

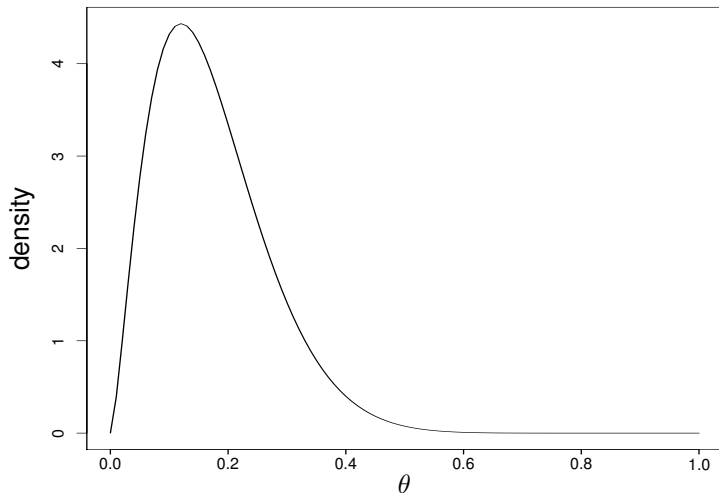
Example 2.3

Max, a video game pirate, is trying to identify the proportion of potential customers θ who might be interested in buying *Call of Duty: Elite* next month.

Based on the proportion of customers who have bought similarly violent games from him in the past, he assumes that $\theta \sim Be(2.5, 12)$ (see Example 3.3, Chapter 3).

Example 2.3

A plot of this prior density is shown in Figure 2.6.



Example 2.3

Max asks five potential customers if they would buy *Call of Duty: Elite* from him, and four say they would.

Using this information, what is Max's posterior distribution for θ ?

Solution to Example 2.3 (1/2)

We have been told that the prior for θ is a $Be(2.5, 12)$ distribution – this has density given by

$$\begin{aligned}\pi(\theta) &= \frac{\theta^{2.5-1}(1-\theta)^{12-1}}{B(2.5, 12)} \\ &= 435.1867\theta^{1.5}(1-\theta)^{11}.\end{aligned}\tag{2.10}$$

We have an observation on the random variable $X|\theta \sim Bin(5, \theta)$. This gives a likelihood function of

$$\begin{aligned}f(x=4|\theta) &= {}^5C_4\theta^4(1-\theta)^1 \\ &= 5\theta^4(1-\theta),\end{aligned}\tag{2.11}$$

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Solution to Example 2.3 (2/2)

We combine our prior information (2.10) with the data (2.11) – to obtain our posterior distribution – using Bayes' Theorem.

The posterior density function is

$$\begin{aligned}\pi(\theta|x=4) &\propto \pi(\theta)f(x=4|\theta) \\ &\propto 435.1867\theta^{1.5}(1-\theta)^{11} \times 5\theta^4(1-\theta), \quad 0 < \theta < 1, \\ &= k\theta^{5.5}(1-\theta)^{12}, \quad 0 < \theta < 1.\end{aligned}\tag{2.12}$$

You should recognise this density function as one from the beta family. In fact, we have a $Be(6.5, 13)$, i.e. $\theta|x=4 \sim Be(6.5, 13)$.

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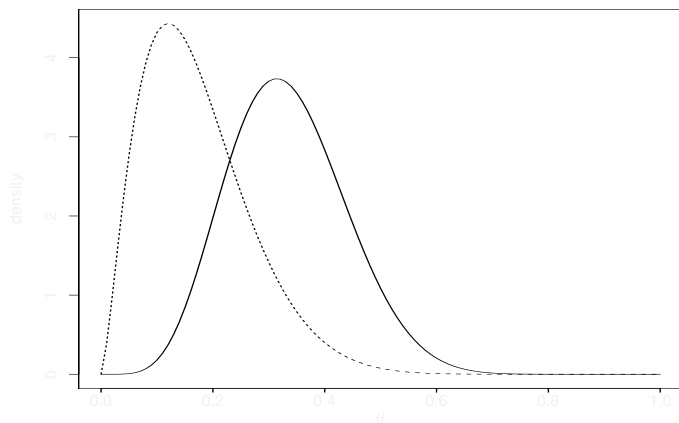
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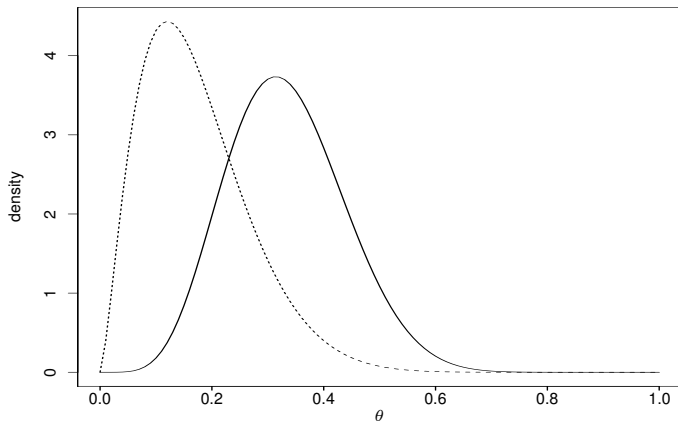
Example 2.3: Summary

The changes in our beliefs about θ are described by the prior and posterior distributions shown in Figure 2.7 and summarised in Table 2.2.



