

Chapter 3

Prior elicitation

In this chapter we will think about how to **construct** a suitable **prior distribution** $\pi(\theta)$ for our parameter of interest θ .

For example:

- Why did we use a $Be(77, 5)$ distribution for θ in the music expert example?
- Why did we use a $Be(2.5, 12)$ distribution for θ in the video game pirate example?
- Why did we assume a $Ga(10, 4000)$ distribution for θ in the earthquake example?

In this chapter we will think about how to **construct** a suitable **prior distribution** $\pi(\theta)$ for our parameter of interest θ .

For example:

- Why did we use a $Be(77, 5)$ distribution for θ in the music expert example?
- Why did we use a $Be(2.5, 12)$ distribution for θ in the video game pirate example?
- Why did we assume a $Ga(10, 4000)$ distribution for θ in the earthquake example?

In this chapter we will think about how to **construct** a suitable **prior distribution** $\pi(\theta)$ for our parameter of interest θ .

For example:

- Why did we use a $Be(77, 5)$ distribution for θ in the music expert example?
- Why did we use a $Be(2.5, 12)$ distribution for θ in the video game pirate example?
- Why did we assume a $Ga(10, 4000)$ distribution for θ in the earthquake example?

In this chapter we will think about how to **construct** a suitable **prior distribution** $\pi(\theta)$ for our parameter of interest θ .

For example:

- Why did we use a $Be(77, 5)$ distribution for θ in the music expert example?
- Why did we use a $Be(2.5, 12)$ distribution for θ in the video game pirate example?
- Why did we assume a $Ga(10, 4000)$ distribution for θ in the earthquake example?

Prior elicitation – the process by which we attempt to construct the most suitable prior distribution for θ – is a huge area of research in Bayesian Statistics.

The aim in this course is to give a brief (and relatively simple) overview.

We will

- have another case study lecture (before Easter)
- have an interactive music session (using the TURNINGPOINT voting system, after Easter)

to demonstrate the techniques.

We will

- have another case study lecture (before Easter)
- have an interactive music session (using the TURNINGPOINT voting system, after Easter)

to demonstrate the techniques.

We will

- have another case study lecture (before Easter)
- have an interactive music session (using the TURNINGPOINT voting system, after Easter)

to demonstrate the techniques.

We will

- have another case study lecture (before Easter)
- have an interactive music session (using the TURNINGPOINT voting system, after Easter)

to demonstrate the techniques.

We will consider the cases of:

- **Substantial prior knowledge**

- turning expert opinion into a probability distribution for θ
- re-visit the examples about the music expert, the video game pirate and earthquakes in Chapter 2.

- **Vague prior knowledge**

- No expert available
- Choose a prior which “makes sense” and keeps the maths simple!

- **Prior ignorance**

- Assume all values of θ are equally likely

We will consider the cases of:

- **Substantial prior knowledge**

- turning expert opinion into a probability distribution for θ
- re-visit the examples about the music expert, the video game pirate and earthquakes in Chapter 2.

- **Vague prior knowledge**

- No expert available
- Choose a prior which “makes sense” and keeps the maths simple!

- **Prior ignorance**

- Assume all values of θ are equally likely

We will consider the cases of:

- **Substantial prior knowledge**

- turning expert opinion into a probability distribution for θ
- re-visit the examples about the music expert, the video game pirate and earthquakes in Chapter 2.

- **Vague prior knowledge**

- No expert available
- Choose a prior which “makes sense” and keeps the maths simple!

- **Prior ignorance**

- Assume all values of θ are equally likely

We will consider the cases of:

- **Substantial prior knowledge**

- turning expert opinion into a probability distribution for θ
- re-visit the examples about the music expert, the video game pirate and earthquakes in Chapter 2.

- **Vague prior knowledge**

- No expert available
- Choose a prior which “makes sense” and keeps the maths simple!

- **Prior ignorance**

- Assume all values of θ are equally likely

We will consider the cases of:

■ Substantial prior knowledge

- turning expert opinion into a probability distribution for θ
- re-visit the examples about the music expert, the video game pirate and earthquakes in Chapter 2.

■ Vague prior knowledge

- No expert available
- Choose a prior which “makes sense” and keeps the maths simple!

