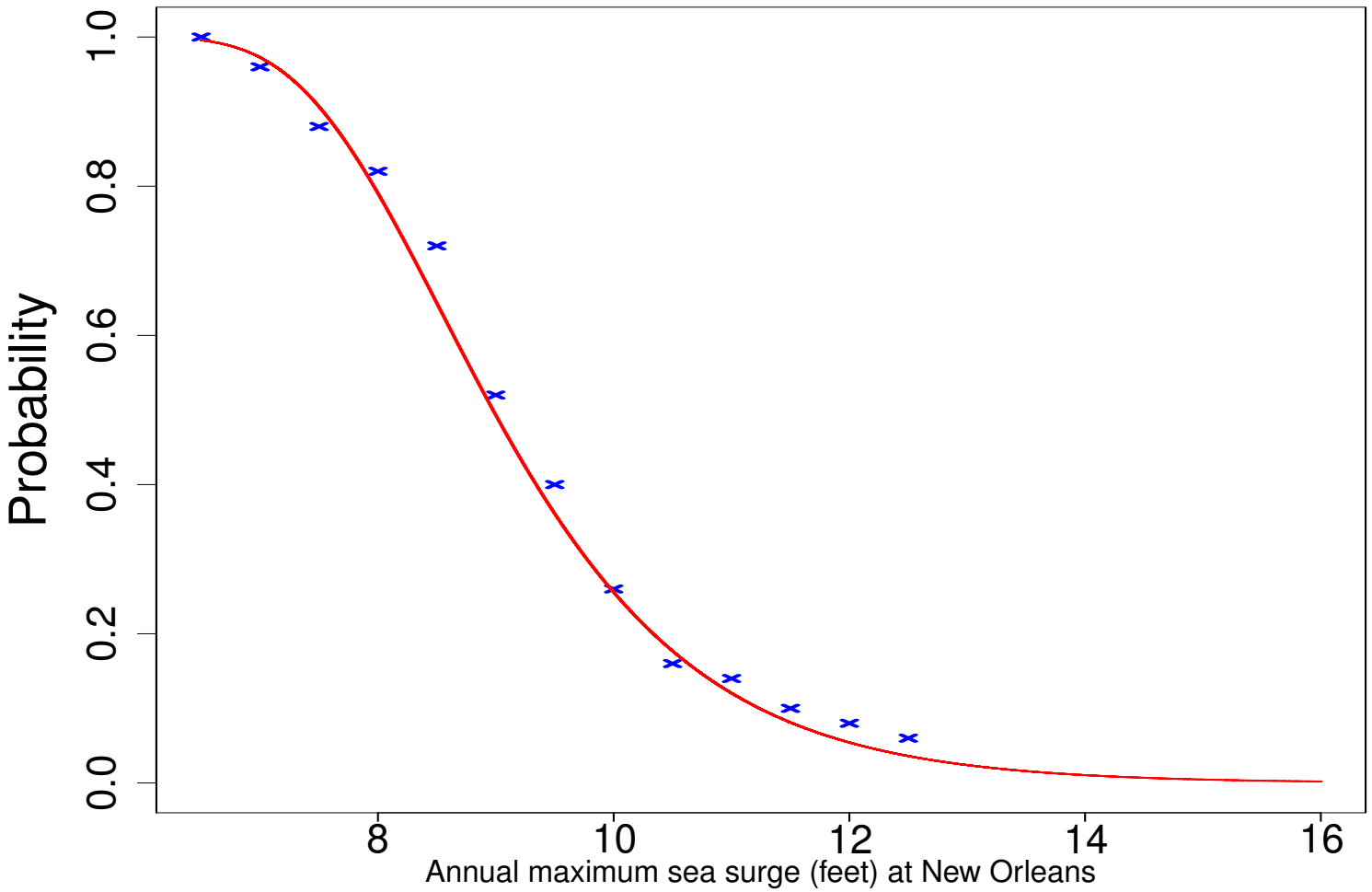

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We can also estimate probabilities of events more extreme than those we have observed via **extrapolation**.

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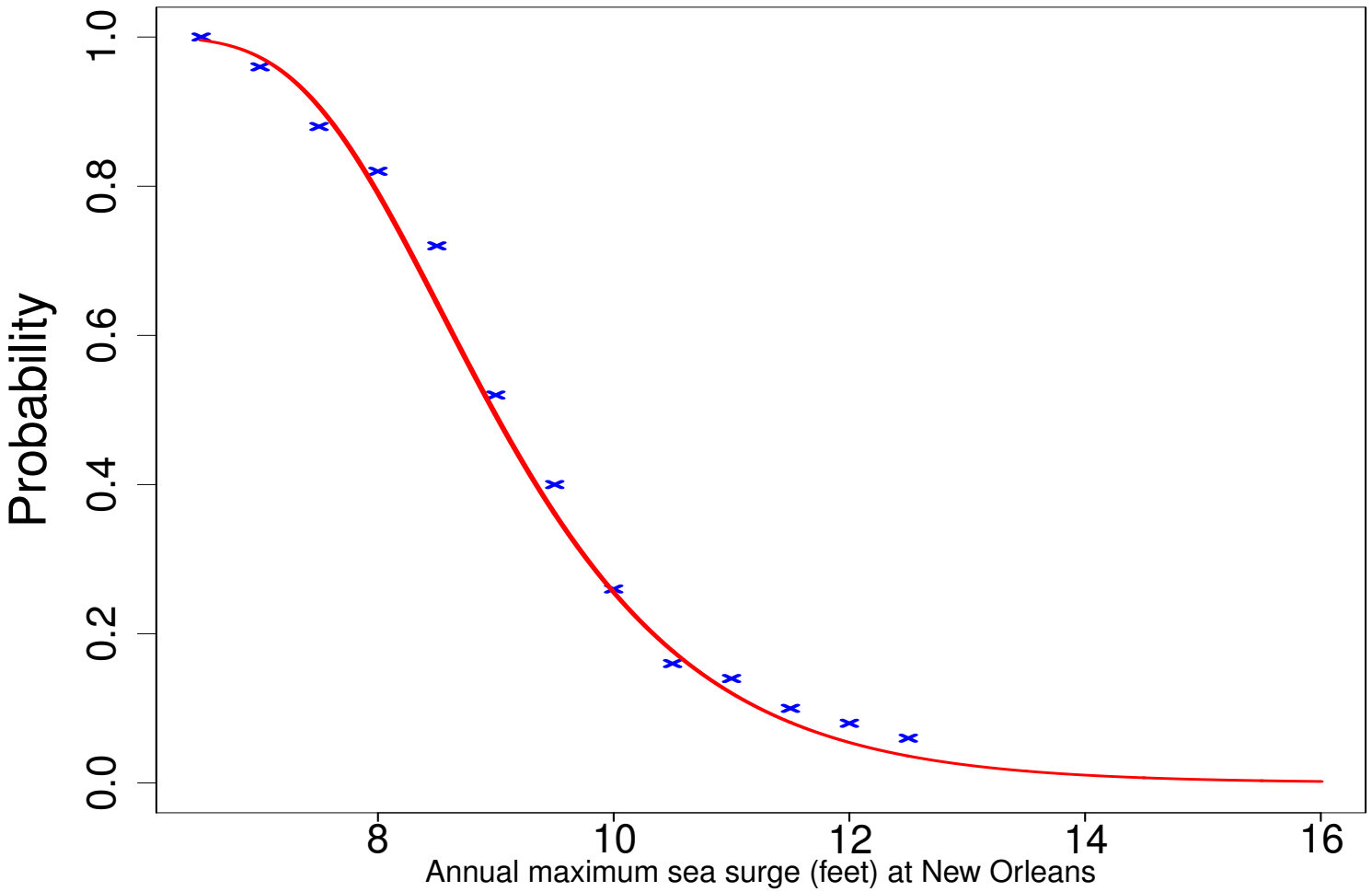

