MAS2906: Computational Probability and Statistics with R

Newcastle University
lee.fawcett@ncl.ac.uk

Semester 1, 2024/25

3.1 The Binomial distribution

If $X \sim Bin(n, p)$ then X has PMF (probability mass function) given by

$$p_X(k) = \binom{n}{k} p^k (1-p)^{n-k}, \quad \text{for } k = 0, 1, \ldots, n.$$

There is no closed formula for the CDF (cumulative distribution function).

R commands:

- `dbinom` – calculate PMF
- `pbinom` – calculate CDF
- `rbinom` – generate random sample

3.1 The Binomial distribution

If $X \sim Bin(n, p)$ then X has PMF (probability mass function) given by

$$p_X(k) = \binom{n}{k} p^k (1-p)^{n-k}, \quad \text{for } k = 0, 1, \ldots, n.$$

There is no closed formula for the CDF (cumulative distribution function).

R commands:

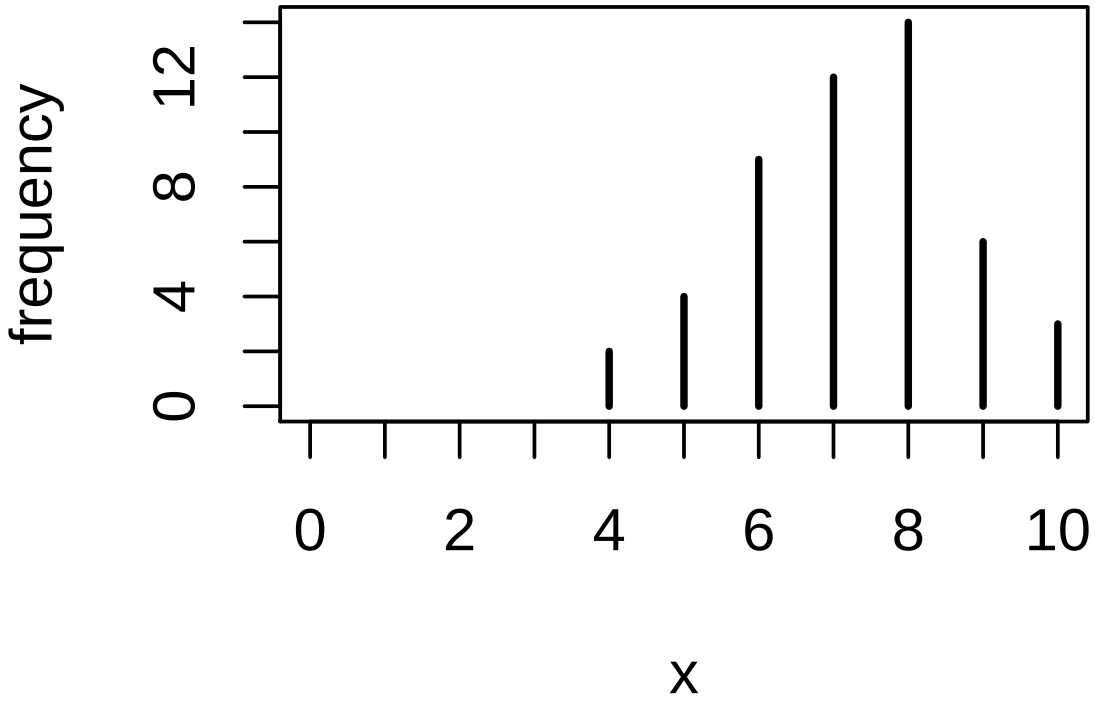
- `dbinom` – calculate PMF
- `pbinom` – calculate CDF
- `rbinom` – generate random sample

3.1 Commands for the Binomial distribution

```
dbinom(5, 10, 0.7)        # Pr(X = 5), X ~ Bin(10, 0.7)
pbinom(4, 10, 0.7)        # Pr(X ≤ 4), X ~ Bin(10, 0.7)
rbinom(50, 10, 0.7)       # Sample a Bin(10, 0.7) distribution 50 times
```

3.1 Creating a bar plot

```
x = rbinom(50, 10, 0.7)
plot(table(x), xlim = c(0,10), xaxt = 'n', xlab = 'x',
    ylab = 'frequency')
axis(1, at = seq(0, 10))
```



3.1 The Geometric distribution

If $Y \sim Geom(p)$ then Y has PMF and CDF given by

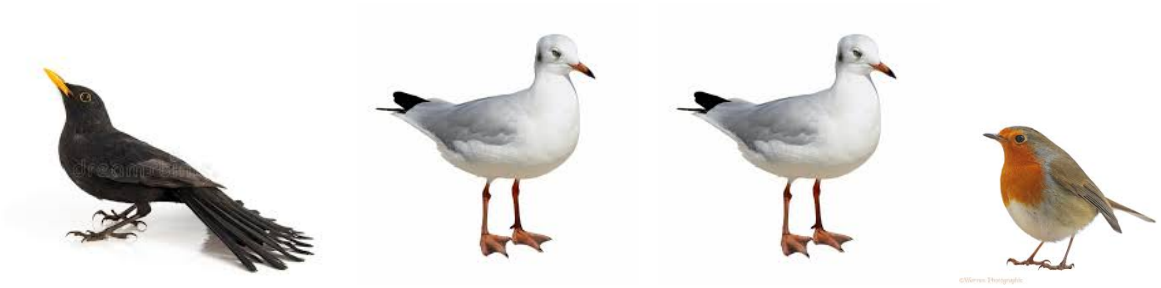
$$p_Y(k) = (1-p)^{k-1}p,$$
$$F_Y(k) = 1-(1-p)^k, \quad \text{for } k = 1, 2, \ldots.$$

Note that Y takes values in $1, 2, 3, \ldots$:

- R uses a slightly different definition
- We use the definition that Y is the number of Bernoulli trials up to and including the first success
- R counts number of trials up to, but **not** including the first success, so in R geometric random variables take values $0, 1, 2, \ldots$.

