

MAS2903

Problems Class 3

Dr. Lee Fawcett

Semester 2, 2018—2019

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$$\begin{aligned}\frac{1}{\sqrt{d + n\tau}} &= 0.1 \\ \longrightarrow n &= \frac{100 - d}{\tau}.\end{aligned}$$

Here, we have $d = 1$ and $\tau = 1/4$, giving $n = 396$.

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The prior variance is $1/d$, which can never be negative; Thus, d cannot be negative. As $d \rightarrow 0$, we have $1/5 = 0.2$ for the posterior standard deviation; as d increases, the posterior standard deviation defined above will always decrease.

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$$D = 1/0.4 + 5 \cdot \frac{1}{4} = 3.75,$$

and

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Thus

$$Y|\mathbf{x} \sim N\left(109.733, \frac{3.75 + 1/4}{3.75/4}\right) = N(109.733, 4.267).$$

Question 34 (ii)

We require $P(Y > 110)$:

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Question 34 (ii)

We require $P(Y > 110)$:

$$\begin{aligned}P(Y > 110) &= P\left(Z > \frac{110 - 109.733}{\sqrt{4.267}}\right) \\&= P(Z > 0.129) \\&= 1 - P(Z < 0.129) \\&= 1 - 0.551 = 0.449.\end{aligned}$$

Question 27 (a)

We have a single observation on a $\text{Bin}(100, \theta)$ distribution, giving the likelihood:

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Then

$$\begin{aligned}\pi(\theta|\mathbf{x}) &\propto \theta^1(1 - \theta)^{199} \times \theta^3(1 - \theta)^{97} \\ &= \theta^4(1 - \theta)^{296},\end{aligned}$$

and so $\theta|\mathbf{x} \sim \text{Beta}(5, 297)$.

Question 27 (b)

Suppose the statistician's posterior is $\theta|\mathbf{x} \sim \text{Beta}(A, B)$. Then

$$E[\theta] = \frac{A}{A+B} = \frac{4}{102},$$

giving $B = 24.5A$.

Question 27 (b)

Suppose the statistician's posterior is $\theta|\mathbf{x} \sim \text{Beta}(A, B)$. Then

$$E[\theta] = \frac{A}{A+B} = \frac{4}{102},$$

giving $B = 24.5A$. Substituting into

$$\text{Var}(\theta) = \frac{AB}{(A+B)^2(A+B+1)} = 0.0003658$$

gives

$$\frac{24.5A^2}{(25.5A)^2(25.5A+1)} = 0.0003658$$

and so $A = 4$. Therefore $B = 98$, and so we have $\theta|\mathbf{x} \sim \text{Beta}(4, 98)$.

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Question 27 (b)

We know the likelihood is

$$f(x|\theta) \propto \theta^3(1 - \theta)^{97},$$

and so

$$\begin{aligned} \text{Posterior} &\propto \text{Prior} \times \text{Likelihood} \\ \theta^3(1 - \theta)^{97} &\propto \theta^{a-1}(1 - \theta)^{b-1} \times \theta^3(1 - \theta)^{97} \end{aligned}$$

meaning that $a = b = 1$ and so $\theta \sim \text{Beta}(1, 1)$ - in other words, a $\theta \sim U(0, 1)$!

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Alternatively, we could just observe that the Posterior is proportional to the likelihood, without any contribution from the prior – so the statistician must have been prior ignorant (hence the Uniform prior).