■ Prior ignorance

- Assume all values of θ are equally likely

We will consider the cases of:

■ Substantial prior knowledge

- turning expert opinion into a probability distribution for θ
- re-visit the examples about the music expert, the video game pirate and earthquakes in Chapter 2.

■ Vague prior knowledge

- No expert available
- Choose a prior which “makes sense” and keeps the maths simple!

■ Prior ignorance

- Assume all values of θ are equally likely

We will consider the cases of:

■ **Substantial prior knowledge**

- turning expert opinion into a probability distribution for θ
- re-visit the examples about the music expert, the video game pirate and earthquakes in Chapter 2.

■ **Vague prior knowledge**

- No expert available
- Choose a prior which “makes sense” and keeps the maths simple!

■ **Prior ignorance**

- Assume all values of θ are equally likely

We will consider the cases of:

- **Substantial prior knowledge**

- turning expert opinion into a probability distribution for θ
- re-visit the examples about the music expert, the video game pirate and earthquakes in Chapter 2.

- **Vague prior knowledge**

- No expert available
- Choose a prior which “makes sense” and keeps the maths simple!

- **Prior ignorance**

- Assume all values of θ are equally likely

We will consider the cases of:

- **Substantial prior knowledge**

- turning expert opinion into a probability distribution for θ
- re-visit the examples about the music expert, the video game pirate and earthquakes in Chapter 2.

- **Vague prior knowledge**

- No expert available
- Choose a prior which “makes sense” and keeps the maths simple!

- **Prior ignorance**

- Assume all values of θ are equally likely

We will consider the cases of:

- **Substantial prior knowledge**

- turning expert opinion into a probability distribution for θ
- re-visit the examples about the music expert, the video game pirate and earthquakes in Chapter 2.

- **Vague prior knowledge**

- No expert available
- Choose a prior which “makes sense” and keeps the maths simple!

- **Prior ignorance**

- Assume all values of θ are equally likely

Substantial prior knowledge

We will consider various methods for constructing prior distributions when we have **substantial prior knowledge**:

- Use of **suggested prior summaries**
- The **trial roulette method**
- The **bisection method**

We will make use of an online elicitation tool called **MATCH**, courtesy of Professor Tony O'Hagan and Dr Jeremy Oakley (Sheffield University).

Substantial prior knowledge

We will consider various methods for constructing prior distributions when we have **substantial prior knowledge**:

- Use of **suggested prior summaries**
 - The **trial roulette method**
 - The **bisection method**

We will make use of an online elicitation tool called **MATCH**, courtesy of Professor Tony O'Hagan and Dr Jeremy Oakley (Sheffield University).

Substantial prior knowledge

We will consider various methods for constructing prior distributions when we have **substantial prior knowledge**:

- Use of **suggested prior summaries**
- The **trial roulette method**
- The **bisection method**

We will make use of an online elicitation tool called **MATCH**, courtesy of Professor Tony O'Hagan and Dr Jeremy Oakley (Sheffield University).

Substantial prior knowledge

We will consider various methods for constructing prior distributions when we have **substantial prior knowledge**:

- Use of **suggested prior summaries**
- The **trial roulette method**
- The **bisection method**

We will make use of an online elicitation tool called **MATCH**, courtesy of Professor Tony O'Hagan and Dr Jeremy Oakley (Sheffield University).

Substantial prior knowledge

We will consider various methods for constructing prior distributions when we have **substantial prior knowledge**:

- Use of **suggested prior summaries**
- The **trial roulette method**
- The **bisection method**

We will make use of an online elicitation tool called `MATCH`, courtesy of Professor Tony O'Hagan and Dr Jeremy Oakley (Sheffield University).

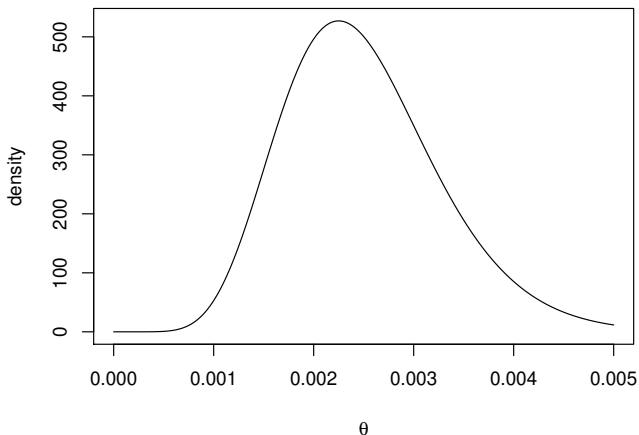
Example 3.1: Using suggested prior summaries

Let us return to **Example 2.4** of Chapter 2.