3.1 The Geometric distribution

Suppose Y: Number of birds I count until I spot a robin.

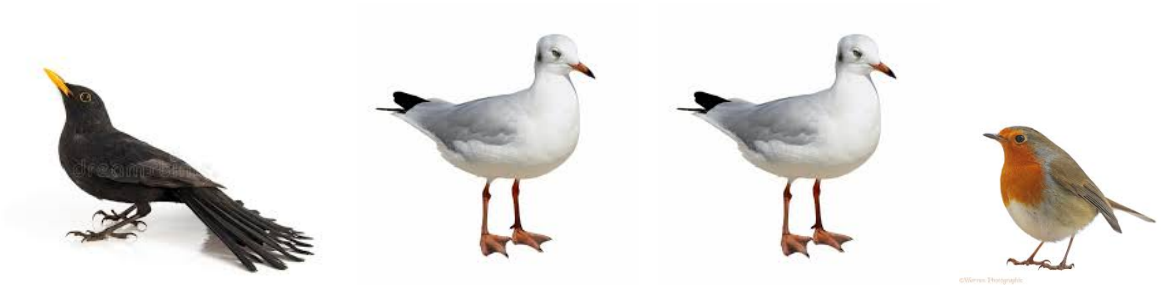


Standard definition: $Y = 4$

R's annoying definition: $Y = 3$

3.1 The Geometric distribution

Suppose Y: Number of birds I count until I spot a robin.

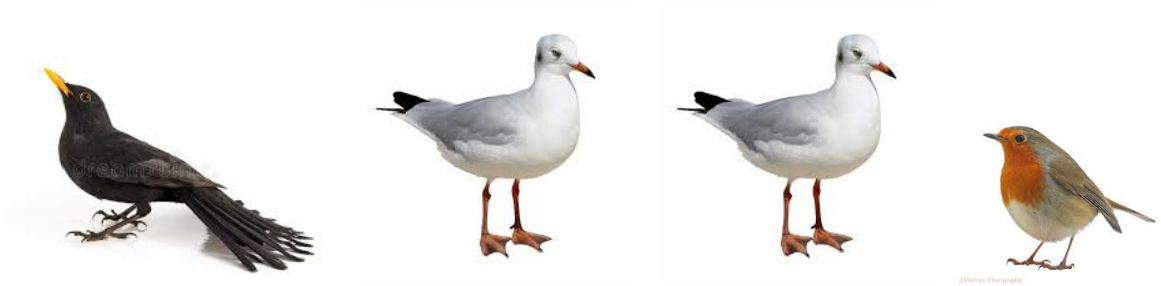


Standard definition: $Y = 4$

R's annoying definition: $Y = 3$

3.1 The Geometric distribution

Suppose Y: Number of birds I count until I spot a robin.



Standard definition: $Y = 4$
R's annoying definition: $Y = 3$

3.1 R commands for the Geometric distribution

Adjust the arguments to account for different definition:

```
dgeom(4, 0.2)              # Pr(Y = 5), Y ~ Geom(0.2)
pgeom(2, 0.2)              # Pr(Y ≤ 3), Y ~ Geom(0.2)
1 + rgeom(100, 0.2)        # Sample a Geom(0.2) distribution 100 times
```

Here's a function to replace `dgeom` with our definition of the geometric distribution:

```
mydgeom = function(x, p) {
    dgeom(x-1, p)}
```

3.1 The Poisson distribution

If $Z \sim Po(\lambda)$ then Z has PMF given by

$$p_Z(k) = \frac{\lambda^k}{k!} e^{-\lambda}, \quad \text{for } k = 0, 1, 2, \ldots.$$

There is no closed formula for the CDF (cumulative distribution function).

R commands:

- `dpois` – calculate PMF
- `ppois` – calculate CDF
- `rpois` – generate random sample

3.1 R commands for the Poisson distribution

```
dpois(5, 3.5)          # Pr(Z = 5), Z ~ Po(3.5)
ppois(2, 3.5)          # Pr(Z ≤ 2), Z ~ Po(3.5)
rpois(100, 3.5)        # Sample a Po(3.5) distribution 100 times
```

3.1 Summary

Distribution	Binomial	Poisson	Geometric ☠
PMF	`dbinom(...)`	`dpois(...)`	`dgeom(...)`
CDF	`pbinom(...)`	`ppois(...)`	`pgeom(...)`
sample	`rbinom(...)`	`rpois(...)`	`rgeom(...)`

3.2 Continuous random variables

R has functions for the uniform, exponential and normal distributions:

Distribution	Uniform	Exponential	Normal
PDF	`dunif(...)`	`dexp(...)`	`dnorm(...)`
CDF	`punif(...)`	`pexp(...)`	`pnorm(...)`
quantile	`qunif(...)`	`qexp(...)`	`qnorm(...)`
sample	`runif(...)`	`rexp(...)`	`rnorm(...)`

3.2 Continuous random variables – R examples

```
dunif(3, 2, 5)        # f_X(3), X ~ U(2,5)
pexp(1.5, 5)          # F_Y(1.5), Y ~ Exp(5)
rnorm(10, 0, 2)       # Sample a N(0,2^2) distribution 10 times
```

For the standard uniform ($U(0,1)$) and standard normal ($N(0,1)$) distributions you don't need to provide the parameters $a = 0$, $b = 1$ and $\mu = 0$, $\sigma = 1$ respectively. For example:

```
runif(20)             # Samples a U(0,1) distribution 20 times
pnorm(1.96)           # Pr(Z < 1.96), Z ~ N(0,1)
[1] 0.9750021
```

