

Suppose that an experiment results in data $\mathbf{x} = (x_1, x_2, \dots, x_n)^T$ and we decide to model the data using a probability (density) function $f(\mathbf{x}|\theta)$.

This p(d)f describes how likely different data $\mathbf{x}$ are to occur given a value of the (unknown) parameter θ .

However, once we have observed the data, $f(\mathbf{x}|\theta)$ tells us how likely different values of the parameters θ are: it is then known as the **likelihood function** for θ .

In other courses you may have seen it written as $L(\theta|\mathbf{x})$ or $L(\theta)$ but, whatever the notation used for the likelihood function, it is simply the joint probability (density) function of the data, $f(\mathbf{x}|\theta)$, regarded as a function of θ rather than of $\mathbf{x}$.

The likelihood function can be simplified if we have further structure in the data.

For example, we may have independent observations, in which case

$$f(\mathbf{x}|\theta) = \prod_{i=1}^n f_{X_i}(x_i|\theta), \quad (1.1)$$

or independent and identically distributed observations (random sample), so that

$$f(\mathbf{x}|\theta) = \prod_{i=1}^n f_X(x_i|\theta). \quad (1.2)$$

Example 1.5

Suppose we have a random sample $\mathbf{x} = (x_1, x_2, \dots, x_n)^T$ of radioactive particle counts.

A typical model for such data would be $X_i|\theta \sim \text{Poisson}(\theta)$, usually abbreviated $X_i|\theta \sim \text{Po}(\theta)$, (independent).

Determine the likelihood function for θ .

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Determine the likelihood function for θ .

Solution to Example 1.5 (1/1)

Using Equation (1.2), the likelihood function is

$$\begin{aligned}f(\mathbf{x}|\theta) &= \prod_{i=1}^n \frac{e^{-\theta} \theta^{x_i}}{x_i!} \\&= \frac{e^{-\theta} \times \dots \times e^{-\theta} \times \theta^{x_1} \times \dots \times \theta^{x_n}}{\prod_{i=1}^n x_i!} \\&= \frac{e^{-\theta-\theta-\dots-\theta} \theta^{x_1+\dots+x_n}}{\prod_{i=1}^n x_i!} \\&= \frac{e^{-n\theta} \theta^{\sum x_i}}{\prod_{i=1}^n x_i!} \\&= \frac{e^{-n\theta} \theta^{n\bar{x}}}{\prod_{i=1}^n x_i!}.\end{aligned}$$

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Example 1.6

Suppose we have a random sample $\mathbf{x} = (x_1, x_2, \dots, x_n)^T$ of times between radioactive particle emissions.

If the emissions occur randomly in time then a plausible model for such data would be $X_i|\theta \sim \text{Exponential}(\theta)$, usually abbreviated $X_i|\theta \sim \text{Exp}(\theta)$, (independent).

Determine the likelihood function for θ .

Solution to Example 1.6 (1/1)

Using Equation (1.2), the likelihood function is

$$\begin{aligned}f(\mathbf{x}|\theta) &= \prod_{i=1}^n \theta e^{-\theta x_i} \\&= \theta \times \dots \times \theta \times e^{-\theta x_1} \times \dots \times e^{-\theta x_n} \\&= \theta^n e^{-\theta x_1 - \theta x_2 - \dots - \theta x_n} \\&= \theta^n e^{-\theta(x_1 + \dots + x_n)} \\&= \theta^n e^{-\theta n \bar{x}}.\end{aligned}$$

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Example 1.7

Suppose we have a random sample $\mathbf{x} = (x_1, x_2, \dots, x_n)^T$ from a Normal distribution: $X_i | \mu, \sigma \sim N(\mu, \sigma^2)$, $i = 1, 2, \dots, n$ (independent).

Determine the likelihood function for (μ, σ) .