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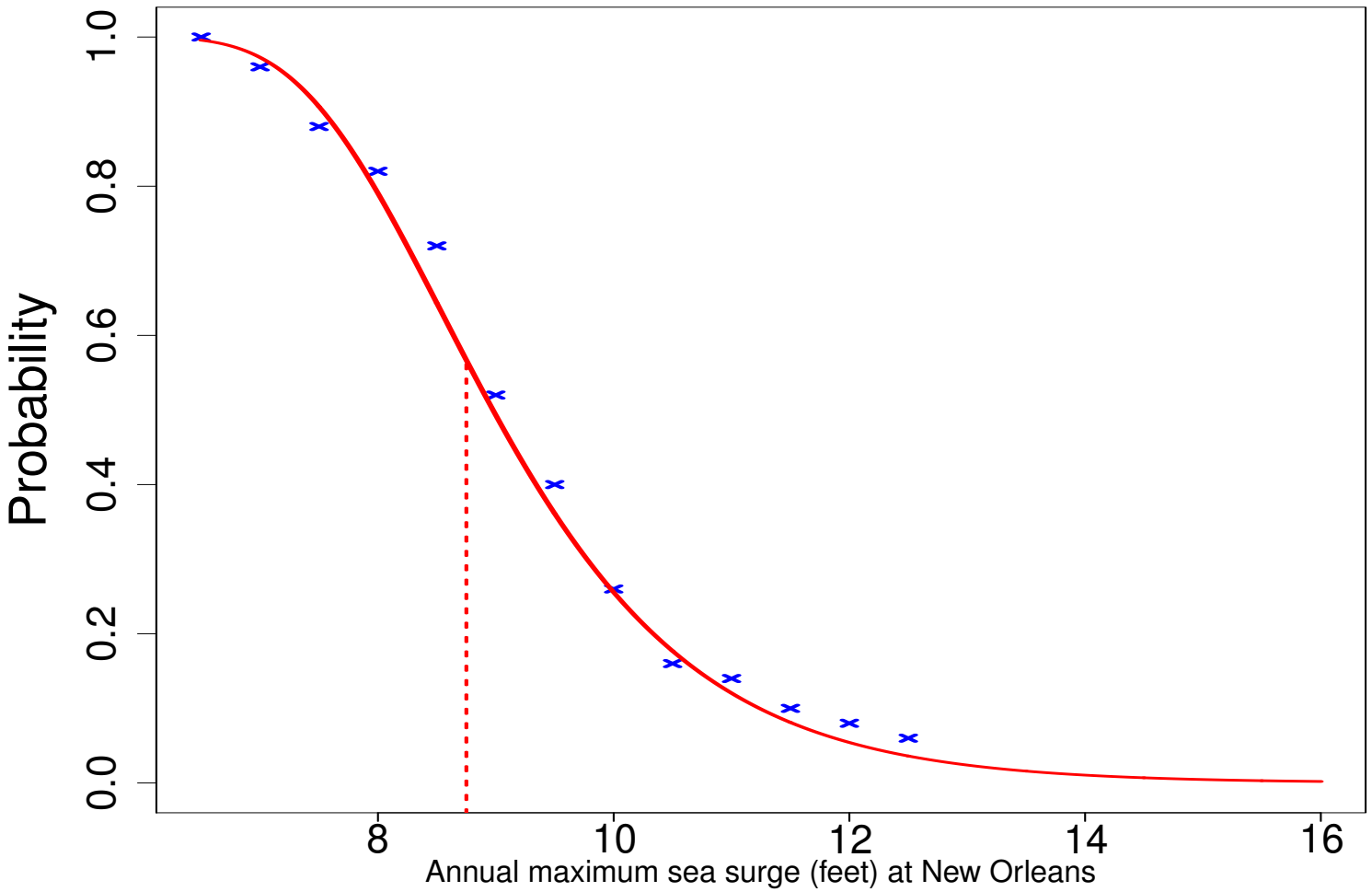

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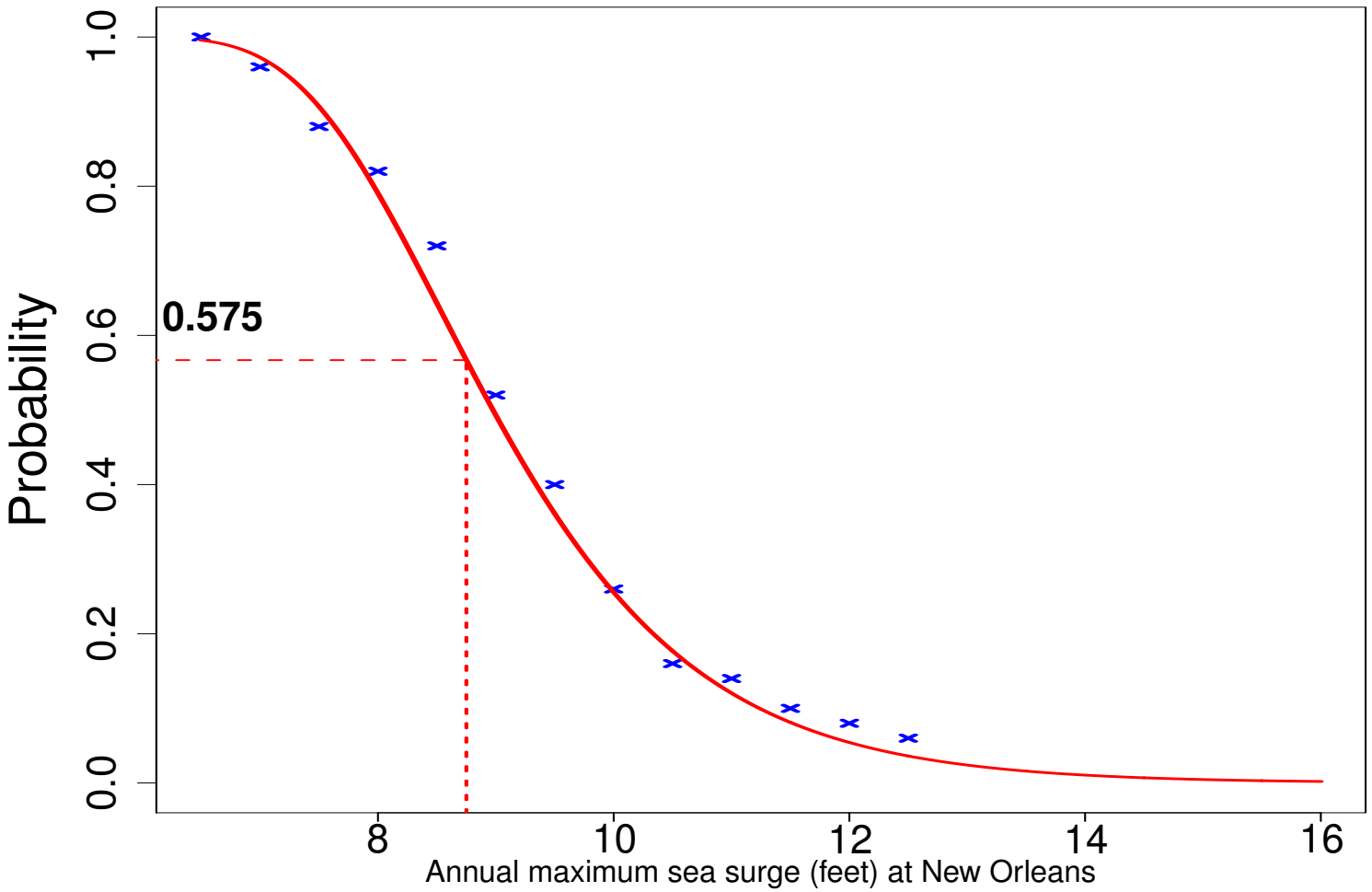

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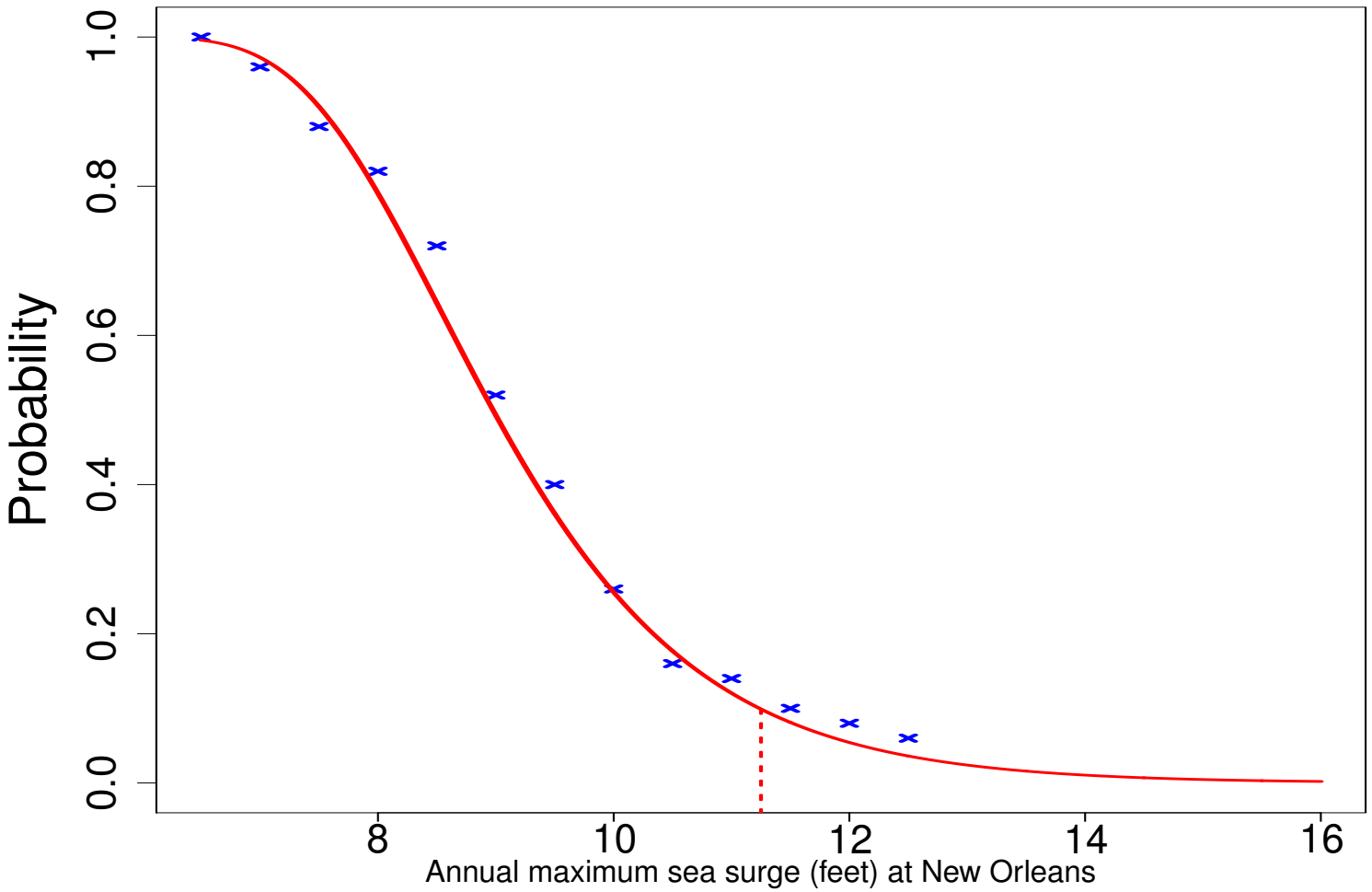