Recall that we were given some data on the “waiting times”, in days, between 21 earthquakes, and we discussed why an exponential distribution $\text{Exp}(\theta)$ might be appropriate to model the waiting times.

Further, we were told that an **expert on earthquakes** has prior beliefs about the rate θ , described by a $\text{Ga}(10, 4000)$ distribution.

Example 3.1: Using suggested prior summaries



How did we get from the expert's beliefs to a $Ga(10, 4000)$?

Example 3.1: Using suggested prior summaries

Suppose the expert tells us that earthquakes in the region we are interested in usually occur **less than once per year**.

In fact, they occur on average **once every 400 days**.

This gives us a rate of occurrence of about $1/400 = 0.0025$ per day.

Further, he is fairly certain about this and specifies a **very small variance** of 6.25×10^{-7} .

Example 3.1: Using suggested prior summaries

Suppose the expert tells us that earthquakes in the region we are interested in usually occur **less than once per year**.

In fact, they occur on average **once every 400 days**.

This gives us a rate of occurrence of about $1/400 = 0.0025$ per day.

Further, he is fairly certain about this and specifies a **very small variance** of 6.25×10^{-7} .

Example 3.1: Using suggested prior summaries

Suppose the expert tells us that earthquakes in the region we are interested in usually occur **less than once per year**.

In fact, they occur on average **once every 400 days**.

This gives us a rate of occurrence of about $1/400 = 0.0025$ per day.

Further, he is fairly certain about this and specifies a **very small variance** of 6.25×10^{-7} .

Example 3.1: Using suggested prior summaries

Suppose the expert tells us that earthquakes in the region we are interested in usually occur **less than once per year**.

In fact, they occur on average **once every 400 days**.

This gives us a rate of occurrence of about $1/400 = 0.0025$ per day.

Further, he is fairly certain about this and specifies a **very small variance** of 6.25×10^{-7} .

Example 3.1: Using suggested prior summaries

A $Ga(a, b)$ distribution seems sensible, since we can't observe a negative daily earthquake rate and the Gamma distribution is specified over positive values only.

Using the information provided by the expert, verify our use of $a = 10$ and $b = 4000$.

Example 3.1: Using suggested prior summaries

A $Ga(a, b)$ distribution seems sensible, since we can't observe a negative daily earthquake rate and the Gamma distribution is specified over positive values only.

Using the information provided by the expert, verify our use of $a = 10$ and $b = 4000$.

Solution to Example 3.1 (1/1)

We know that, if $\theta \sim \text{Ga}(a, b)$, then $E(\theta) = a/b$ and $\text{Var}(\theta) = a/b^2$. Thus

$$\frac{a}{b} = 0.0025 \implies a = 0.0025b.$$

Substituting into $a/b^2 = 0.000000625$ gives

$$\frac{0.0025b}{b^2} = 0.000000625, \quad \text{giving}$$

$$b = 4000.$$

Thus $a = 0.0025 \times 4000 = 10$.

Solution to Example 3.1 (1/1)

We know that, if $\theta \sim \text{Ga}(a, b)$, then $E(\theta) = a/b$ and $\text{Var}(\theta) = a/b^2$. Thus

$$\frac{a}{b} = 0.0025 \implies a = 0.0025b.$$

Substituting into $a/b^2 = 0.000000625$ gives

$$\frac{0.0025b}{b^2} = 0.000000625, \quad \text{giving}$$

$$b = 4000.$$

Thus $a = 0.0025 \times 4000 = 10$.

Solution to Example 3.1 (1/1)

We know that, if $\theta \sim \text{Ga}(a, b)$, then $E(\theta) = a/b$ and $\text{Var}(\theta) = a/b^2$. Thus

$$\frac{a}{b} = 0.0025 \implies a = 0.0025b.$$

Substituting into $a/b^2 = 0.000000625$ gives

$$\frac{0.0025b}{b^2} = 0.000000625, \quad \text{giving}$$

$$b = 4000.$$

Thus $a = 0.0025 \times 4000 = 10$.