3.2 Quantiles

The quantile functions `qunif`, `qexp`, `qnorm` solve equations like

$$F_X(\alpha) = p$$

for α given a probability p. For example:

```
qnorm(0.9750021)
[1] 1.96
```

Example 3.1

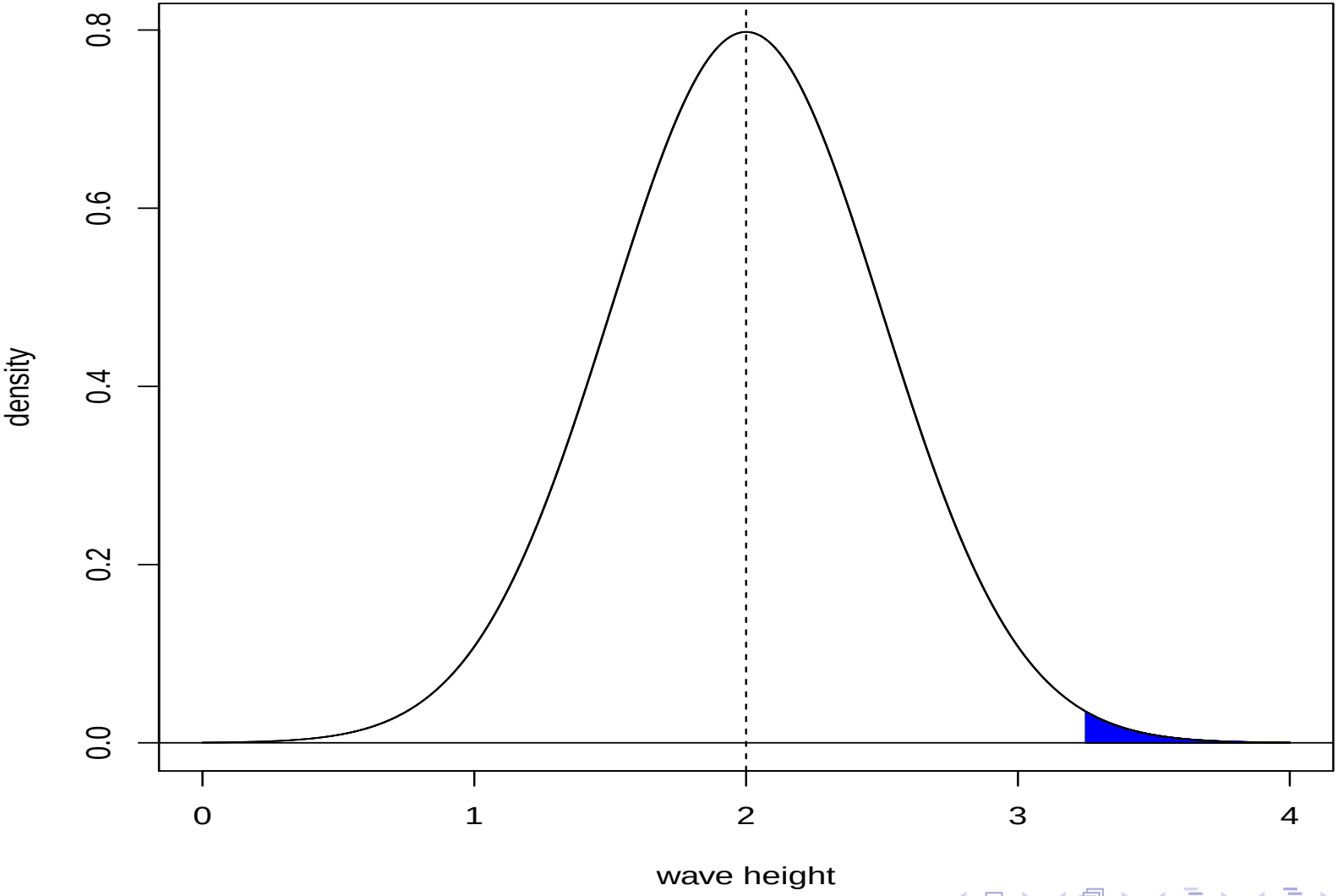
Example

Suppose annual maximum wave heights observed off the coast at a flood-prone town are assumed Normally distributed, with mean 2 metres and standard deviation 0.5 metres.

(a) Write down the R command to find the probability that the largest wave height next year will exceed 3.25 metres.

(b) Write down the R command to estimate the height of a new sea wall such that we might expect the town to be flooded, on average, once per century. Why might our modelling assumption be invalid?

Example 3.1: Solution (a)



Example 3.1: Solution (a)

Could work out the old-fashioned way:

$$\begin{aligned}
\Pr(X > 3.25) &= \Pr\left(Z > \frac{3.25 - 2}{0.5}\right) \\
&= \Pr(Z > 2.5) \\
&= 1 - \Pr(Z \leq 2.5) = 1 - 0.994 = 0.006.
\end{aligned}$$

Or could just use R:

```
1-pnorm(3.25, 2, 0.5)
[1] 0.006209665
```

Example 3.1: Solution (a)

Could work out the old-fashioned way:

$$\begin{aligned}
\Pr(X > 3.25) &= \Pr\left(Z > \frac{3.25 - 2}{0.5}\right) \\
&= \Pr(Z > 2.5) \\
&= 1 - \Pr(Z \leq 2.5) = 1 - 0.994 = 0.006.
\end{aligned}$$

Or could just use R:

```
1-pnorm(3.25, 2, 0.5)
[1] 0.006209665
```

Example 3.1: Solution (a)

