

# *Home Study 7: To be completed by the end of Friday 4th December*

## MAS2905: Probability and Bayesian Methods with R

Newcastle University  
`lee.fawcett@ncl.ac.uk`

Semester 1, 2020/21

# The story so far...

- You should now have completed **all of Chapter 5**
- You should have completed the video **Home Study 6** earlier this week
- You should have attended the synchronous online lecture on Wednesday 2nd December
- We'll now go through the proof of the **transformation method**, and work through a bonus example!

# The story so far...

- You should now have completed **all of Chapter 5**
- You should have completed the video **Home Study 6** earlier this week
- You should have attended the synchronous online lecture on Wednesday 2nd December
- We'll now go through the proof of the **transformation method**, and work through a bonus example!

# The story so far...

- You should now have completed **all of Chapter 5**
- You should have completed the video **Home Study 6** earlier this week
- You should have attended the synchronous online lecture on Wednesday 2nd December
- We'll now go through the proof of the **transformation method**, and work through a bonus example!

# The story so far...

- You should now have completed **all of Chapter 5**
- You should have completed the video **Home Study 6** earlier this week
- You should have attended the synchronous online lecture on Wednesday 2nd December
- We'll now go through the proof of the **transformation method**, and work through a bonus example!

## 5. Transformation of random variables

- In Chapter 5 of the course notes we look at how to obtain the distribution of a **transformation** of a random variable
- We can do this two ways:
  - Informally, using **simulation** in R
  - More formally, by deriving the distribution of the transformed random variable through the **Transformation Method** (see proof on page 42)
- We also thought about **why** we might want to explore the distribution of a transformed random variable (e.g. variables that might only be defined over the positive real line)

## 5. Transformation of random variables

- In Chapter 5 of the course notes we look at how to obtain the distribution of a **transformation** of a random variable
- We can do this two ways:
  - Informally, using **simulation** in R
  - More formally, by deriving the distribution of the transformed random variable through the **Transformation Method** (see proof on page 42)
- We also thought about **why** we might want to explore the distribution of a transformed random variable (e.g. variables that might only be defined over the positive real line)

## 5. Transformation of random variables

- In Chapter 5 of the course notes we look at how to obtain the distribution of a **transformation** of a random variable
- We can do this two ways:
  - Informally, using **simulation** in R
  - More formally, by deriving the distribution of the transformed random variable through the **Transformation Method** (see proof on page 42)
- We also thought about **why** we might want to explore the distribution of a transformed random variable (e.g. variables that might only be defined over the positive real line)

## 5. Transformation of random variables

- In Chapter 5 of the course notes we look at how to obtain the distribution of a **transformation** of a random variable
- We can do this two ways:
  - Informally, using **simulation** in R
  - More formally, by deriving the distribution of the transformed random variable through the **Transformation Method** (see proof on page 42)
- We also thought about **why** we might want to explore the distribution of a transformed random variable (e.g. variables that might only be defined over the positive real line)

## 5. Transformation of random variables

- In Chapter 5 of the course notes we look at how to obtain the distribution of a **transformation** of a random variable
- We can do this two ways:
  - Informally, using **simulation** in R
  - More formally, by deriving the distribution of the transformed random variable through the **Transformation Method** (see proof on page 42)
- We also thought about **why** we might want to explore the distribution of a transformed random variable (e.g. variables that might only be defined over the positive real line)

## 5.3 The transformation method

Recall (from page 42) that if  $X$  has PDF  $f_X(x)$  and  $h(x)$  is a monotonically increasing or monotonically decreasing function then

- $h$  is invertible and
- the PDF of  $Y = h(X)$  is

$$f_Y(y) = f_X(h^{-1}(y)) \times \left| \frac{d(h^{-1}(y))}{dy} \right|$$

when  $y$  is in the range of  $h(x)$  and zero otherwise.

This result allows direct calculation of  $f_Y(y)$  and is called the **univariate transformation method**.

# Bonus example

Hourly wind speed maxima ( $X$  miles per hour), observed on the roof of Claremont Tower as part of the University's *Urban Observatory*, are assumed exponentially distributed with rate 0.1.

Unfortunately, the data recorded are the logged wind speeds.

Use the transformation method to find the PDF of  $Y = \log X$ .