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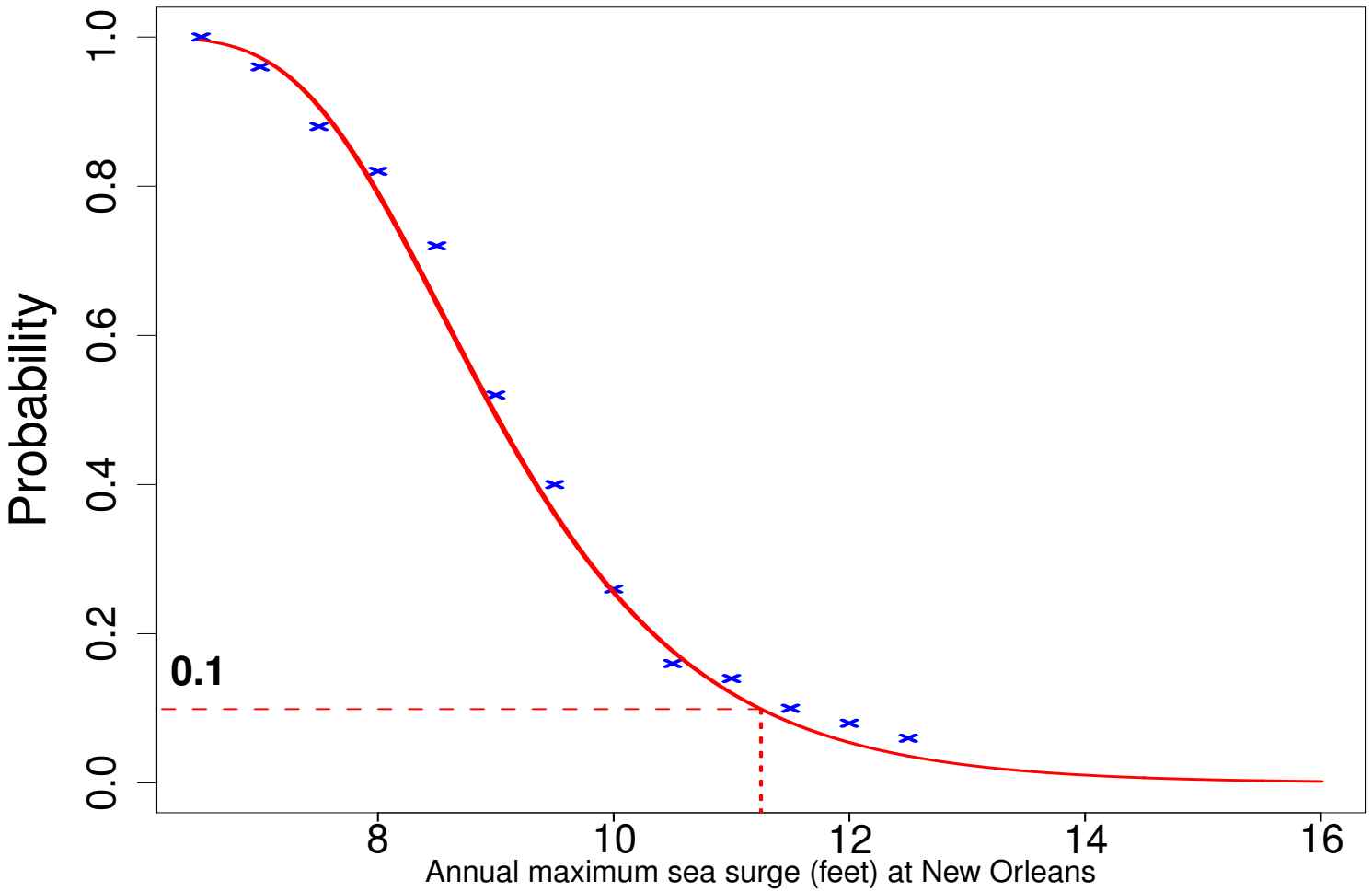

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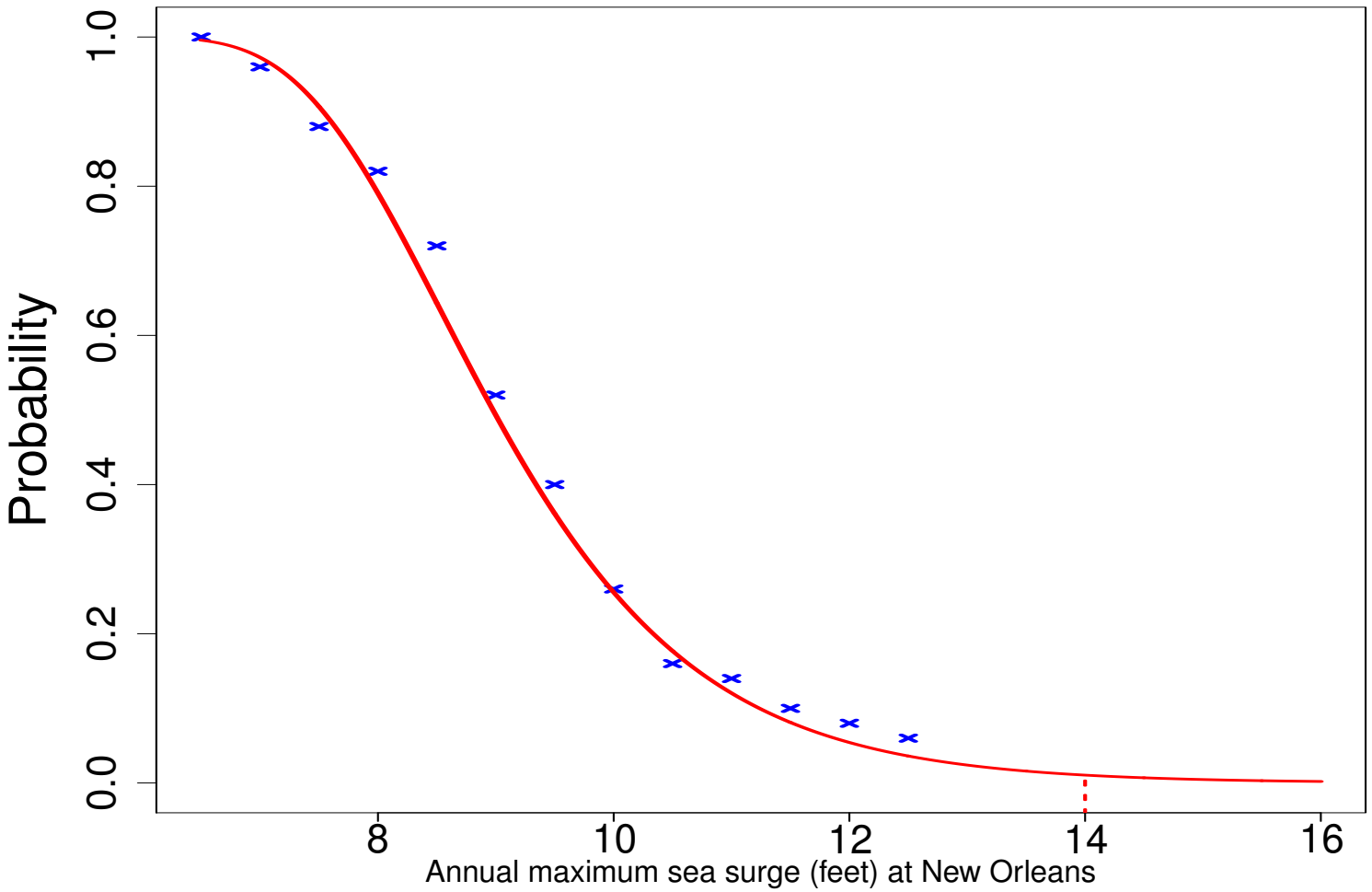