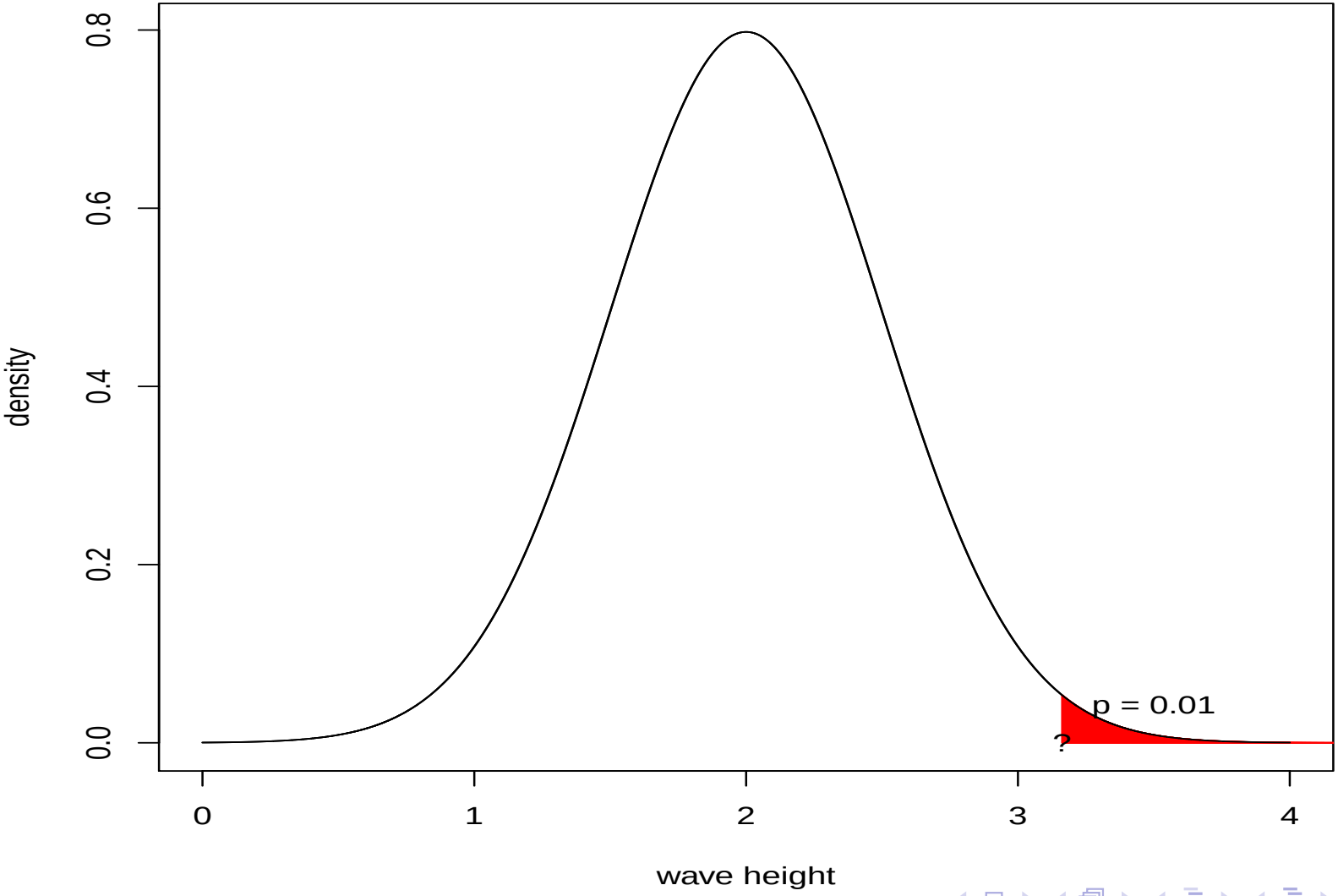
Could work out the old-fashioned way:

$$\begin{aligned}
\Pr(X > 3.25) &= \Pr\left(Z > \frac{3.25 - 2}{0.5}\right) \\
&= \Pr(Z > 2.5) \\
&= 1 - \Pr(Z \leq 2.5) = 1 - 0.994 = 0.006.
\end{aligned}$$

Or could just use R:

```
1-pnorm(3.25, 2, 0.5)
[1] 0.006209665
```

Example 3.1: Solution (b)



Example 3.1: Solution (b)

Similarly:

```
qnorm(0.99, 2, 0.5)
[1] 3.163174
```

Quiz #1

What is the correct R code to find $\Pr(X < 4)$, when $X \sim Bin(12, 0.3)$?

1. `rbinom(4, 12, 0.3)`
2. `pbinom(3, 12, 0.3)`
3. `pbinom(4, 12, 0.3)`
4. `dbinom(3, 0.3, 12)`

Quiz #1

What is the correct R code to find $\Pr(X < 4)$, when $X \sim Bin(12, 0.3)$?

1. `rbinom(4, 12, 0.3)`
2. `pbinom(3, 12, 0.3)`
3. `pbinom(4, 12, 0.3)`
4. `dbinom(3, 0.3, 12)`

Quiz #2

Suppose the time (in hours) between collision 'near misses' at a known road safety hotspot in Newcastle is exponentially distributed with rate 0.01.

What is the correct R code to find the probability that the time between 'near misses' will exceed 5 days?

1. `rexp(5, 0.01)`
2. `1 - pexp(5, 0.01)`
3. `1 - pexp(120, 0.01)`
4. `pexp(120, 0.99)`

Quiz #2

Suppose the time (in hours) between collision 'near misses' at a known road safety hotspot in Newcastle is exponentially distributed with rate 0.01.

What is the correct R code to find the probability that the time between 'near misses' will exceed 5 days?

1. `rexp(5, 0.01)`
2. `1 - pexp(5, 0.01)`
3. `1 - pexp(120, 0.01)`
4. `pexp(120, 0.99)`

Example 3.2

Use R to generate a sample of size 10,000 from an exponential distribution with rate 2.5, and store this sample in the vector x.

(a) Use your vector x to estimate $\Pr(X \leq 0.5)$, where $X \sim Exp(2.5)$.

(b) How does your estimate in part (a) compare to the true probability? Use R here – do not perform any calculations by hand.