## Bonus example: Solution

Recall that if  $X \sim \text{Exp}(\lambda)$ , then it has PDF

$$f_X(x) = \lambda e^{-\lambda x}, \quad x > 0, \quad \lambda > 0.$$

Now here, we have

$$h(x) = \log x, \quad \text{and} \quad h^{-1}(y) = e^y.$$

Therefore

$$\begin{aligned} f_Y(y) &= f_X(h^{-1}(y)) \times \left| \frac{d(h^{-1}(y))}{dy} \right| \\ &= \lambda \exp \{-\lambda \exp \{y\}\} \times \exp \{y\} \\ &= \lambda \exp \{y - \lambda e^y\}, \end{aligned}$$

with  $\lambda = 0.1$ .

## Bonus example: Solution

Recall that if  $X \sim \text{Exp}(\lambda)$ , then it has PDF

$$f_X(x) = \lambda e^{-\lambda x}, \quad x > 0, \quad \lambda > 0.$$

Now here, we have

$$h(x) = \log x, \quad \text{and} \quad h^{-1}(y) = e^y.$$

Therefore

$$\begin{aligned} f_Y(y) &= f_X(h^{-1}(y)) \times \left| \frac{d(h^{-1}(y))}{dy} \right| \\ &= \lambda \exp \{-\lambda \exp \{y\}\} \times \exp \{y\} \\ &= \lambda \exp \{y - \lambda e^y\}, \end{aligned}$$

with  $\lambda = 0.1$ .

## Bonus example: Solution

Recall that if  $X \sim \text{Exp}(\lambda)$ , then it has PDF

$$f_X(x) = \lambda e^{-\lambda x}, \quad x > 0, \quad \lambda > 0.$$

Now here, we have

$$h(x) = \log x, \quad \text{and} \quad h^{-1}(y) = e^y.$$

Therefore

$$\begin{aligned} f_Y(y) &= f_X(h^{-1}(y)) \times \left| \frac{d(h^{-1}(y))}{dy} \right| \\ &= \lambda \exp \{-\lambda \exp \{y\}\} \times \exp \{y\} \\ &= \lambda \exp \{y - \lambda e^y\}, \end{aligned}$$

with  $\lambda = 0.1$ .

## Bonus example: Solution

Recall that if  $X \sim \text{Exp}(\lambda)$ , then it has PDF

$$f_X(x) = \lambda e^{-\lambda x}, \quad x > 0, \quad \lambda > 0.$$

Now here, we have

$$h(x) = \log x, \quad \text{and} \quad h^{-1}(y) = e^y.$$

Therefore

$$\begin{aligned} f_Y(y) &= f_X(h^{-1}(y)) \times \left| \frac{d(h^{-1}(y))}{dy} \right| \\ &= \lambda \exp\{-\lambda \exp\{y\}\} \times \exp\{y\} \\ &= \lambda \exp\{y - \lambda e^y\}, \end{aligned}$$

with  $\lambda = 0.1$ .

## Bonus example: Solution

Recall that if  $X \sim \text{Exp}(\lambda)$ , then it has PDF

$$f_X(x) = \lambda e^{-\lambda x}, \quad x > 0, \quad \lambda > 0.$$

Now here, we have

$$h(x) = \log x, \quad \text{and} \quad h^{-1}(y) = e^y.$$

Therefore

$$\begin{aligned} f_Y(y) &= f_X(h^{-1}(y)) \times \left| \frac{d(h^{-1}(y))}{dy} \right| \\ &= \lambda \exp\{-\lambda \exp\{y\}\} \times \exp\{y\} \\ &= \lambda \exp\{y - \lambda e^y\}, \end{aligned}$$

with  $\lambda = 0.1$ .

## Bonus example: Solution

Recall that if  $X \sim \text{Exp}(\lambda)$ , then it has PDF

$$f_X(x) = \lambda e^{-\lambda x}, \quad x > 0, \quad \lambda > 0.$$

Now here, we have

$$h(x) = \log x, \quad \text{and} \quad h^{-1}(y) = e^y.$$

Therefore

$$\begin{aligned} f_Y(y) &= f_X(h^{-1}(y)) \times \left| \frac{d(h^{-1}(y))}{dy} \right| \\ &= \lambda \exp\{-\lambda \exp\{y\}\} \times \exp\{y\} \\ &= \lambda \exp\{y - \lambda e^y\}, \end{aligned}$$

with  $\lambda = 0.1$ .

# Bonus example: Solution

We can check this in R:

```
1 X = rexp(1000, 0.1)
2 Y = log(X)
3 hist(Y, prob=TRUE, main="Y=log(Wind Speeds)", xlab="
  Miles per hour")
4 lambda = 0.1
5 curve(lambda*exp(x-lambda*exp(x)), add=TRUE, col="red",
  lty=2, lwd=2)
```