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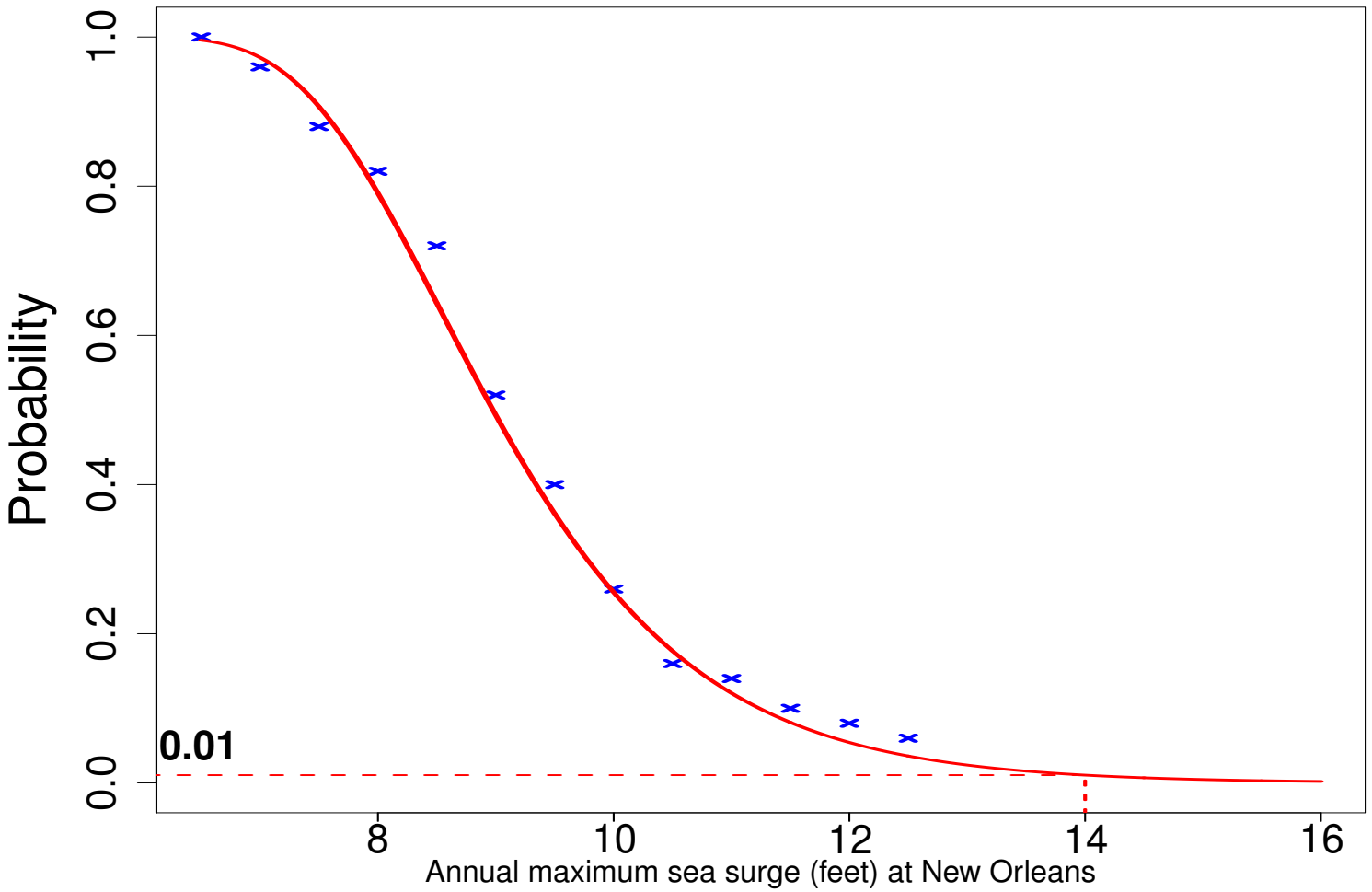

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So we get:

Probabilities	Exceeds		
	8.75 feet	11.25 feet	14 feet
Data alone	0.66	0.12	0
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Can you see why Katrina was billed the "**storm of the century**"?

Uncertainty!

So far, so good?!

The problem is, the 50 sea-surge annual maxima we have analysed are from **one** particular location in the Gulf of Mexico.

As you can imagine, there are **lots** of other data collection points in the Gulf of Mexico.

The Gumbel curve we have constructed will change depending on the dataset we use: it is defined by μ and σ...

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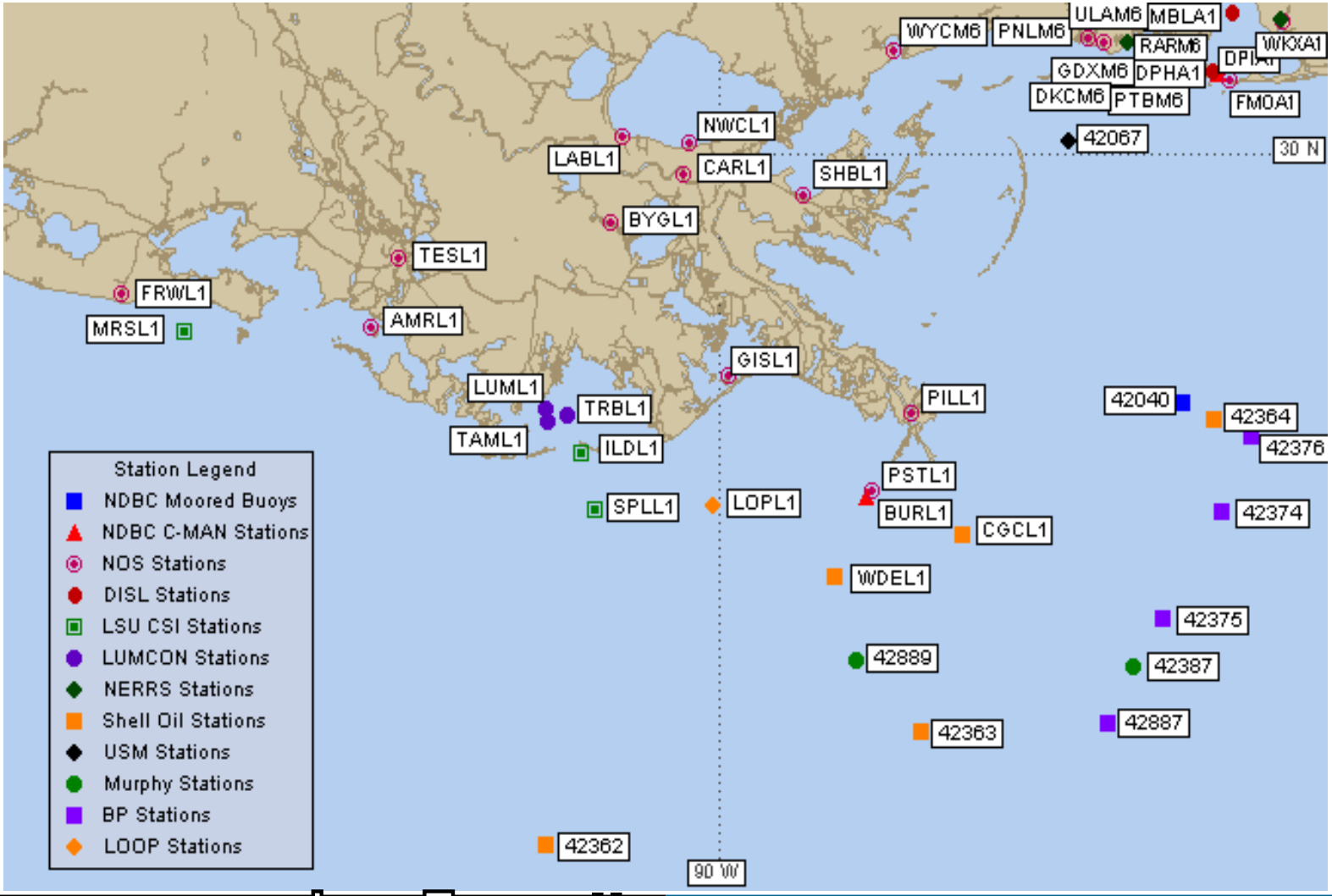