Example 3.3: Solution

Model	Question	R code	Answer
$X \sim Po(3)$	Find $\Pr(X = 6)$.	`dpois(6,3)`	0.0504
	Find $\Pr(X < 5)$.	`ppois(4,3)`	0.8153
	Generate 3 observations.	`rpois(3,3)`	2, 9, 3
$Y \sim Bin(10, 0.2)$	Find $\Pr(Y \leq 4)$.	`pbinom(4,10,0.2)`	0.9672
	Find $\Pr(Y \geq 4)$.	`1-pbinom(3,10,0.2)`	0.1209
	Find y such that $\Pr(Y \leq y) = 0.677$	`qbinom(0.677,10,0.2)`	2
$Z \sim N(0, 1)$	Find $\Pr(Z \leq 0.2)$.	`pnorm(0.2)`	0.5793
	Find $\Pr(Z > -2.5)$.	`1-pnorm(-2.5)`	0.9938
	Generate 3 observations.	`rnorm(3)`	0.993 -2.523 -2.468

Example 3.3: Solution

Model	Question	R code	Answer
$X \sim Po(3)$	Find $\Pr(X = 6)$.	`dpois(6,3)`	0.0504
	Find $\Pr(X < 5)$.	`ppois(4,3)`	0.8153
	Generate 3 observations.	`rpois(3,3)`	2, 9, 3
$Y \sim Bin(10, 0.2)$	Find $\Pr(Y \leq 4)$.	`pbinom(4,10,0.2)`	0.9672
	Find $\Pr(Y \geq 4)$.	`1-pbinom(3,10,0.2)`	0.1209
	Find y such that $\Pr(Y \leq y) = 0.677$	`qbinom(0.677,10,0.2)`	2
$Z \sim N(0, 1)$	Find $\Pr(Z \leq 0.2)$.	`pnorm(0.2)`	0.5793
	Find $\Pr(Z > -2.5)$.	`1-pnorm(-2.5)`	0.9938
	Generate 3 observations.	`rnorm(3)`	0.993 -2.523 -2.468

Example 3.3: Solution

Model	Question	R code	Answer
$X \sim Po(3)$	Find $\Pr(X = 6)$.	`dpois(6,3)`	0.0504
	Find $\Pr(X < 5)$.	`ppois(4,3)`	0.8153
	Generate 3 observations.	`rpois(3,3)`	2, 9, 3
$Y \sim Bin(10, 0.2)$	Find $\Pr(Y \leq 4)$.	`pbinom(4,10,0.2)`	0.9672
	Find $\Pr(Y \geq 4)$.	`1-pbinom(3,10,0.2)`	0.1209
	Find y such that $\Pr(Y \leq y) = 0.677$	`qbinom(0.677,10,0.2)`	2
$Z \sim N(0, 1)$	Find $\Pr(Z \leq 0.2)$.	`pnorm(0.2)`	0.5793
	Find $\Pr(Z > -2.5)$.	`1-pnorm(-2.5)`	0.9938
	Generate 3 observations.	`rnorm(3)`	0.993 -2.523 -2.468

Example 3.3: Solution

Model	Question	R code	Answer
$X \sim Po(3)$	Find $\Pr(X = 6)$.	`dpois(6,3)`	0.0504
	Find $\Pr(X < 5)$.	`ppois(4,3)`	0.8153
	Generate 3 observations.	`rpois(3,3)`	2, 9, 3
$Y \sim Bin(10, 0.2)$	Find $\Pr(Y \leq 4)$.	`pbinom(4,10,0.2)`	0.9672
	Find $\Pr(Y \geq 4)$.	`1-pbinom(3,10,0.2)`	0.1209
	Find y such that $\Pr(Y \leq y) = 0.677$	`qbinom(0.677,10,0.2)`	2
$Z \sim N(0, 1)$	Find $\Pr(Z \leq 0.2)$.	`pnorm(0.2)`	0.5793
	Find $\Pr(Z > -2.5)$.	`1-pnorm(-2.5)`	0.9938
	Generate 3 observations.	`rnorm(3)`	0.993 -2.523 -2.468

Example 3.3: Solution

Model	Question	R code	Answer
$X \sim Po(3)$	Find $\Pr(X = 6)$.	`dpois(6,3)`	0.0504
	Find $\Pr(X < 5)$.	`ppois(4,3)`	0.8153
	Generate 3 observations.	`rpois(3,3)`	2, 9, 3
$Y \sim Bin(10, 0.2)$	Find $\Pr(Y \leq 4)$.	`pbinom(4,10,0.2)`	0.9672
	Find $\Pr(Y \geq 4)$.	`1-pbinom(3,10,0.2)`	0.1209
	Find y such that $\Pr(Y \leq y) = 0.677$	`qbinom(0.677,10,0.2)`	2
$Z \sim N(0, 1)$	Find $\Pr(Z \leq 0.2)$.	`pnorm(0.2)`	0.5793
	Find $\Pr(Z > -2.5)$.	`1-pnorm(-2.5)`	0.9938
	Generate 3 observations.	`rnorm(3)`	0.993 -2.523 -2.468

Example 3.3: Solution

Model	Question	R code	Answer
$X \sim Po(3)$	Find $\Pr(X = 6)$.	`dpois(6,3)`	0.0504
	Find $\Pr(X < 5)$.	`ppois(4,3)`	0.8153
	Generate 3 observations.	`rpois(3,3)`	2, 9, 3
$Y \sim Bin(10, 0.2)$	Find $\Pr(Y \leq 4)$.	`pbinom(4,10,0.2)`	0.9672
	Find $\Pr(Y \geq 4)$.	`1-pbinom(3,10,0.2)`	0.1209
	Find y such that $\Pr(Y \leq y) = 0.677$	`qbinom(0.677,10,0.2)`	2
$Z \sim N(0, 1)$	Find $\Pr(Z \leq 0.2)$.	`pnorm(0.2)`	0.5793
	Find $\Pr(Z > -2.5)$.	`1-pnorm(-2.5)`	0.9938
	Generate 3 observations.	`rnorm(3)`	0.993 -2.523 -2.468