Example 3.2: Using suggested prior summaries

Now let us return to **Example 2.2** of Chapter 2.

We considered an experiment to determine how good a music expert is at distinguishing between pages from Haydn and Mozart scores.

When presented with a score from each composer, the expert makes the correct choice with probability θ .

Example 3.2: Using suggested prior summaries

Before conducting the experiment, we were told that the expert is very competent. In fact, we were told that

- θ should have a prior distribution **peaking at around 0.95**
- $\Pr(\theta < 0.8)$ should be **very small**

To achieve this, we assumed that $\theta \sim Be(77, 5)$, with density given in Figure 2.4.

Example 3.2: Using suggested prior summaries

Before conducting the experiment, we were told that the expert is very competent. In fact, we were told that

- θ should have a prior distribution **peaking at around 0.95**
- $\Pr(\theta < 0.8)$ should be **very small**

To achieve this, we assumed that $\theta \sim \text{Be}(77, 5)$, with density given in Figure 2.4.

Example 3.2: Using suggested prior summaries

Before conducting the experiment, we were told that the expert is very competent. In fact, we were told that

- θ should have a prior distribution **peaking at around 0.95**
- $\Pr(\theta < 0.8)$ should be **very small**

To achieve this, we assumed that $\theta \sim Be(77, 5)$, with density given in Figure 2.4.

Example 3.2: Using suggested prior summaries

Before conducting the experiment, we were told that the expert is very competent. In fact, we were told that

- θ should have a prior distribution **peaking at around 0.95**
- $\Pr(\theta < 0.8)$ should be **very small**

To achieve this, we assumed that $\theta \sim \text{Be}(77, 5)$, with density given in Figure 2.4.

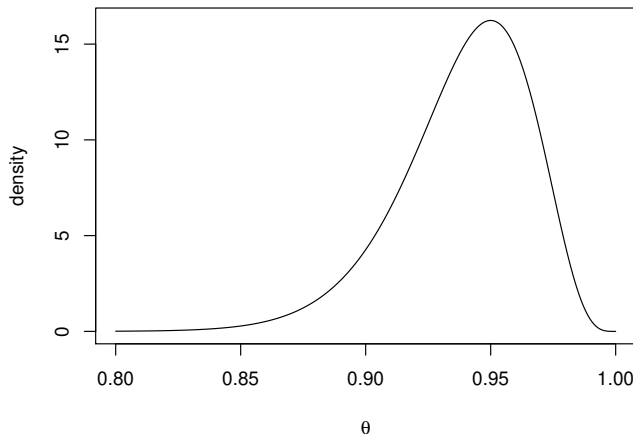
Example 3.2: Using suggested prior summaries

Before conducting the experiment, we were told that the expert is very competent. In fact, we were told that

- θ should have a prior distribution **peaking at around 0.95**
- $\Pr(\theta < 0.8)$ should be **very small**

To achieve this, we assumed that $\theta \sim Be(77, 5)$, with density given in Figure 2.4.

Example 3.2: Using suggested prior summaries



How did we know a *Be*(77, 5) would work?

Example 3.2: Using suggested prior summaries

We are told that the mode of the distribution should be around 0.95. Using the formulae on **page 23**, we can write

$$\frac{a-1}{a+b-2} = 0.95$$

$$\Rightarrow a-1 = 0.95(a+b-2)$$

$$\Rightarrow a - 0.95a = 0.95b - 1.9 + 1$$

$$\Rightarrow 0.05a = 0.95b - 0.9$$

$$\Rightarrow a = 19b - 18.$$

Example 3.2: Using suggested prior summaries

We are told that the mode of the distribution should be around 0.95. Using the formulae on **page 23**, we can write

$$\frac{a-1}{a+b-2} = 0.95$$

$$\Rightarrow a-1 = 0.95(a+b-2)$$

$$\Rightarrow a - 0.95a = 0.95b - 1.9 + 1$$

$$\Rightarrow 0.05a = 0.95b - 0.9$$

$$\Rightarrow a = 19b - 18.$$

Example 3.2: Using suggested prior summaries

We are told that the mode of the distribution should be around 0.95. Using the formulae on **page 23**, we can write