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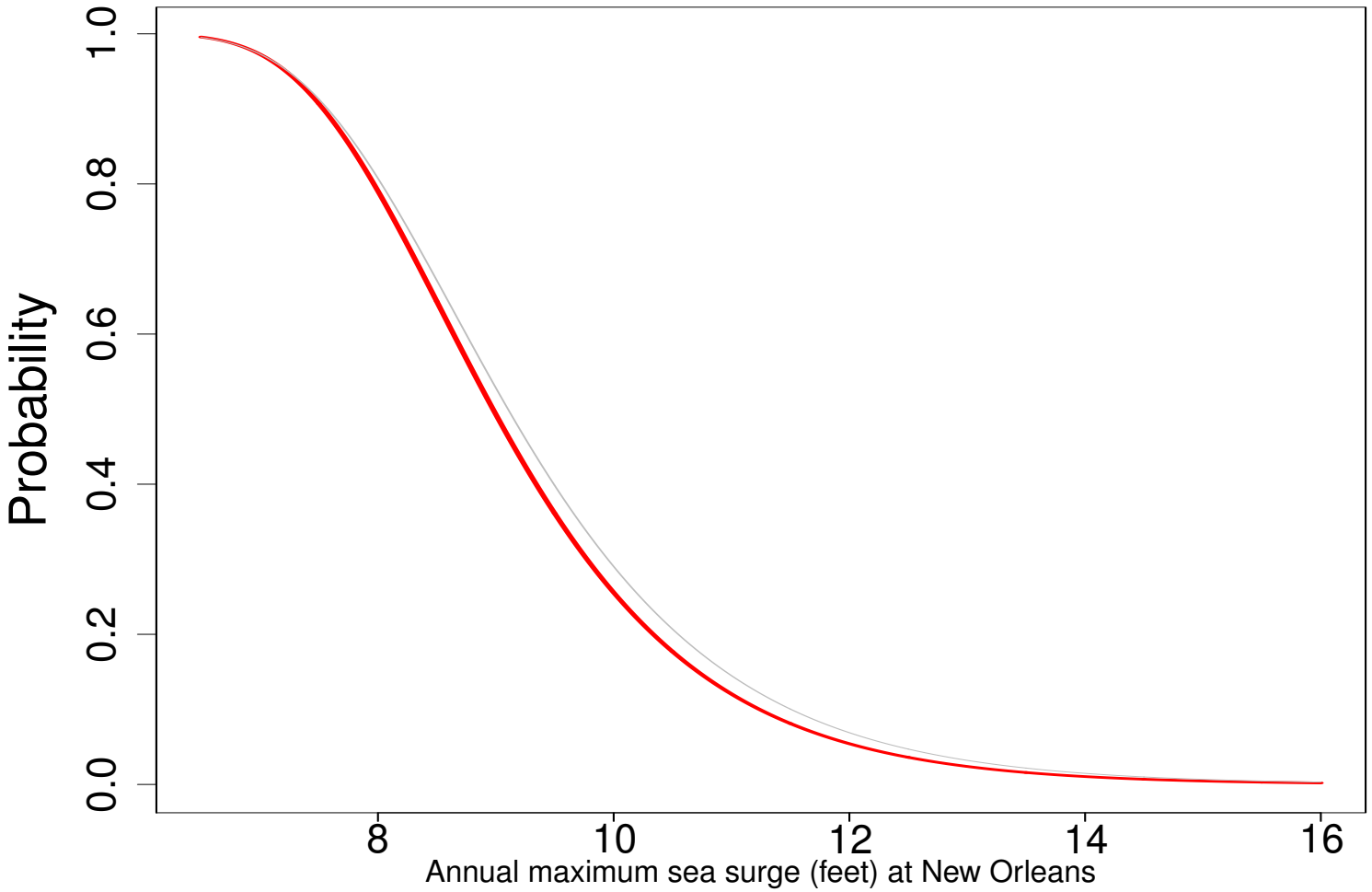

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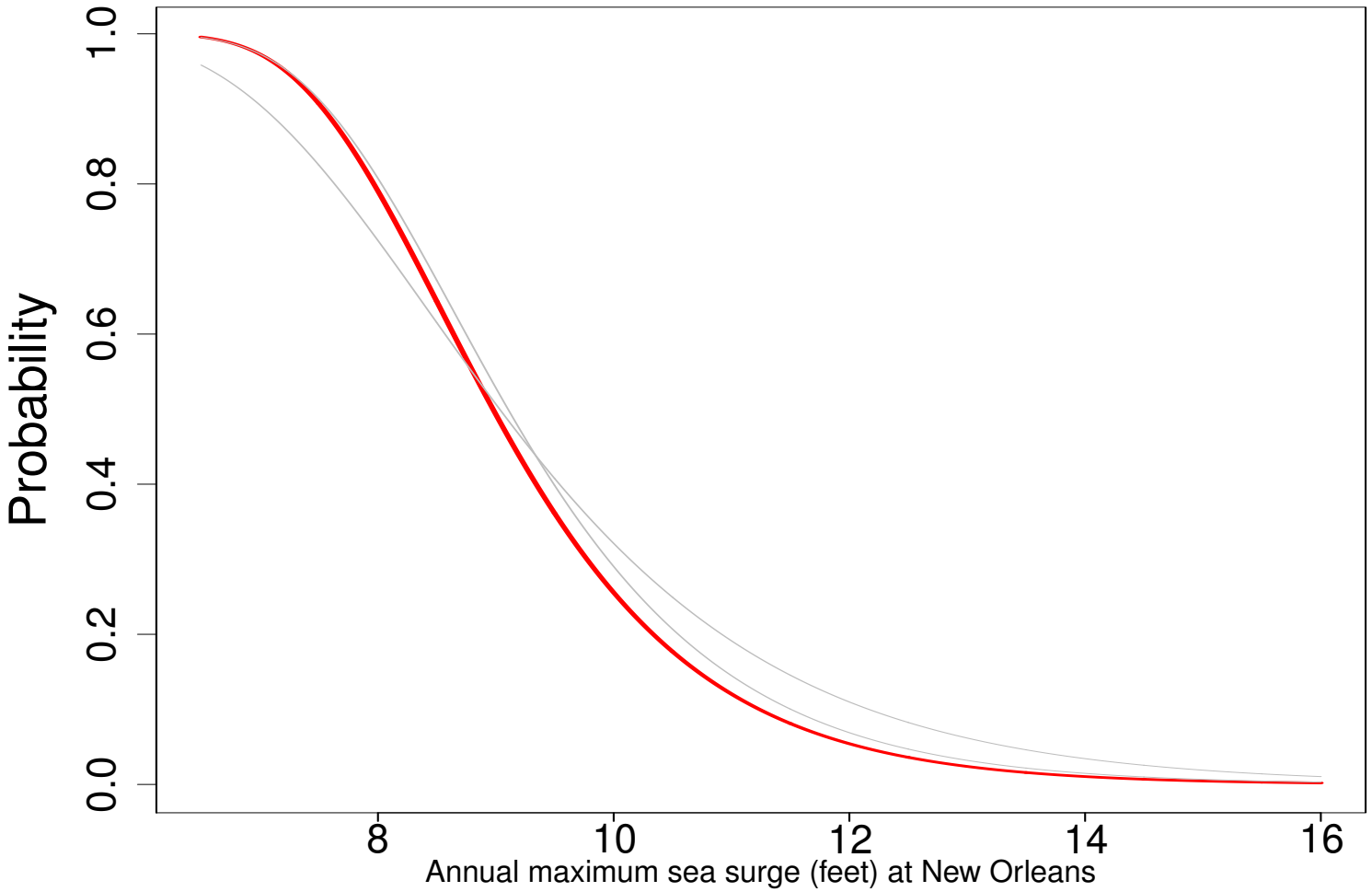