Example 3.3: Solution

Model	Question	R code	Answer
$X \sim Po(3)$	Find $\Pr(X = 6)$.	`dpois(6,3)`	0.0504
	Find $\Pr(X < 5)$.	`ppois(4,3)`	0.8153
	Generate 3 observations.	`rpois(3,3)`	2, 9, 3
$Y \sim Bin(10, 0.2)$	Find $\Pr(Y \leq 4)$.	`pbinom(4,10,0.2)`	0.9672
	Find $\Pr(Y \geq 4)$.	`1-pbinom(3,10,0.2)`	0.1209
	Find y such that $\Pr(Y \leq y) = 0.677$	`qbinom(0.677,10,0.2)`	2
$Z \sim N(0, 1)$	Find $\Pr(Z \leq 0.2)$.	`pnorm(0.2)`	0.5793
	Find $\Pr(Z > -2.5)$.	`1-pnorm(-2.5)`	0.9938
	Generate 3 observations.	`rnorm(3)`	0.993 -2.523 -2.468

Example 3.3: Solution

Model	Question	R code	Answer
$X \sim Po(3)$	Find $\Pr(X = 6)$.	`dpois(6,3)`	0.0504
	Find $\Pr(X < 5)$.	`ppois(4,3)`	0.8153
	Generate 3 observations.	`rpois(3,3)`	2, 9, 3
$Y \sim Bin(10, 0.2)$	Find $\Pr(Y \leq 4)$.	`pbinom(4,10,0.2)`	0.9672
	Find $\Pr(Y \geq 4)$.	`1-pbinom(3,10,0.2)`	0.1209
	Find y such that $\Pr(Y \leq y) = 0.677$	`qbinom(0.677,10,0.2)`	2
$Z \sim N(0, 1)$	Find $\Pr(Z \leq 0.2)$.	`pnorm(0.2)`	0.5793
	Find $\Pr(Z > -2.5)$.	`1-pnorm(-2.5)`	0.9938
	Generate 3 observations.	`rnorm(3)`	0.993 -2.523 -2.468

Example 3.3: Solution

Model	Question	R code	Answer
$X \sim Po(3)$	Find $\Pr(X = 6)$.	`dpois(6,3)`	0.0504
	Find $\Pr(X < 5)$.	`ppois(4,3)`	0.8153
	Generate 3 observations.	`rpois(3,3)`	2, 9, 3
$Y \sim Bin(10, 0.2)$	Find $\Pr(Y \leq 4)$.	`pbinom(4,10,0.2)`	0.9672
	Find $\Pr(Y \geq 4)$.	`1-pbinom(3,10,0.2)`	0.1209
	Find y such that $\Pr(Y \leq y) = 0.677$	`qbinom(0.677,10,0.2)`	2
$Z \sim N(0, 1)$	Find $\Pr(Z \leq 0.2)$.	`pnorm(0.2)`	0.5793
	Find $\Pr(Z > -2.5)$.	`1-pnorm(-2.5)`	0.9938
	Generate 3 observations.	`rnorm(3)`	0.993 -2.523 -2.468

Example 3.3: Solution

Model	Question	R code	Answer
$X \sim Po(3)$	Find $\Pr(X = 6)$.	`dpois(6,3)`	0.0504
	Find $\Pr(X < 5)$.	`ppois(4,3)`	0.8153
	Generate 3 observations.	`rpois(3,3)`	2, 9, 3
$Y \sim Bin(10, 0.2)$	Find $\Pr(Y \leq 4)$.	`pbinom(4,10,0.2)`	0.9672
	Find $\Pr(Y \geq 4)$.	`1-pbinom(3,10,0.2)`	0.1209
	Find y such that $\Pr(Y \leq y) = 0.677$.	`qbinom(0.677,10,0.2)`	2
$Z \sim N(0, 1)$	Find $\Pr(Z \leq 0.2)$.	`pnorm(0.2)`	0.5793
	Find $\Pr(Z > -2.5)$.	`1-pnorm(-2.5)`	0.9938
	Generate 3 observations.	`rnorm(3)`	0.993 -2.523 -2.468

Example 3.3: Solution

Model	Question	R code	Answer
$X \sim Po(3)$	Find $\Pr(X = 6)$.	`dpois(6,3)`	0.0504
	Find $\Pr(X < 5)$.	`ppois(4,3)`	0.8153
	Generate 3 observations.	`rpois(3,3)`	2, 9, 3
$Y \sim Bin(10, 0.2)$	Find $\Pr(Y \leq 4)$.	`pbinom(4,10,0.2)`	0.9672
	Find $\Pr(Y \geq 4)$.	`1-pbinom(3,10,0.2)`	0.1209
	Find y such that $\Pr(Y \leq y) = 0.677$.	`qbinom(0.677,10,0.2)`	2
$Z \sim N(0, 1)$	Find $\Pr(Z \leq 0.2)$.	`pnorm(0.2)`	0.5793
	Find $\Pr(Z > -2.5)$.	`1-pnorm(-2.5)`	0.9938
	Generate 3 observations.	`rnorm(3)`	0.993, -2.523, -2.468

Example 3.3: Solution

Model	Question	R code	Answer
$X \sim Po(3)$	Find $\Pr(X = 6)$.	`dpois(6,3)`	0.0504
	Find $\Pr(X < 5)$.	`ppois(4,3)`	0.8153
	Generate 3 observations.	`rpois(3,3)`	2, 9, 3
$Y \sim Bin(10, 0.2)$	Find $\Pr(Y \leq 4)$.	`pbinom(4,10,0.2)`	0.9672
	Find $\Pr(Y \geq 4)$.	`1-pbinom(3,10,0.2)`	0.1209
	Find y such that $\Pr(Y \leq y) = 0.677$.	`qbinom(0.677,10,0.2)`	2
$Z \sim N(0, 1)$	Find $\Pr(Z \leq 0.2)$.	`pnorm(0.2)`	0.5793
	Find $\Pr(Z > -2.5)$.	`1-pnorm(-2.5)`	0.9938
	Generate 3 observations.	`rnorm(3)`	0.993, -2.523, -2.468

