

In this short video, I just wanted to pull a few things together from the last few lectures.

In particular, the notion of:

- Subjective probabilities
- Prior probabilities
- Posterior probabilities

Probability interpretations

Example: E : Get a six when you roll a die.

1 Frequency interpretation:

Roll the die 1000 times, observe 175 sixes:

$$\begin{aligned}\Pr(E) &= \frac{\text{No. of times } E \text{ occurs}}{\text{No. of times experiment performed}} \\ &= \frac{175}{1000} = \frac{7}{40} = 0.175.\end{aligned}$$

2 Classical interpretation:

Assumes all outcomes are equally likely:

$$\Pr(E) = \frac{1}{6} = 0.167.$$

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3 Subjective interpretation:

I think this die is biased towards getting a six. I reckon

$$\Pr(E) \approx \frac{1}{3}.$$

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Subjective interpretation

Your **subjective probability** for an outcome A represents your own judgement of the likelihood that the outcome will occur.

This judgement will be based on the beliefs and information H you have at the time.

One way of determining (or quantifying) a subjective value for $\Pr_H(A)$ is to consider a series of possible bets with outcome

win $\pounds c$ if A occurs and $\pounds 0$ if A^c occurs.

How much would you be prepared to pay (stake) for placing such a bet?

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How much would you be prepared to pay (stake) for placing such a bet?

In terms of expected winnings, you should be prepared to stake $\pounds p$ if you believe that $\Pr_H(A) = p$. Why?

Suppose you would win $\pounds 10$ if A occurs and $\pounds 0$ if it does not.
Suppose further that you believe A is likely to occur with probability $p = 0.3$.

Then the **expected monetary value** of the bet is
 $\pounds 10 \times 0.3 = \pounds 3$;

that is, on average, we can expect to win $\pounds 3$ from this bet.
Thus, staking any more than $\pounds 3$ would not be wise!

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One problem with this approach is that, in general, p will depend on c .

A person who is willing to bet £1 on the spin of a coin to win £2 if it lands heads may refuse to bet if the stakes are raised to £1000: most people are **risk-averse**.

Therefore, we shall restrict our attention to the $c = 1$ case: pay £ p for the bet

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Subjective interpretation

You can make sure that your bet is “honest” by randomising between whether you “**host**” the bet or “**place**” the bet.

For example, suppose you believe that $\Pr_H(A) = 0.5$.

An “honest” bet would mean that you would buy the bet for a maximum stake of £0.50.

However, if you weren't honest you might try to buy the bet for any amount less than £0.50, say £0.20.

If you were hosting the bet, you would take the bet for any amount more than £ p , say for £0.80.

These conflicting interests can be offset if, when choosing p , it is equally likely that you are hosting the bet or placing the bet.

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Bruno di Finetti (1906–1985)

- Italian mathematical probabilist, developed his ideas on subjective probability in the 1920s.

- Famed for saying

“Probability does not exist”

- By this, he meant it has no *objective* existence – i.e. it is not a feature of the world around us.
- It is a measure of degree-of-belief – your belief, my belief, someone else's belief – all of which could be different.

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Dennis Lindley (1923—2013)

- Worked hard to find a mathematical basis for the subject of Statistics
- With Leonard Savage, he found a deeper justification for Statistics in **Bayesian theory**
- Turned into a critic of the classical statistical inference he had hoped to justify
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“If a decision-maker thinks something cannot be true and interprets this to mean it has zero probability, he will never be influenced by any data, which is surely absurd. So leave a little probability for the moon being made of green cheese; it can be as small as 1 in a million, but have it there since otherwise an army of astronauts returning with samples of the said cheese will leave you unmoved”

Example 1.1 (page 4)

- 1** The probability
 $\Pr(\text{Newcastle Utd win the Championship this season})$
could only be determined using a subjective assessment.
- 2** The probability
 $\Pr(\text{M\&S student chosen at random was born in January})$
could be determined using a frequency or classical interpretation (with a list of all M\&S students and their birth dates) or a subjective interpretation.
- 3** The probability
 $\Pr(\text{England win the toss at a given Test Match})$ could be determined using either the classical or subjective interpretation.

Example 1.2: Blood test

We were told that 0.5% of a population has a disease.
Therefore our **prior probabilities** are:

$$\begin{aligned}\Pr(D) &= 0.005 \quad \text{and} \\ \Pr(D^c) &= 0.995.\end{aligned}$$

We then found the **posterior probabilities**:

$$\begin{aligned}\Pr(D|+ve) &= 0.323 \quad \text{and} \\ \Pr(D^c|-ve) &= 0.9997.\end{aligned}$$

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Example 1.3: Car diagnosis

θ_1 : Fault in gearbox

θ_2 : Fault in engine

Also, there were three symptoms:

x_1 : Overheating

x_2 : Irregular traction

x_3 : Both

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$$\Pr(\theta_1) = 0.4 \quad \text{and}$$

$$\Pr(\theta_2) = 0.6.$$

We then found the **posterior probabilities**:

$$\Pr(\theta_1|x_3) = 0.625 \quad \text{and}$$

$$\Pr(\theta_2|x_3) = 0.375.$$

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Example 1.3: Car diagnosis

In terms of **odds**:

$$\text{Prior odds : } \frac{\Pr(\theta_1)}{\Pr(\theta_2)} = \frac{0.4}{0.6} = \frac{2}{3} \quad \text{or 3:2 in favour of } \theta_2$$

$$\text{Posterior odds : } \frac{\Pr(\theta_1|x_3)}{\Pr(\theta_2|x_3)} = \frac{0.625}{0.375} = \frac{5}{3} \quad \text{or 5:3 in favour of } \theta_1.$$

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Example 1.4: Multiple choice exam

$\Pr(K)$ $= \theta$	$\Pr(K C)$ $= 5\theta/(1 + 4\theta)$
0.0	0.000
0.1	0.357
0.2	0.556
0.3	0.682
0.4	0.769
0.5	0.833
0.6	0.882
0.7	0.921
0.8	0.952
0.9	0.978
1.0	1.000