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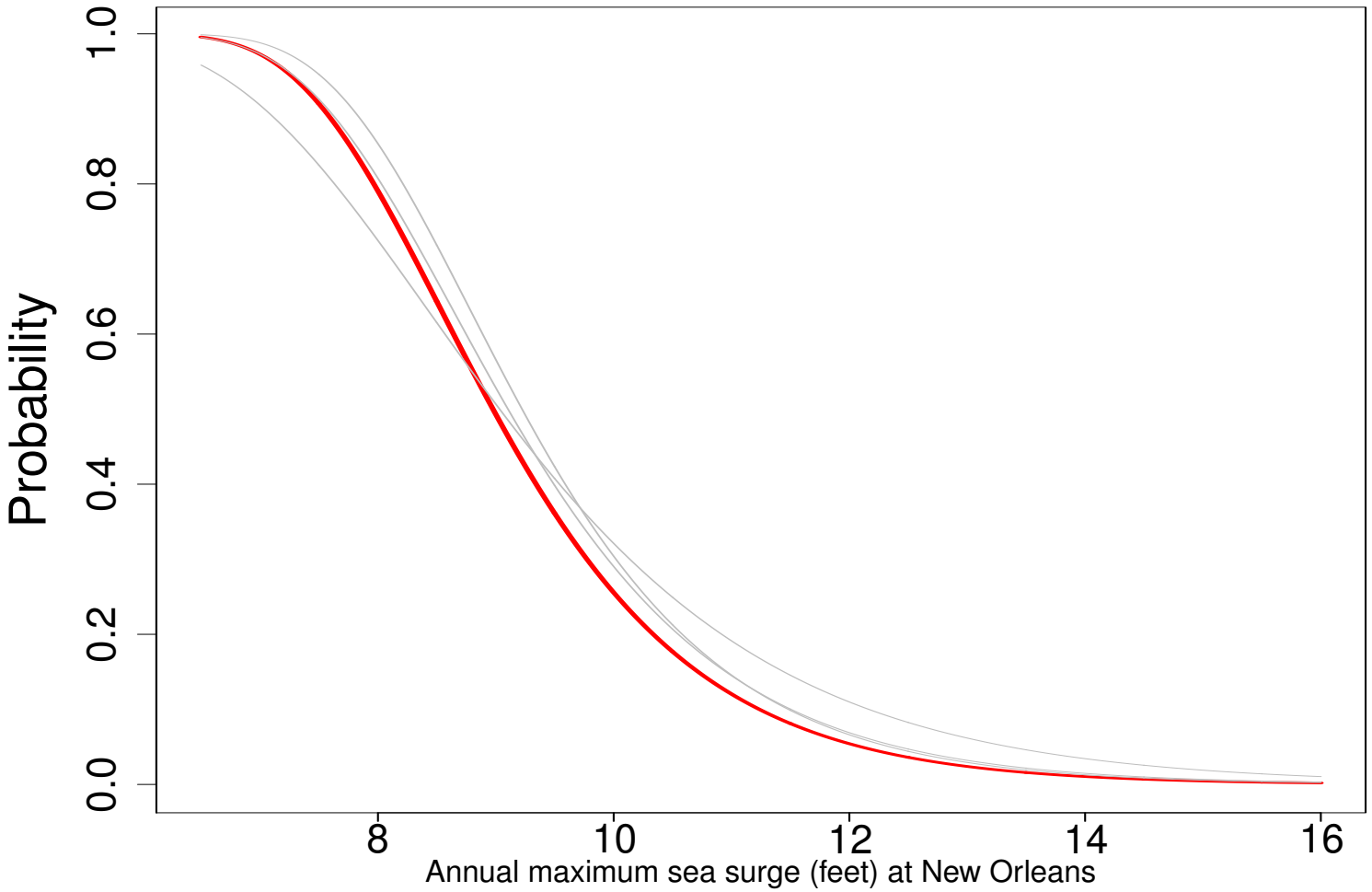

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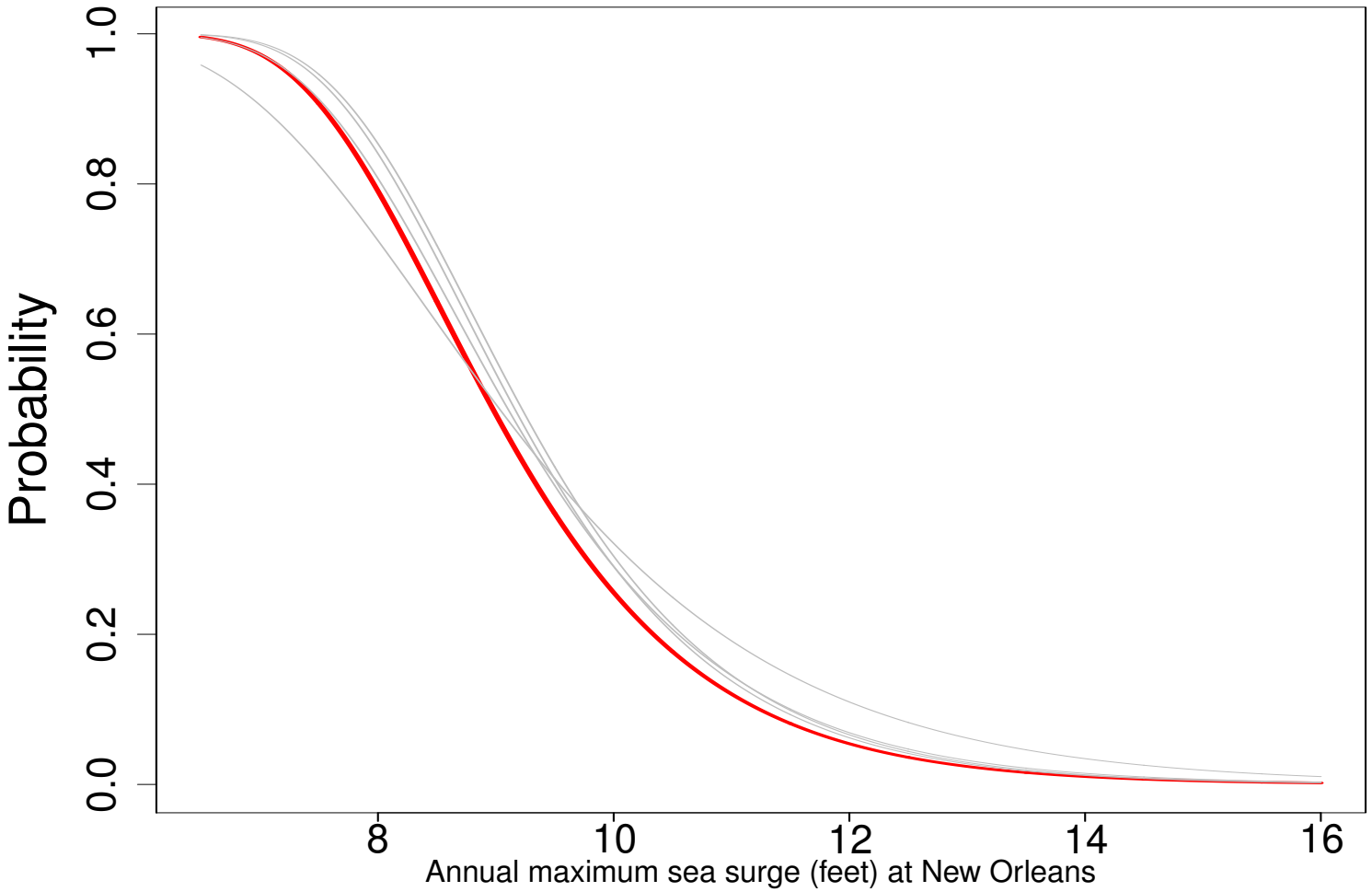

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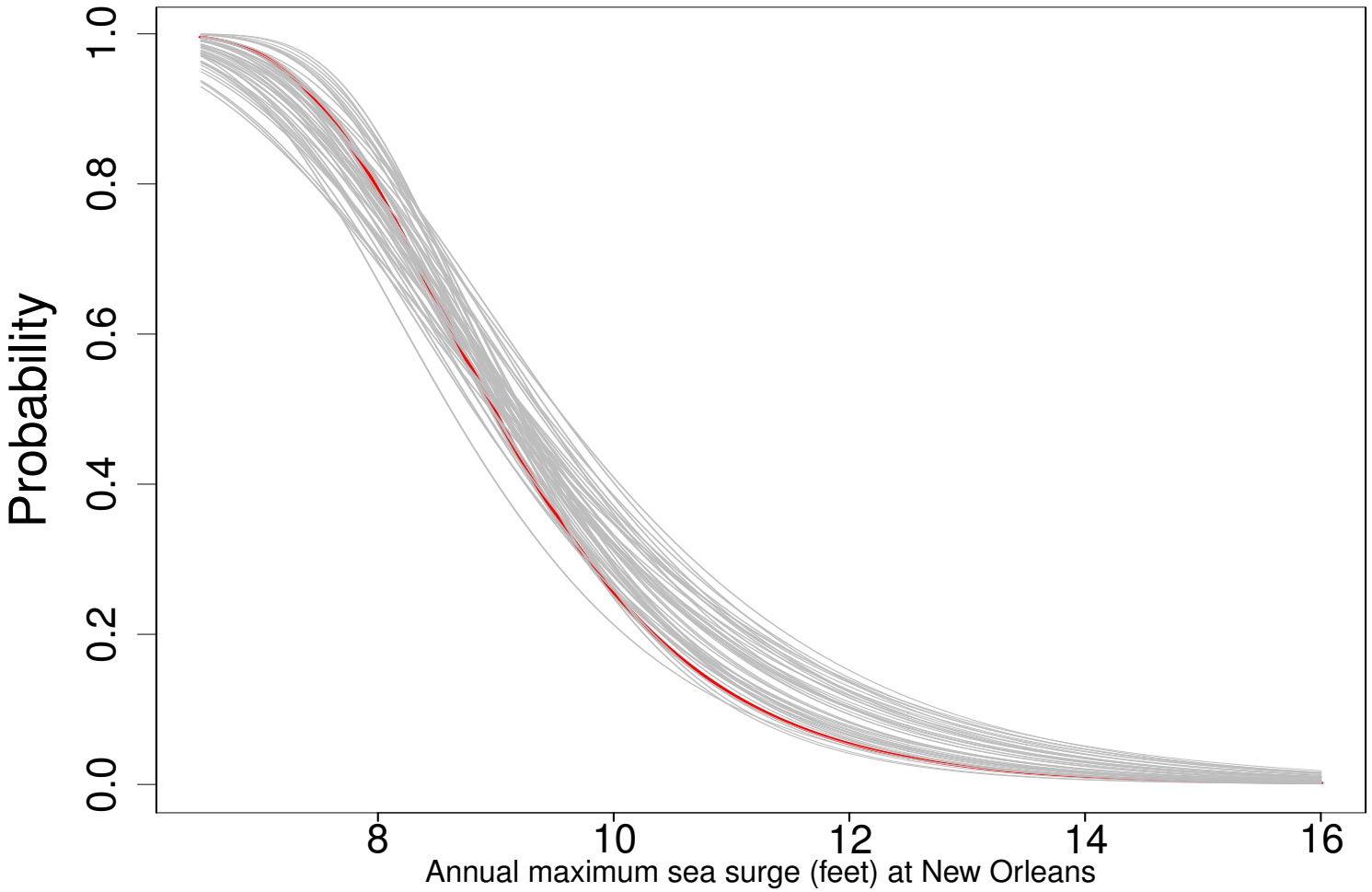