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	Prior (2.10)	Likelihood (2.11)	Posterior (2.12)
$Mode(\theta)$	0.12	0.8	0.314
$E(\theta)$	0.172	—	0.333
$SD(\theta)$	0.096	—	0.104

Notice how the posterior has been “pulled” from the prior towards the observed value: the mode has moved up from 0.12 to 0.314, and the mean has moved up from 0.172 to 0.333.

Having just one observation in the likelihood, we see that there is hardly any change in the standard deviation from prior to posterior: we would expect to see a decrease in standard deviation with the addition of more data values.

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Example 2.4

Table 2.3 shows some data on the times between serious earthquakes.

An earthquake is included if its magnitude is at least 7.5 on the Richter scale or if over 1000 people were killed.

Recording starts on 16 December 1902 (4500 killed in Turkistan).

The table includes data on 21 earthquakes, that is, 20 “waiting times” between earthquakes.

840	157	145	44	33	121	150	280	434	736
584	887	263	1901	695	294	562	721	76	710

Example 2.4

It is believed that earthquakes happen in a random haphazard kind of way and that times between earthquakes can be described by an exponential distribution.

Data over a much longer period suggest that this exponential assumption is plausible.

Therefore, we will assume that these data are a random sample from an exponential distribution with rate θ (and mean $1/\theta$). The parameter θ describes the rate at which earthquakes occur.

Example 2.4

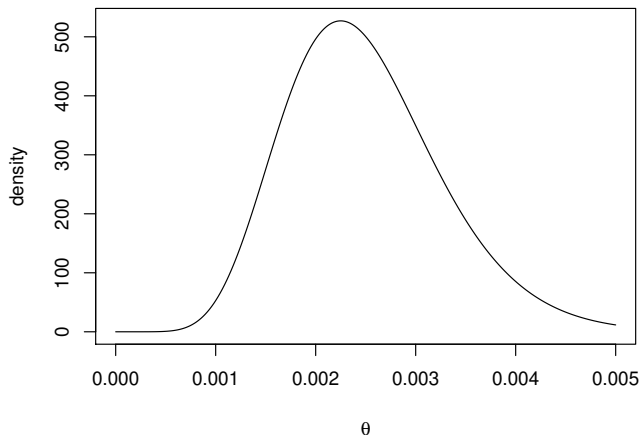
An expert on earthquakes has prior beliefs about the rate of earthquakes, θ , described by a $Ga(10, 4000)$ distribution (see Example 3.1, Chapter 3), which has density density

$$\pi(\theta) = \frac{4000^{10} \theta^9 e^{-4000\theta}}{\Gamma(10)}, \quad \theta > 0, \quad (2.13)$$

and mean $E(\theta) = 0.0025$.

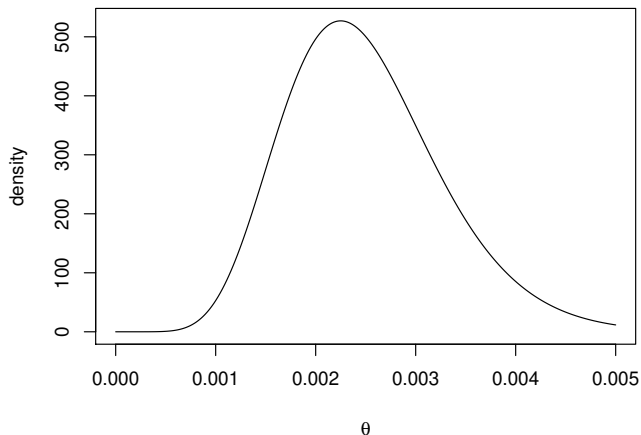
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A plot of this prior distribution can be found in Figure 2.8. As you might expect, the expert believes that only very small values of θ are likely, though larger values are not ruled out!



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Solution to Example 2.4 (1/2)

The data are observations on $X_i|\theta \sim \text{Exp}(\theta)$, $i = 1, 2, \dots, 20$ (independent).