$$\frac{a-1}{a+b-2} = 0.95$$

$$\Rightarrow a-1 = 0.95(a+b-2)$$

$$\Rightarrow a - 0.95a = 0.95b - 1.9 + 1$$

$$\Rightarrow 0.05a = 0.95b - 0.9$$

$$\Rightarrow a = 19b - 18.$$

Example 3.2: Using suggested prior summaries

We are told that the mode of the distribution should be around 0.95. Using the formulae on **page 23**, we can write

$$\frac{a-1}{a+b-2} = 0.95$$

$$\Rightarrow a-1 = 0.95(a+b-2)$$

$$\Rightarrow a - 0.95a = 0.95b - 1.9 + 1$$

$$\Rightarrow 0.05a = 0.95b - 0.9$$

$$\Rightarrow a = 19b - 18.$$

Example 3.2: Using suggested prior summaries

We are told that the mode of the distribution should be around 0.95. Using the formulae on **page 23**, we can write

$$\frac{a-1}{a+b-2} = 0.95$$

$$\Rightarrow a-1 = 0.95(a+b-2)$$

$$\Rightarrow a - 0.95a = 0.95b - 1.9 + 1$$

$$\Rightarrow 0.05a = 0.95b - 0.9$$

$$\Rightarrow a = 19b - 18.$$

Example 3.2: Using suggested prior summaries

We are told that the mode of the distribution should be around 0.95. Using the formulae on **page 23**, we can write

$$\frac{a-1}{a+b-2} = 0.95$$

$$\Rightarrow a-1 = 0.95(a+b-2)$$

$$\Rightarrow a - 0.95a = 0.95b - 1.9 + 1$$

$$\Rightarrow 0.05a = 0.95b - 0.9$$

$$\Rightarrow a = 19b - 18.$$

Example 3.2: Using suggested prior summaries

We are told that the mode of the distribution should be around 0.95. Using the formulae on **page 23**, we can write

$$\frac{a-1}{a+b-2} = 0.95$$

$$\Rightarrow a-1 = 0.95(a+b-2)$$

$$\Rightarrow a - 0.95a = 0.95b - 1.9 + 1$$

$$\Rightarrow 0.05a = 0.95b - 0.9$$

$$\Rightarrow a = 19b - 18.$$

Example 3.2: Using suggested prior summaries

We are also told that $\Pr(\theta < 0.8)$ must be small.

In fact, suppose we are told that $\theta < 0.8$ might occur with probability 0.0001.

This means that if we integrate the probability density function for our beta distribution between 0 and 0.8, we would get 0.0001; from Equation (2.1) on **page 23**, we can write this as

$$\int_0^{0.8} \frac{\theta^{a-1}(1-\theta)^{b-1}}{B(a,b)} d\theta = 0.0001, \quad \text{i.e.}$$

$$\int_0^{0.8} \frac{\theta^{(19b-18)-1}(1-\theta)^{b-1}}{B(19b-18,b)} d\theta = 0.0001.$$

Example 3.2: Using suggested prior summaries

We are also told that $\Pr(\theta < 0.8)$ must be small.

In fact, suppose we are told that $\theta < 0.8$ might occur with probability 0.0001.

This means that if we integrate the probability density function for our beta distribution between 0 and 0.8, we would get 0.0001; from Equation (2.1) on **page 23**, we can write this as

$$\int_0^{0.8} \frac{\theta^{a-1}(1-\theta)^{b-1}}{B(a,b)} d\theta = 0.0001, \quad \text{i.e.}$$

$$\int_0^{0.8} \frac{\theta^{(19b-18)-1}(1-\theta)^{b-1}}{B(19b-18,b)} d\theta = 0.0001.$$

Example 3.2: Using suggested prior summaries

We are also told that $\Pr(\theta < 0.8)$ must be small.

In fact, suppose we are told that $\theta < 0.8$ might occur with probability 0.0001.