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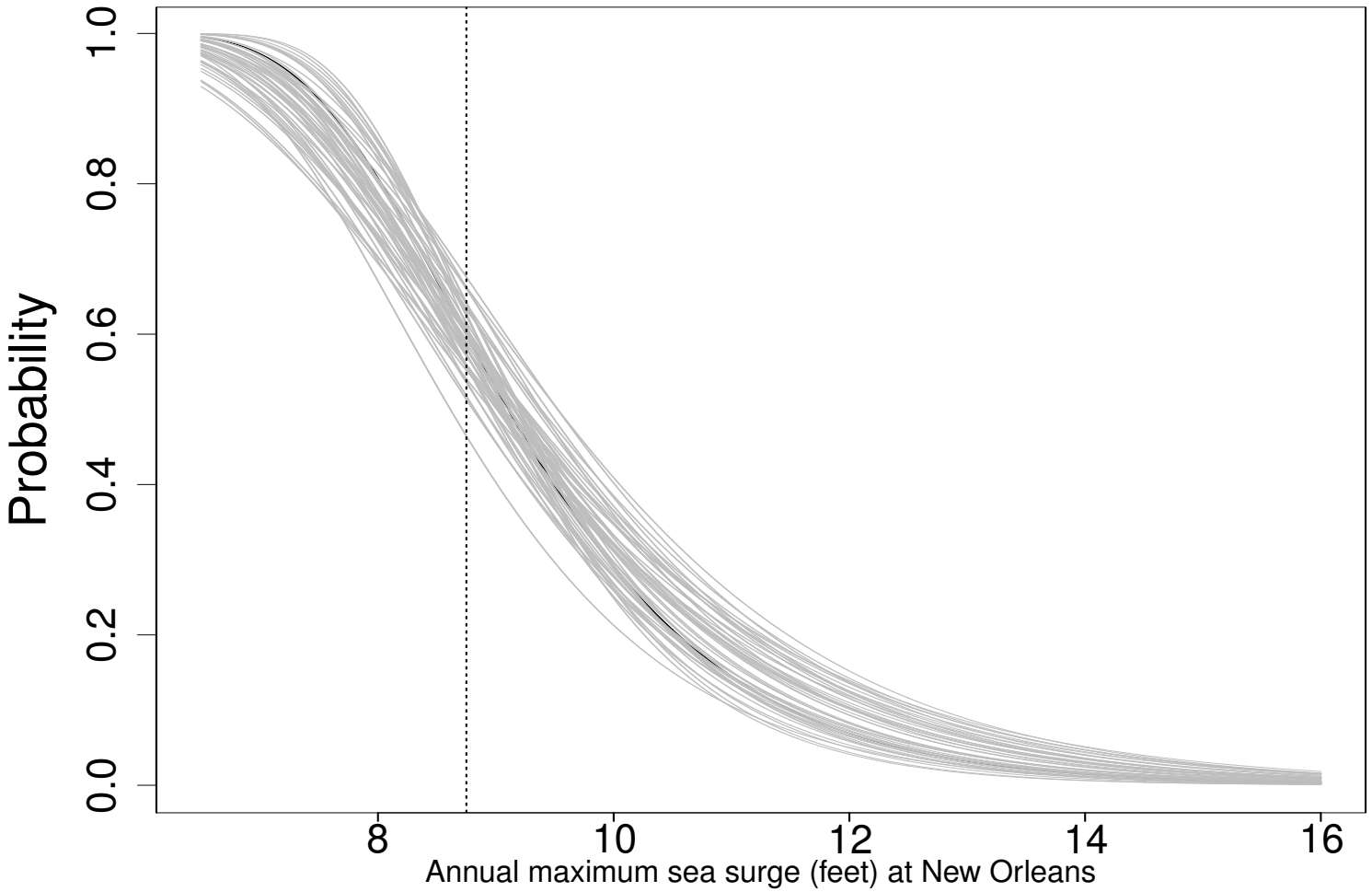

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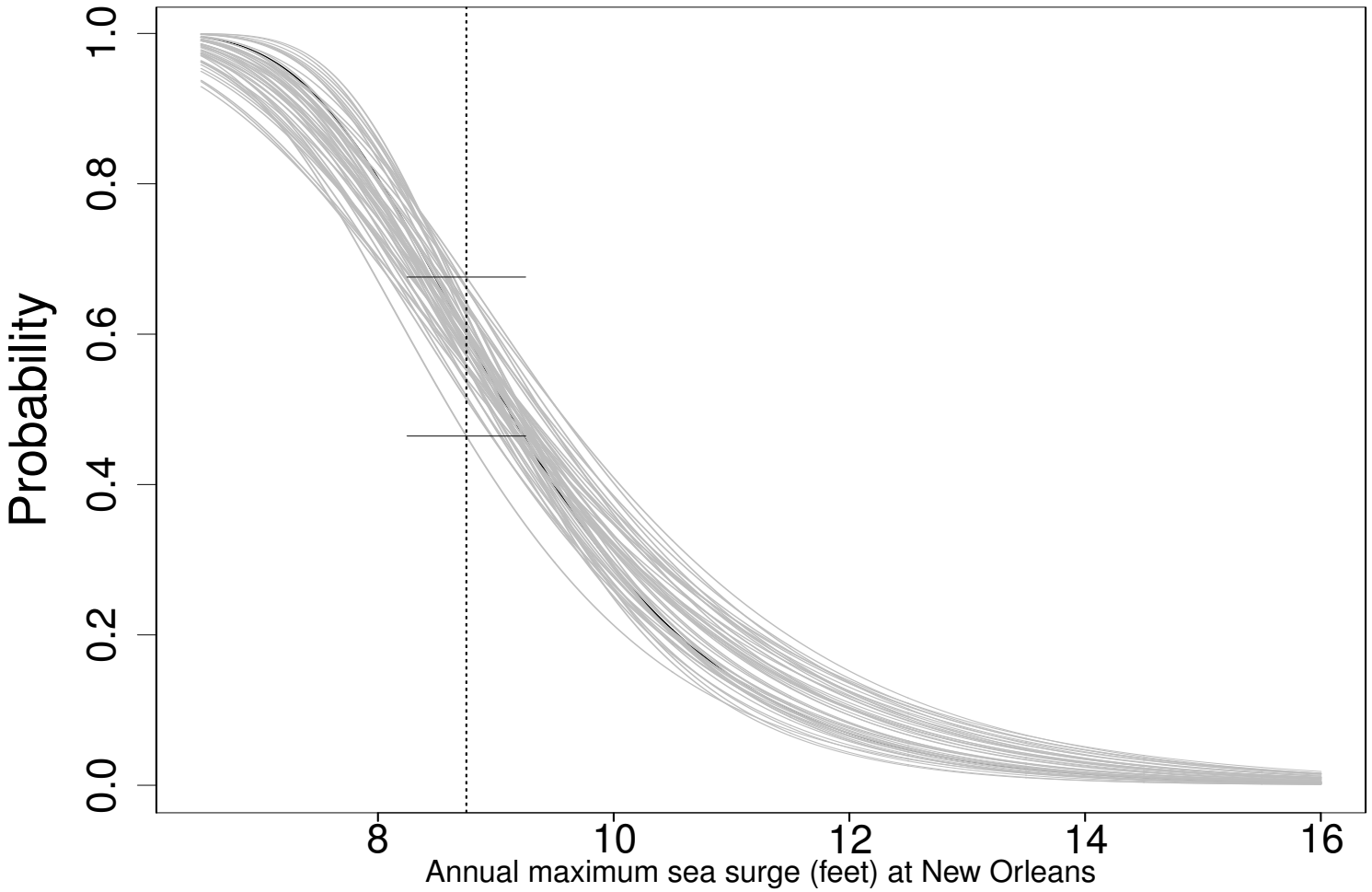

