Lecture: 1pm, Tuesday 23rd November

MAS2906: Computational Probability and Statistics with R

Newcastle University
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Semester 1, 2021/22

Assessments

- Numbas test 1 – will make a short video going through the questions very soon, with feedback on your work
- Mini Project
  - Now available in Canvas
  - Due 4pm, Friday 10th December
  - You can work in pairs if you like!
  - Standard submission via Canvas; no Numbas; any code should be included in your solutions document
  - Will give you help with this in the practical sessions this Thursday – do come along!

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Example 5.2

Example

If $X \sim N(0,1)$, what is the PDF of $Y = X^2$?

Can look at this informally in R:

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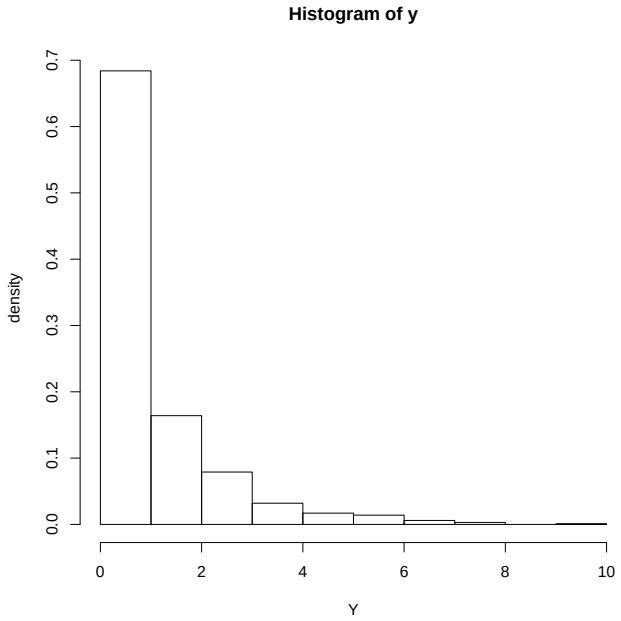
y = (rnorm(1000))^2
hist(y, xlab="Y", ylab="density", prob=TRUE)
```



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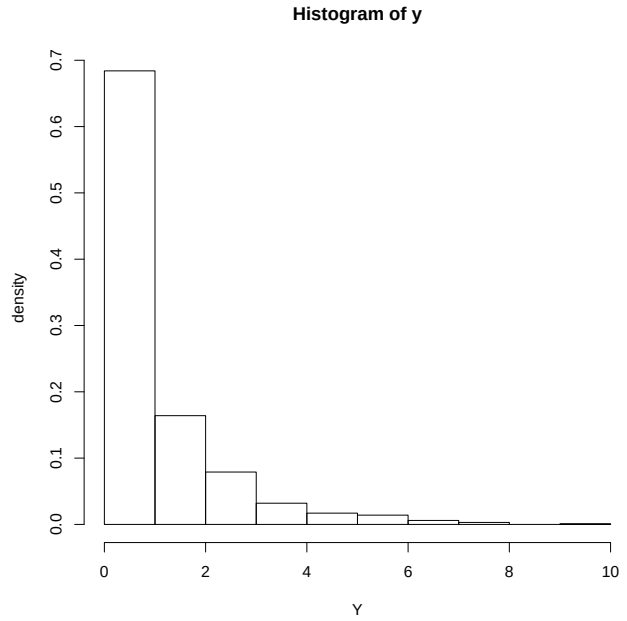


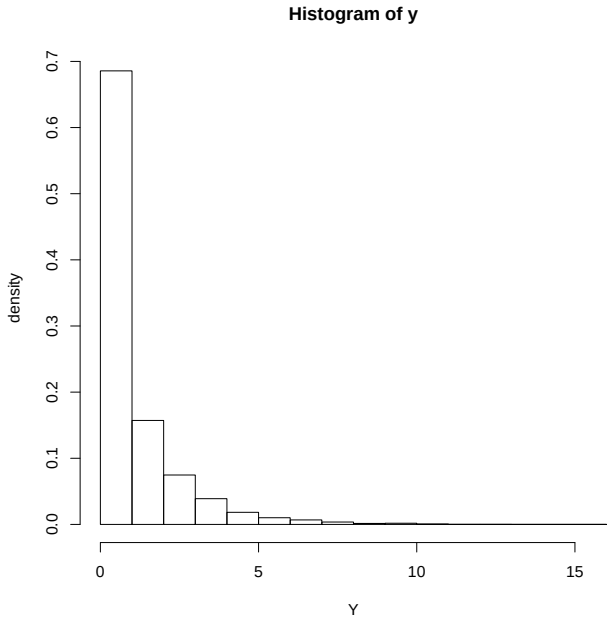
Example 5.2

Example

If $X \sim N(0,1)$, what is the PDF of $Y = X^2$?

Can look at this informally in R:

```
y = (rnorm(10000))^2
hist(y, xlab="Y", ylab="density", prob=TRUE)
```



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Example

If $X \sim N(0,1)$, what is the PDF of $Y = X^2$?

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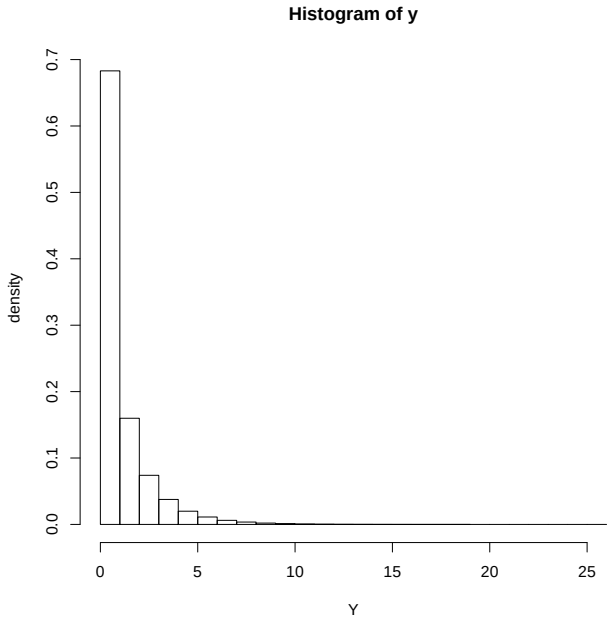
```
y = (rnorm(1000000))^2
hist(y, xlab="Y", ylab="density", prob=TRUE)
```



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```

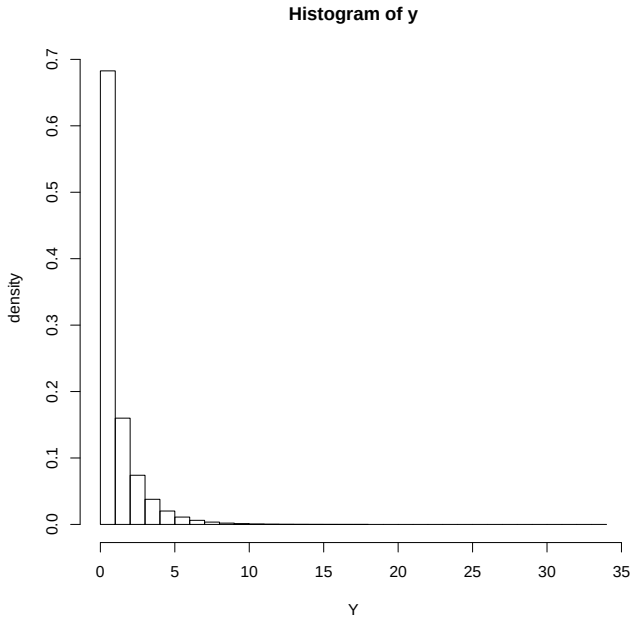

Example 5.2

What about the mathematical solution?

$$
\begin{aligned}
F_Y(y) &= \Pr(Y \leq y) \\
&= \Pr(X^2 \leq y) \\
&= \Pr(-\sqrt{y} \leq X \leq \sqrt{y}) \\
&= \Pr(X \leq \sqrt{y}) - \Pr(X \leq -\sqrt{y}) \\
&= \Pr(X \leq \sqrt{y}) - \{1 - \Pr(X \leq \sqrt{y})\} \\
&= 2 \cdot \Pr(X \leq \sqrt{y}) - 1 \\
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Then

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\begin{aligned}
f_Y(y) &= \frac{d}{dy}\left(2 \cdot F_X(y^{1/2}) - 1\right) \\
&= 2 f_X(y^{1/2}) \times \frac{1}{2} y^{-1/2} \quad \text{by chain rule} \\
&= \frac{1}{\sqrt{y}} f_X(\sqrt{y}) \\
&= \frac{1}{\sqrt{y}} \times \frac{1}{\sqrt{2\pi}} \exp\left\{-\frac{1}{2}(\sqrt{y})^2\right\} \\
&= \frac{1}{\sqrt{2\pi y}} e^{-y/2}, \quad \text{for } y \geq 0, \text{ zero otherwise.}
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This is the PDF for a **chi-squared distribution on 1 degree of freedom**.

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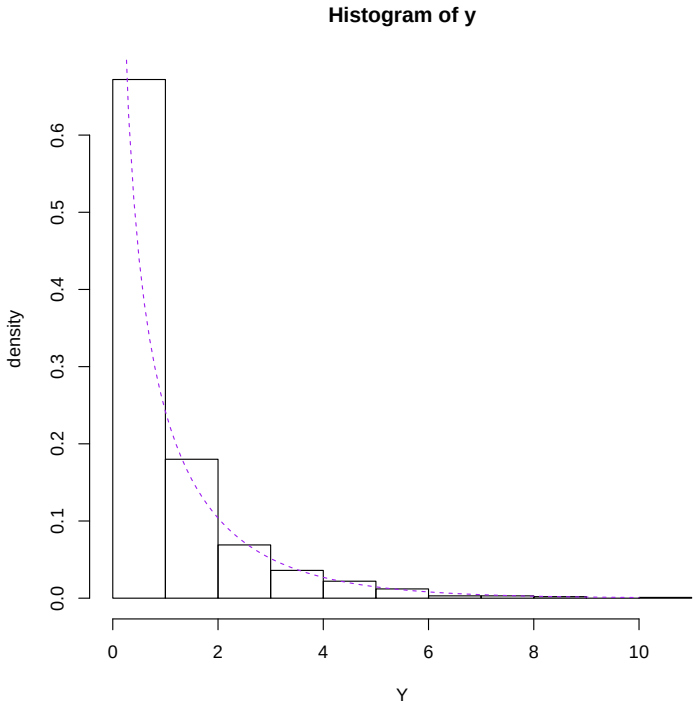
Example 5.2

To confirm this result we can simulte Y:

```
y = (rnorm(1000))^2
hist(y, xlab="Y", ylab="density", prob=TRUE)
curve((1/sqrt(2*pi*x))*exp(-x/2), from=0, to=10, col="
    purple", lty=2, add=TRUE)
```



Example 5.3: To try in your own time!

Example

If $X \sim N(\mu, \sigma^2)$, what is the PDF of $Y = \exp(X)$?
This is the example used at the start of this section of the notes.
We say Y has the **log-normal** distribution and write
$Y \sim LN(\mu, \sigma^2)$.

$$
\begin{aligned}
F_Y(y) &= F_X(\log y) \quad \text{by usual argument with CDF} \\
f_Y(y) &= \frac{d}{dy} F_X(\log y) \\
&= f_X(\log y) \times \frac{1}{y} \quad \text{for } y \geq 0 \\
&= \frac{1}{\sigma y \sqrt{2\pi}} \exp\left\{ -\frac{1}{2} \left(\frac{(\log y - \mu)^2}{\sigma^2} \right) \right\}
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5.3 The transformation method

If X has PDF $f_X(x)$ and $h(x)$ is a monotonically increasing or monotonically decreasing function then

- h is invertible and
- the PDF of $Y = h(X)$ is

$$f_Y(y) = f_X(h^{-1}(y)) \times \left| \frac{d\left(h^{-1}(y)\right)}{dy} \right|$$

when y is in the range of $h(x)$ and zero otherwise.

This result allows direct calculation of $f_Y(y)$ and is called the univariate transformation method.

See Lee's scribbles for Proof.

Some Examples

Example

When $X \sim U(0,1)$, use the transformation method to find the PDF of $Y = X^3$.

$$h(x) = x^3, \quad h^{-1}(y) = y^{1/3}.$$

$$\begin{aligned} f_Y(y) &= 1 \times \frac{d}{dy} y^{1/3} \\ &= \frac{1}{3} y^{-2/3}, \quad 0 \le y \le 1. \end{aligned}$$

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Some Examples

Example

When $X \sim U(-1, 1)$ and $Y = \exp(X)$, use the transformation method to find the PDF of Y.

$$h(x) = e^x, \quad h^{-1}(y) = \log(y).$$

$$\begin{aligned} f_Y(y) &= \frac{1}{2} \times \left| \frac{d}{dy} \log(y) \right| \\ &= \frac{1}{2|y|} = \frac{1}{2y}, \quad e^{-1} \leq y \leq e^1, \text{ zero otherwise.} \end{aligned}$$

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Example

Use the transformation method to find the PDF of the log-normal distribution.

$$h(x) = e^x, \quad h^{-1}(y) = \log(y).$$

$$\begin{aligned} f_Y(y) &= f_X(\log(y)) \times \frac{1}{y} \qquad \text{for } y > 0 \\ &= \frac{1}{y} \times \frac{1}{\sigma\sqrt{2\pi}} \exp\left[-\frac{1}{2}\left(\frac{\log(y) - \mu}{\sigma}\right)^2\right]. \end{aligned}$$

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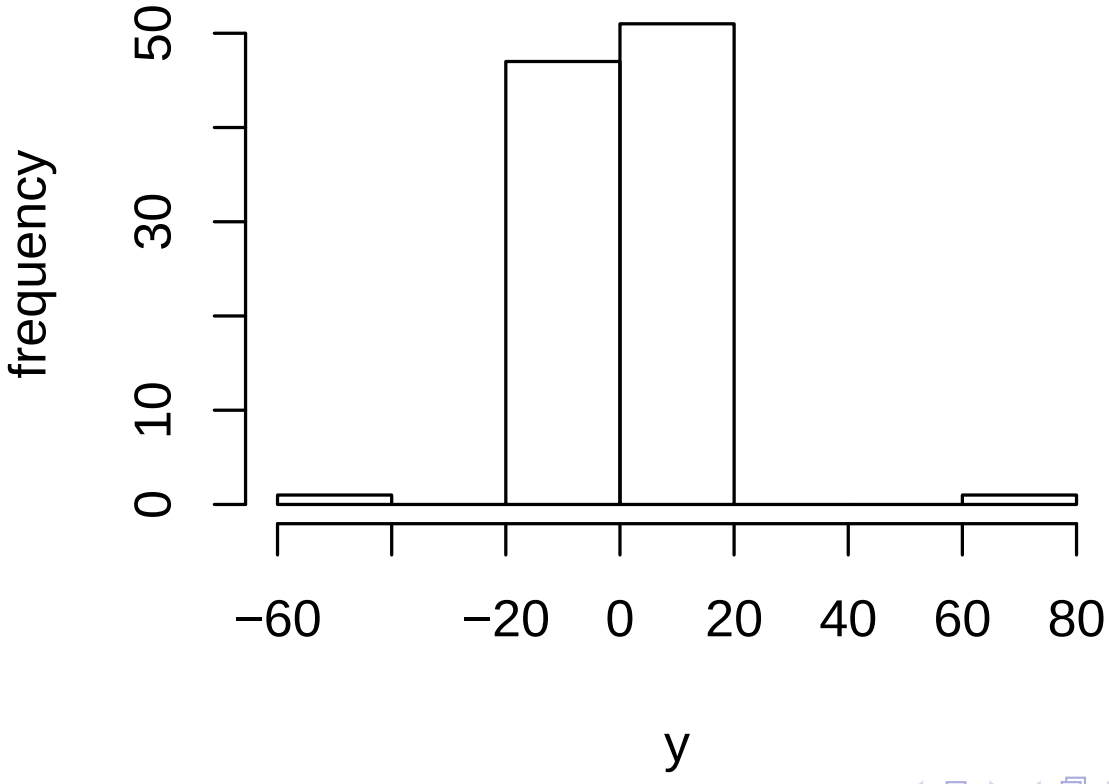
Cauchy example

Example

Suppose $X \sim U(-\frac{\pi}{2}, \frac{\pi}{2})$ and $Y = \tan(X)$.
Use simulation to study the distribution of Y and explicitly compute the PDF.

Solution