This means that if we integrate the probability density function for our beta distribution between 0 and 0.8, we would get 0.0001; from Equation (2.1) on **page 23**, we can write this as

$$\int_0^{0.8} \frac{\theta^{a-1}(1-\theta)^{b-1}}{B(a,b)} d\theta = 0.0001, \quad \text{i.e.}$$
$$\int_0^{0.8} \frac{\theta^{(19b-18)-1}(1-\theta)^{b-1}}{B(19b-18,b)} d\theta = 0.0001.$$

Example 3.2: Using suggested prior summaries

We are also told that $\Pr(\theta < 0.8)$ must be small.

In fact, suppose we are told that $\theta < 0.8$ might occur with probability 0.0001.

This means that if we integrate the probability density function for our beta distribution between 0 and 0.8, we would get 0.0001; from Equation (2.1) on **page 23**, we can write this as

$$\int_0^{0.8} \frac{\theta^{a-1}(1-\theta)^{b-1}}{B(a,b)} d\theta = 0.0001, \quad \text{i.e.}$$

$$\int_0^{0.8} \frac{\theta^{(19b-18)-1}(1-\theta)^{b-1}}{B(19b-18,b)} d\theta = 0.0001.$$

Example 3.2: Using suggested prior summaries

We are also told that $\Pr(\theta < 0.8)$ must be small.

In fact, suppose we are told that $\theta < 0.8$ might occur with probability 0.0001.

This means that if we integrate the probability density function for our beta distribution between 0 and 0.8, we would get 0.0001; from Equation (2.1) on **page 23**, we can write this as

$$\int_0^{0.8} \frac{\theta^{a-1}(1-\theta)^{b-1}}{B(a,b)} d\theta = 0.0001, \quad \text{i.e.}$$

$$\int_0^{0.8} \frac{\theta^{(19b-18)-1}(1-\theta)^{b-1}}{B(19b-18,b)} d\theta = 0.0001.$$

Example 3.2: Using suggested prior summaries

In other words, we set the cumulative distribution function for a $Be(19b - 18, b)$, evaluated at 0.8, equal to 0.0001 and solve for b .

Although this would be tricky to do by hand, we can do this quite easily in R.

Recall that the R command:

- `dbeta(x, a, b)` evaluates the **density** of the $Be(a, b)$ distribution at the point x
- `pbeta(x, a, b)` evaluates the **cumulative distribution function** at x

Example 3.2: Using suggested prior summaries

In other words, we set the cumulative distribution function for a $Be(19b - 18, b)$, evaluated at 0.8, equal to 0.0001 and solve for b .

Although this would be tricky to do by hand, we can do this quite easily in R.

Recall that the R command:

- `dbeta(x, a, b)` evaluates the **density** of the $Be(a, b)$ distribution at the point x
- `pbeta(x, a, b)` evaluates the **cumulative distribution function** at x

Example 3.2: Using suggested prior summaries

In other words, we set the cumulative distribution function for a $Be(19b - 18, b)$, evaluated at 0.8, equal to 0.0001 and solve for b .

Although this would be tricky to do by hand, we can do this quite easily in R.

Recall that the R command:

- `dbeta(x, a, b)` evaluates the **density** of the $Be(a, b)$ distribution at the point x
- `pbeta(x, a, b)` evaluates the **cumulative distribution function** at x

Example 3.2: Using suggested prior summaries

In other words, we set the cumulative distribution function for a $Be(19b - 18, b)$, evaluated at 0.8, equal to 0.0001 and solve for b .

Although this would be tricky to do by hand, we can do this quite easily in R.

Recall that the R command:

- `dbeta(x, a, b)` evaluates the **density** of the $Be(a, b)$ distribution at the point x
- `pbeta(x, a, b)` evaluates the **cumulative distribution function** at x

Example 3.2: Using suggested prior summaries

In other words, we set the cumulative distribution function for a $Be(19b - 18, b)$, evaluated at 0.8, equal to 0.0001 and solve for b .

Although this would be tricky to do by hand, we can do this quite easily in R.

Recall that the R command:

- `dbeta(x, a, b)` evaluates the **density** of the $Be(a, b)$ distribution at the point x
- `pbeta(x, a, b)` evaluates the **cumulative distribution function** at x

Example 3.2: Using suggested prior summaries

In other words, we set the cumulative distribution function for a $Be(19b - 18, b)$, evaluated at 0.8, equal to 0.0001 and solve for b .

Although this would be tricky to do by hand, we can do this quite easily in R.