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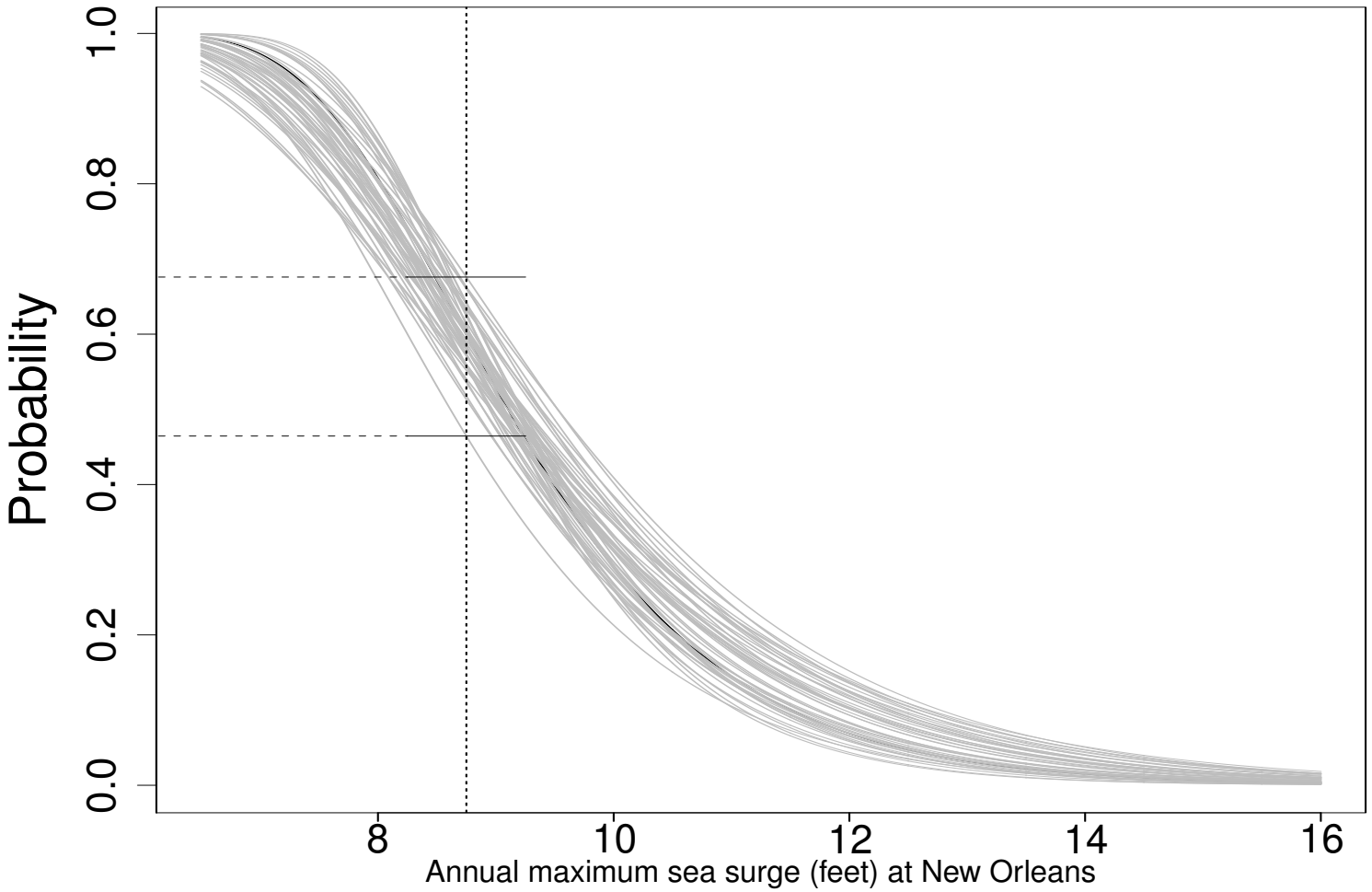

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So we get an updated table of results to **reflect our uncertainty**:

Probabilities	Exceeds		
	8.75 feet	11.25 feet	14 feet
Data alone	0.66	0.12	0
Gumbel model	0.575 (0.464, 0.676)	0.1 (0.083, 0.225)	0.01 (0.006, 0.051)

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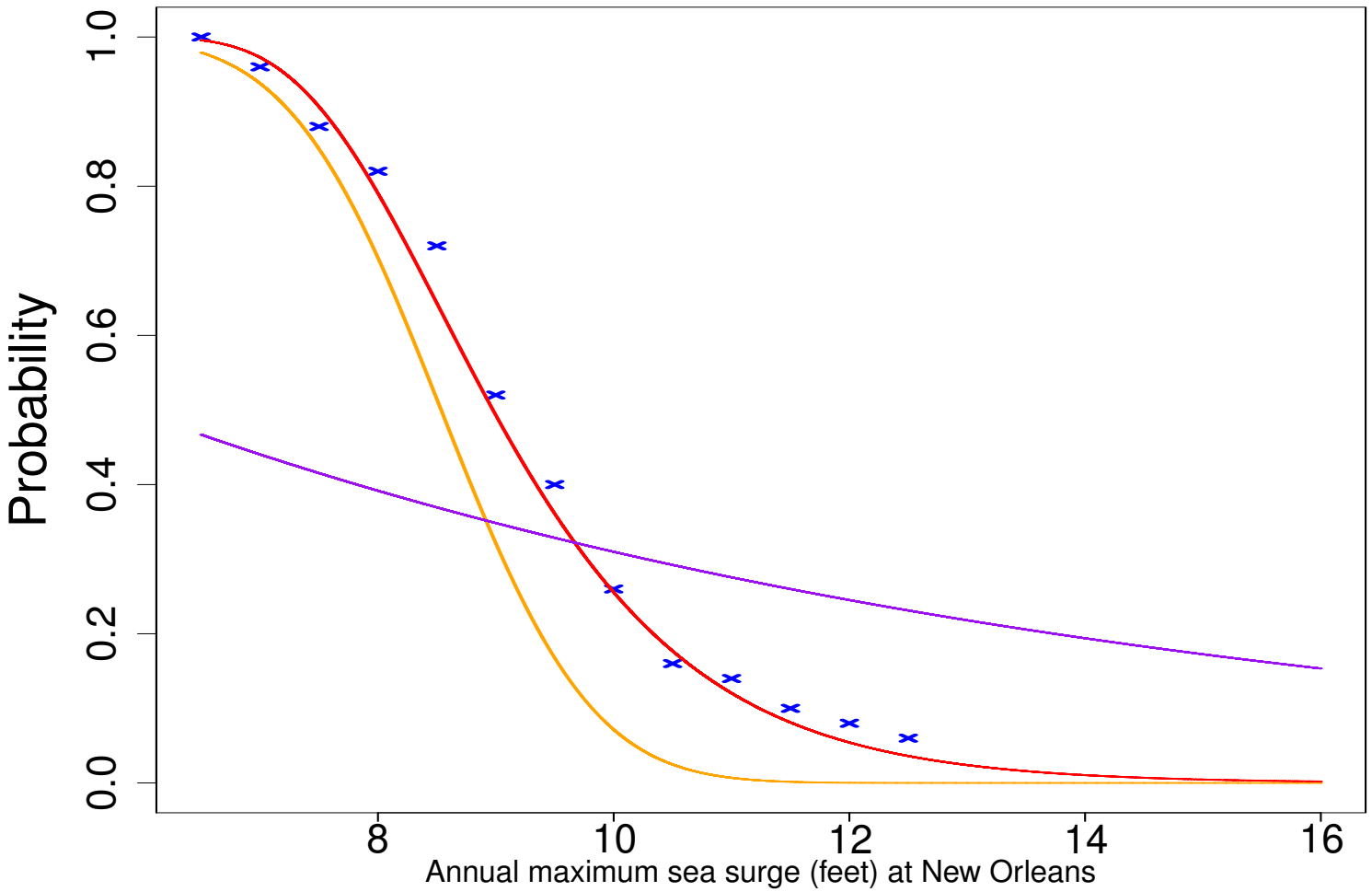

Structural design

As you might remember from the first part of this session, during Hurricane Katrina sea-surges exceeded 14 feet and parts of the sea wall system protecting the city were breached.

A new flood defence system is to be built; as the mathematician, you are asked how tall the sea wall should be to protect against the storm we might expect to see, on average, **once every 500 years**.

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Structural design

Use the Gumbel model to help estimate the height of the new sea wall.

We want x such that

$$\boldsymbol{P}(\text{Sea-surge} > x) = \frac{1}{500}.$$

Using Gumbel's formula, this gives

$$1 - \exp\left[-\exp\left\{-\left(\frac{x - 8.536}{1.2}\right)\right\}\right] = \frac{1}{500}.$$

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- High school mathematics gives you grounding in the basics
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  - Lecturers are also often **researchers**
  - Real–life research problems brought into the classroom
  - Excellent employment prospects if you do well!
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References

- Fawcett, L. and Newman, K. (2018). The Storm of the Century! Promoting Student Enthusiasm for Practical Statistics. *Teaching Statistics*, **3**(1).
- Fawcett, L. and Walshaw, D. (2016). Sea-surge and Wind Speed Extremes: Optimal Estimation Strategies for Planners and Engineers. *Stochastic Environmental Research and Risk Assessment*, **30**, pp. 463–480.