Example 3.3: Solution

Model	Question	R code	Answer
$X \sim Po(3)$	Find $\Pr(X = 6)$.	`dpois(6,3)`	0.0504
	Find $\Pr(X < 5)$.	`ppois(4,3)`	0.8153
	Generate 3 observations.	`rpois(3,3)`	2, 9, 3
$Y \sim Bin(10, 0.2)$	Find $\Pr(Y \leq 4)$.	`pbinom(4,10,0.2)`	0.9672
	Find $\Pr(Y \geq 4)$.	`1-pbinom(3,10,0.2)`	0.1209
	Find y such that $\Pr(Y \leq y) = 0.677$.	`qbinom(0.677,10,0.2)`	2
$Z \sim N(0, 1)$	Find $\Pr(Z \leq 0.2)$.	`pnorm(0.2)`	0.5793
	Find $\Pr(Z > -2.5)$.	`1-pnorm(-2.5)`	0.9938
	Generate 3 observations.	`rnorm(3)`	0.993, -2.523, -2.468

Example 3.3: Solution

Model	Question	R code	Answer
$X \sim Po(3)$	Find $\Pr(X = 6)$.	`dpois(6,3)`	0.0504
	Find $\Pr(X < 5)$.	`ppois(4,3)`	0.8153
	Generate 3 observations.	`rpois(3,3)`	2, 9, 3
$Y \sim Bin(10, 0.2)$	Find $\Pr(Y \leq 4)$.	`pbinom(4,10,0.2)`	0.9672
	Find $\Pr(Y \geq 4)$.	`1-pbinom(3,10,0.2)`	0.1209
	Find y such that $\Pr(Y \leq y) = 0.677$.	`qbinom(0.677,10,0.2)`	2
$Z \sim N(0, 1)$	Find $\Pr(Z \leq 0.2)$.	`pnorm(0.2)`	0.5793
	Find $\Pr(Z > -2.5)$.	`1-pnorm(-2.5)`	0.9938
	Generate 3 observations.	`rnorm(3)`	0.993 -2.523 -2.468

Example 3.3: Solution

Model	Question	R code	Answer
$X \sim Po(3)$	Find $\Pr(X = 6)$.	`dpois(6,3)`	0.0504
	Find $\Pr(X < 5)$.	`ppois(4,3)`	0.8153
	Generate 3 observations.	`rpois(3,3)`	2, 9, 3
$Y \sim Bin(10, 0.2)$	Find $\Pr(Y \leq 4)$.	`pbinom(4,10,0.2)`	0.9672
	Find $\Pr(Y \geq 4)$.	`1-pbinom(3,10,0.2)`	0.1209
	Find y such that $\Pr(Y \leq y) = 0.677$.	`qbinom(0.677,10,0.2)`	2
$Z \sim N(0, 1)$	Find $\Pr(Z \leq 0.2)$.	`pnorm(0.2)`	0.5793
	Find $\Pr(Z > -2.5)$.	`1-pnorm(-2.5)`	0.9938
	Generate 3 observations.	`rnorm(3)`	0.993, -2.523, -2.468

Example 3.3: Solution

Model	Question	R code	Answer
$X \sim Po(3)$	Find $\Pr(X = 6)$.	`dpois(6,3)`	0.0504
	Find $\Pr(X < 5)$.	`ppois(4,3)`	0.8153
	Generate 3 observations.	`rpois(3,3)`	2, 9, 3
$Y \sim Bin(10, 0.2)$	Find $\Pr(Y \leq 4)$.	`pbinom(4,10,0.2)`	0.9672
	Find $\Pr(Y \geq 4)$.	`1-pbinom(3,10,0.2)`	0.1209
	Find y such that $\Pr(Y \leq y) = 0.677$.	`qbinom(0.677,10,0.2)`	2
$Z \sim N(0, 1)$	Find $\Pr(Z \leq 0.2)$.	`pnorm(0.2)`	0.5793
	Find $\Pr(Z > -2.5)$.	`1-pnorm(-2.5)`	0.9938
	Generate 3 observations.	`rnorm(3)`	0.993 -2.523 -2.468

Example 3.3: Solution

Model	Question	R code	Answer
$X \sim Po(3)$	Find $\Pr(X = 6)$.	`dpois(6,3)`	0.0504
	Find $\Pr(X < 5)$.	`ppois(4,3)`	0.8153
	Generate 3 observations.	`rpois(3,3)`	2, 9, 3
$Y \sim Bin(10, 0.2)$	Find $\Pr(Y \leq 4)$.	`pbinom(4,10,0.2)`	0.9672
	Find $\Pr(Y \geq 4)$.	`1-pbinom(3,10,0.2)`	0.1209
	Find y such that $\Pr(Y \leq y) = 0.677$.	`qbinom(0.677,10,0.2)`	2
$Z \sim N(0, 1)$	Find $\Pr(Z \leq 0.2)$.	`pnorm(0.2)`	0.5793
	Find $\Pr(Z > -2.5)$.	`1-pnorm(-2.5)`	0.9938
	Generate 3 observations.	`rnorm(3)`	0.993 -2.523 -2.468

Bonus example

MMathStat students at Newcastle produce an average of 1 typo per four pages in their MMathStat dissertations. An MMathStat dissertation can be no longer than 44 pages, and students lose 5 presentation marks if the assessors detect more than 10 typos.

(a) Ben's dissertation is the maximum allowed length. What is the probability that he will lose 5 presentation marks for exceeding the number of typos allowed?

(b) Of six students whose dissertations were the maximum allowed length, what is the probability that at least four will lose 5 presentation marks for exceeding the number of typos allowed?