Therefore, the likelihood function for θ is

$$\begin{aligned} f(\mathbf{x}|\theta) &= \prod_{i=1}^{20} \theta e^{-\theta x_i}, \quad \theta > 0 \\ &= \theta^{20} \exp \left(-\theta \sum_{i=1}^{20} x_i \right), \quad \theta > 0 \\ &= \theta^{20} e^{-9633\theta}, \quad \theta > 0. \end{aligned} \tag{2.14}$$

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Solution to Example 2.4 (2/2)

We now apply Bayes Theorem to combine the expert opinion with the observed data. The posterior density function is

$$\begin{aligned}\pi(\theta|\mathbf{x}) &\propto \pi(\theta)f(\mathbf{x}|\theta) \\ &\propto \frac{4000^{10} \theta^9 e^{-4000\theta}}{\Gamma(10)} \times \theta^{20} e^{-9633\theta}, \quad \theta > 0 \\ &= k \theta^{30-1} e^{-13633\theta}, \quad \theta > 0.\end{aligned}\tag{2.15}$$

The only continuous distribution which takes the form $k\theta^{g-1}e^{-h\theta}$, $\theta > 0$ is the $Ga(g, h)$ distribution.

Therefore, the posterior distribution must be $\theta|\mathbf{x} \sim Ga(30, 13633)$.

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The data have updated our beliefs about θ from a $Ga(10, 4000)$ distribution to a $Ga(30, 13633)$ distribution.

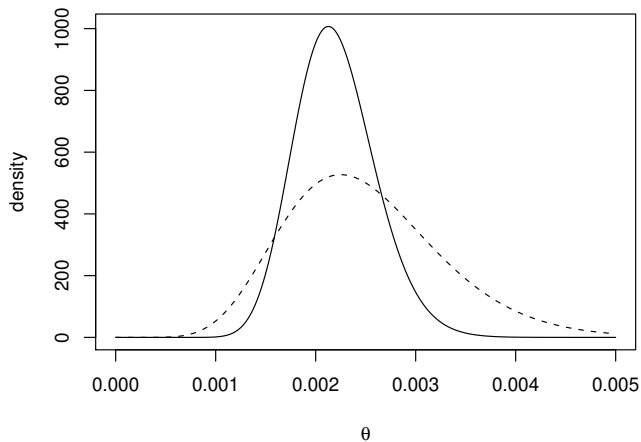
Plots of these distributions are given in Figure 2.9, and Table 2.4 gives a summary of the main changes induced by incorporating the data.

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$Mode(\theta)$	0.00225	0.00208	0.00213
$E(\theta)$	0.00250	—	0.00220
$SD(\theta)$	0.00079	—	0.00040

Notice that, as the mode of the likelihood function is close to that of the prior distribution, the information in the data is consistent with that in the prior distribution.

This results in a reduction in variability from the prior to the posterior distributions.

The similarity between the prior beliefs and the data has reduced the uncertainty we have about the likely earthquake rate θ .

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Example 2.5

We now consider the general case of the problem discussed in Example 2.4.

Suppose $X_i|\theta \sim \text{Exp}(\theta)$, $i = 1, 2, \dots, n$ (independent) and our prior beliefs about θ are summarised by a $\text{Ga}(g, h)$ distribution (with g and h known), with density

$$\pi(\theta) = \frac{h^g \theta^{g-1} e^{-h\theta}}{\Gamma(g)}, \quad \theta > 0. \quad (2.16)$$

Determine the posterior distribution for θ .

Solution to Example 2.5 (1/2)

The likelihood function for θ is

$$\begin{aligned}f(\mathbf{x}|\theta) &= \prod_{i=1}^n \theta e^{-\theta x_i}, \quad \theta > 0 \\&= \theta^n e^{-\theta x_1 - \theta x_2 - \dots - \theta x_n} \\&= \theta^n e^{-\theta \sum x} \\&= \theta^n e^{-n\bar{x}\theta}, \quad \theta > 0.\end{aligned}\tag{2.17}$$

Solution to Example 2.5 (2/2)

We now apply Bayes Theorem. The posterior density function is

$$\pi(\theta|\mathbf{x}) \propto \pi(\theta)f(\mathbf{x}|\theta)$$

$$\propto \frac{h^g \theta^{g-1} e^{-h\theta}}{\Gamma(g)} \times \theta^n e^{-n\bar{x}\theta}, \quad \theta > 0.$$

$$\pi(\theta|\mathbf{x}) = k\theta^{g+n-1} e^{-(h+n\bar{x})\theta}, \quad \theta > 0, \quad (2.18)$$

where k is a constant that does not depend on θ .

Therefore, the posterior distribution takes the form $k\theta^{g-1} e^{-h\theta}$, $\theta > 0$ and so must be a gamma distribution.

Thus we have $\theta|\mathbf{x} \sim Ga(g+n, h+n\bar{x})$.

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Example 2.5: Summary

If

- $X_i \sim \text{Exp}(\theta)$ and
- $\theta \sim \text{Ga}(g, h)$, then
- $\theta | \mathbf{x} \sim \text{Ga}(g + n, h + n\bar{x})$

The changes in our beliefs about θ are summarised in Table 2.5, taking $g \geq 1$.

	Prior (2.16)	Likelihood (2.17)	Posterior (2.18)
$\text{Mode}(\theta)$	$(g - 1)/h$	$1/\bar{x}$	$(g + n - 1)/(h + n\bar{x})$
$E(\theta)$	g/h	—	$(g + n)/(h + n\bar{x})$
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