Recall that the R command:

- `dbeta(x, a, b)` evaluates the **density** of the $Be(a, b)$ distribution at the point x
- `pbeta(x, a, b)` evaluates the **cumulative distribution function** at x

Example 3.2: Using suggested prior summaries

First of all, we re-write (3.3) to set it equal to zero:

$$\int_0^{0.8} \frac{\theta^{(19b-18)-1} (1-\theta)^{b-1}}{B(19b-18, b)} d\theta - 0.0001 = 0. \quad (3.4)$$

We then write a function in R which computes the left-hand-side of Equation (3.4):

```
f=function(b) {  
  answer=pbeta(0.8, ((19*b)-18), b)-0.0001  
  return(answer)}  
}
```

Example 3.2: Using suggested prior summaries

First of all, we re-write (3.3) to set it equal to zero:

$$\int_0^{0.8} \frac{\theta^{(19b-18)-1} (1-\theta)^{b-1}}{B(19b-18, b)} d\theta - 0.0001 = 0. \quad (3.4)$$

We then write a function in R which computes the left-hand-side of Equation (3.4):

```
f=function(b) {  
  answer=pbeta(0.8, ((19*b)-18), b)-0.0001  
  return(answer)}  
}
```

Example 3.2: Using suggested prior summaries

First of all, we re-write (3.3) to set it equal to zero:

$$\int_0^{0.8} \frac{\theta^{(19b-18)-1} (1-\theta)^{b-1}}{B(19b-18, b)} d\theta - 0.0001 = 0. \quad (3.4)$$

We then write a function in R which computes the left-hand-side of Equation (3.4):

```
f=function(b) {  
  answer=pbeta(0.8, ((19*b)-18), b)-0.0001  
  return(answer)}  
}
```

Example 3.2: Using suggested prior summaries

First of all, we re-write (3.3) to set it equal to zero:

$$\int_0^{0.8} \frac{\theta^{(19b-18)-1} (1-\theta)^{b-1}}{B(19b-18, b)} d\theta - 0.0001 = 0. \quad (3.4)$$

We then write a function in R which computes the left-hand-side of Equation (3.4):

```
f=function(b) {  
  answer=pbeta(0.8, ((19*b)-18), b)-0.0001  
  return(answer)}  
}
```

Example 3.2: Using suggested prior summaries

The trick now is to use a **numerical procedure** to find the root of `answer` in our R function.

In other words, find the value `b` which equates `answer` to zero.

The R function `uniroot(f, lower=, upper=)`

- uses a numerical search algorithm to find the root of the expression provided by the function `f`
- requires the user to provide a `lower` and `upper` bound to search within

We know from the formulae on **page 23** that $a, b > 1$ when using expression (3.1) for the mode $\rightarrow$ so we search for a root over some specified domain > 1 .

Example 3.2: Using suggested prior summaries

The trick now is to use a **numerical procedure** to find the root of `answer` in our R function.

In other words, find the value `b` which equates `answer` to zero.

The R function `uniroot(f, lower=, upper=)`

- uses a numerical search algorithm to find the root of the expression provided by the function `f`
- requires the user to provide a `lower` and `upper` bound to search within

We know from the formulae on **page 23** that $a, b > 1$ when using expression (3.1) for the mode $\rightarrow$ so we search for a root over some specified domain > 1 .

Example 3.2: Using suggested prior summaries

The trick now is to use a **numerical procedure** to find the root of `answer` in our R function.

In other words, find the value `b` which equates `answer` to zero.

The R function `uniroot(f, lower=, upper=)`

- uses a numerical search algorithm to find the root of the expression provided by the function `f`
- requires the user to provide a `lower` and `upper` bound to search within

We know from the formulae on **page 23** that $a, b > 1$ when using expression (3.1) for the mode $\rightarrow$ so we search for a root over some specified domain > 1 .

Example 3.2: Using suggested prior summaries

The trick now is to use a **numerical procedure** to find the root of `answer` in our R function.

In other words, find the value `b` which equates `answer` to zero.

The R function `uniroot(f, lower=, upper=)`

- uses a numerical search algorithm to find the root of the expression provided by the function `f`
- requires the user to provide a `lower` and `upper` bound to search within

We know from the formulae on **page 23** that $a, b > 1$ when using expression (3.1) for the mode $\rightarrow$ so we search for a root over some specified domain > 1 .