Bonus example: Solution

(a) We have X: No. of typos per 44 pages, and $X \sim Po(11)$. Then we need

$$\begin{aligned}\Pr(X > 10) &= 1 - \Pr(X \leq 10) \\ &= \texttt{1-ppois(10, 11)} = 0.5401113.\end{aligned}$$

(b) Let Y: No of students who lose 5 presentation marks, and $Y \sim Bin(6, 0.5401113)$. Then we need

$$\begin{aligned}\Pr(Y \geq 4) &= 1 - \Pr(Y \leq 3) \\ &= \texttt{1-pbinom(3, 6, 0.5401113)} = 0.421634.\end{aligned}$$

Bonus example: Solution

(a) We have X: No. of typos per 44 pages, and $X \sim Po(11)$. Then we need

$$\begin{aligned} \Pr(X > 10) &= 1 - \Pr(X \leq 10) \\ &= \texttt{1-ppois(10, 11)} = 0.5401113. \end{aligned}$$

(b) Let Y: No of students who lose 5 presentation marks, and $Y \sim Bin(6, 0.5401113)$. Then we need

$$\begin{aligned} \Pr(Y \geq 4) &= 1 - \Pr(Y \leq 3) \\ &= \texttt{1-pbinom(3, 6, 0.5401113)} = 0.421634. \end{aligned}$$

Bonus example: Solution

(a) We have X: No. of typos per 44 pages, and $X \sim Po(11)$. Then we need

$$\begin{aligned} \Pr(X > 10) &= 1 - \Pr(X \leq 10) \\ &= \texttt{1-ppois(10, 11)} = 0.5401113. \end{aligned}$$

(b) Let Y: No of students who lose 5 presentation marks, and $Y \sim Bin(6, 0.5401113)$. Then we need

$$\begin{aligned} \Pr(Y \geq 4) &= 1 - \Pr(Y \leq 3) \\ &= \texttt{1-pbinom(3, 6, 0.5401113)} = 0.421634. \end{aligned}$$

Bonus example: Solution

(a) We have X: No. of typos per 44 pages, and $X \sim Po(11)$. Then we need

$$\begin{aligned} \Pr(X > 10) &= 1 - \Pr(X \leq 10) \\ &= \texttt{1-ppois(10, 11)} = 0.5401113. \end{aligned}$$

(b) Let Y: No of students who lose 5 presentation marks, and $Y \sim Bin(6, 0.5401113)$. Then we need

$$\begin{aligned} \Pr(Y \geq 4) &= 1 - \Pr(Y \leq 3) \\ &= \texttt{1-pbinom(3, 6, 0.5401113)} = 0.421634. \end{aligned}$$

Bonus example: Solution

(a) We have X: No. of typos per 44 pages, and $X \sim Po(11)$. Then we need

$$\begin{aligned} \Pr(X > 10) &= 1 - \Pr(X \leq 10) \\ &= \texttt{1-ppois(10, 11)} = 0.5401113. \end{aligned}$$

(b) Let Y: No of students who lose 5 presentation marks, and $Y \sim Bin(6, 0.5401113)$. Then we need

$$\begin{aligned} \Pr(Y \geq 4) &= 1 - \Pr(Y \leq 3) \\ &= \texttt{1-pbinom(3, 6, 0.5401113)} = 0.421634. \end{aligned}$$

Bonus example: Solution

(a) We have X: No. of typos per 44 pages, and $X \sim Po(11)$. Then we need

$$\begin{aligned} \Pr(X > 10) &= 1 - \Pr(X \leq 10) \\ &= \texttt{1-ppois(10, 11)} = 0.5401113. \end{aligned}$$

(b) Let Y: No of students who lose 5 presentation marks, and $Y \sim Bin(6, 0.5401113)$. Then we need

$$\begin{aligned} \Pr(Y \geq 4) &= 1 - \Pr(Y \leq 3) \\ &= \texttt{1-pbinom(3, 6, 0.5401113)} = 0.421634. \end{aligned}$$

Bonus example: Solution

(a) We have X: No. of typos per 44 pages, and $X \sim Po(11)$. Then we need

$$\begin{aligned} \Pr(X > 10) &= 1 - \Pr(X \leq 10) \\ &= \texttt{1-ppois(10, 11)} = 0.5401113. \end{aligned}$$

(b) Let Y: No of students who lose 5 presentation marks, and $Y \sim Bin(6, 0.5401113)$. Then we need

$$\begin{aligned} \Pr(Y \geq 4) &= 1 - \Pr(Y \leq 3) \\ &= \texttt{1-pbinom(3, 6, 0.5401113)} = 0.421634. \end{aligned}$$

Bonus example: Solution

(a) We have X: No. of typos per 44 pages, and $X \sim Po(11)$. Then we need

$$\begin{aligned} \Pr(X > 10) &= 1 - \Pr(X \leq 10) \\ &= \texttt{1-ppois(10, 11)} = 0.5401113. \end{aligned}$$

(b) Let Y: No of students who lose 5 presentation marks, and $Y \sim Bin(6, 0.5401113)$. Then we need

$$\begin{aligned} \Pr(Y \geq 4) &= 1 - \Pr(Y \leq 3) \\ &= \texttt{1-pbinom(3, 6, 0.5401113)} = 0.421634. \end{aligned}$$

Bonus example: Solution

(a) We have X: No. of typos per 44 pages, and $X \sim Po(11)$. Then we need

$$\begin{aligned} \Pr(X > 10) &= 1 - \Pr(X \leq 10) \\ &= \texttt{1-ppois(10, 11)} = 0.5401113. \end{aligned}$$

(b) Let Y: No of students who lose 5 presentation marks, and $Y \sim Bin(6, 0.5401113)$. Then we need

$$\begin{aligned} \Pr(Y \geq 4) &= 1 - \Pr(Y \leq 3) \\ &= \texttt{1-pbinom(3, 6, 0.5401113)} = 0.421634. \end{aligned}$$