```r
y = tan(runif(100, -pi/2, pi/2))
hist(y, xlab = 'y', ylab = 'frequency', main = '')
```



Solution

This distribution looks odd! Calculate the PDF:

$$h(x) = \tan(x), \quad h^{-1}(y) = \tan^{-1}(y),$$

$$\begin{aligned} f_Y(y) &= \frac{1}{\pi}\frac{d}{dy}\left(\tan^{-1} y\right) \quad -\infty < y < \infty, \\ &= \frac{1}{\pi(1+y^2)} \quad -\infty < y < \infty. \end{aligned}$$

This is the **Cauchy distribution**. It has *very* heavy tails.

Solution

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Bonus example

Hourly wind speed maxima (X miles per hour), observed on the roof of Claremont Tower as part of the University's *Urban Observatory*, are assumed exponentially distributed with rate 0.1.

Unfortunately, the data recorded are the logged wind speeds.

Use the transformation method to find the PDF of $Y = \log X$.

Bonus example: Solution

Recall that if $X \sim Exp(\lambda)$, then it has PDF

$$f_X(x) = \lambda e^{-\lambda x}, \quad x > 0, \quad \lambda > 0.$$

Now here, we have

$$h(x) = \log x, \quad \text{and} \quad h^{-1}(y) = e^y.$$

Therefore

$$\begin{aligned} f_Y(y) &= f_X(h^{-1}(y)) \times \left| \frac{d\left(h^{-1}(y)\right)}{dy} \right| \\ &= \lambda \exp\left\{-\lambda \exp\{y\}\right\} \times \exp\{y\} \\ &= \lambda \exp\left\{y - \lambda e^y\right\}, \end{aligned}$$

with $\lambda = 0.1$.

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We can check this in R:

```
X = rexp(1000,  0.1)
Y = log(X)
hist(Y, prob=TRUE, main="Y=log(Wind Speeds)", xlab="
    Miles per hour")
lambda = 0.1
curve(lambda*exp(x-lambda*exp(x)), add=TRUE, col="red",
     lty=2, lwd=2)
```