Example 3.2: Using suggested prior summaries

The trick now is to use a **numerical procedure** to find the root of `answer` in our R function.

In other words, find the value `b` which equates `answer` to zero.

The R function `uniroot(f, lower=, upper=)`

- uses a numerical search algorithm to find the root of the expression provided by the function `f`
- requires the user to provide a `lower` and `upper` bound to search within

We know from the formulae on **page 23** that $a, b > 1$ when using expression (3.1) for the mode $\rightarrow$ so we search for a root over some specified domain > 1 .

Example 3.2: Using suggested prior summaries

The trick now is to use a **numerical procedure** to find the root of `answer` in our R function.

In other words, find the value `b` which equates `answer` to zero.

The R function `uniroot(f, lower=, upper=)`

- uses a numerical search algorithm to find the root of the expression provided by the function `f`
- requires the user to provide a `lower` and `upper` bound to search within

We know from the formulae on **page 23** that $a, b > 1$ when using expression (3.1) for the mode → so we search for a root over some specified domain > 1 .

Example 3.2: Using suggested prior summaries

The trick now is to use a **numerical procedure** to find the root of `answer` in our R function.

In other words, find the value `b` which equates `answer` to zero.

The R function `uniroot(f, lower=, upper=)`

- uses a numerical search algorithm to find the root of the expression provided by the function `f`
- requires the user to provide a `lower` and `upper` bound to search within

We know from the formulae on **page 23** that $a, b > 1$ when using expression (3.1) for the mode $\rightarrow$ so we search for a root over some specified domain > 1 .

Example 3.2: Using suggested prior summaries

The trick now is to use a **numerical procedure** to find the root of `answer` in our R function.

In other words, find the value `b` which equates `answer` to zero.

The R function `uniroot(f, lower=, upper=)`

- uses a numerical search algorithm to find the root of the expression provided by the function `f`
- requires the user to provide a `lower` and `upper` bound to search within

We know from the formulae on **page 23** that $a, b > 1$ when using expression (3.1) for the mode $\rightarrow$ so we search for a root over some specified domain > 1 .

Example 3.2: Using suggested prior summaries

For example, we might use `lower=1` and `upper=100`, giving:

```
> uniroot(f, lower=1, upper=100)
$root
[1] 5.06513
$f.root
[1] 6.008134e-09
$iter
[1] 14
$estim.prec
[1] 6.103516e-05
```

Example 3.2: Using suggested prior summaries

For example, we might use `lower=1` and `upper=100`, giving:

```
> uniroot(f, lower=1, upper=100)
```

```
$root
```

```
[1] 5.06513
```

```
$f.root
```

```
[1] 6.008134e-09
```

```
$iter
```

```
[1] 14
```

```
$estim.prec
```

```
[1] 6.103516e-05
```

Example 3.2: Using suggested prior summaries

Thus, the solution to Equation (3.3) is $b = 5.06513$.

For simplicity, rounding down to $b = 5$ and then substituting into (3.2) gives

$$a = 19 \times 5 - 18 = 77,$$

hence the use of $\theta \sim Be(77, 5)$ in Example 2.2 in Chapter 2.

Example 3.2: Using suggested prior summaries

Thus, the solution to Equation (3.3) is $b = 5.06513$.

For simplicity, rounding down to $b = 5$ and then substituting into (3.2) gives

$$a = 19 \times 5 - 18 = 77,$$

hence the use of $\theta \sim Be(77, 5)$ in Example 2.2 in Chapter 2.

Example 3.2: Using suggested prior summaries

Thus, the solution to Equation (3.3) is $b = 5.06513$.

For simplicity, rounding down to $b = 5$ and then substituting into (3.2) gives

$$a = 19 \times 5 - 18 = 77,$$

hence the use of $\theta \sim Be(77, 5)$ in Example 2.2 in Chapter 2.

Example 3.2: Using suggested prior summaries

Thus, the solution to Equation (3.3) is $b = 5.06513$.

For simplicity, rounding down to $b = 5$ and then substituting into (3.2) gives

$$a = 19 \times 5 - 18 = 77,$$

hence the use of $\theta \sim Be(77, 5)$ in Example 2.2 in Chapter 2.