

1. (a) Let  $y = 1 - e^{-x/\sigma}$ . Then

$$\begin{aligned} -x/\sigma &= \log(1 - y) \\ x &= -\sigma \log(1 - y), \end{aligned}$$

with  $\hat{\sigma} = 12$ .

Code:

```
inv.cdf = function(y){
  ans = -12*log(1-y)
  return(ans)}
```

- (b) Code:

```
X = inv.cdf(runif(1000))
```

- (c) Code:

```
hist(X, probability=TRUE, col="ForestGreen", xlab="Whatever", ylab="Whatever")
curve((1/12)*exp(-x/12), from=0, to=80, col="red", lty=2, add=TRUE)
```

- (d) Code:

```
summary(X)
```

- (e) Code:

```
X1 = rexp(1000, 1/12)
```

Summaries for X and X1 very similar - this form of the Generalised Pareto Distribution is equivalent to an exponential distribution with rate  $1/\sigma$ .

2. (a) Code:

```
Y1 = sqrt(runif(1000,1,5))
Y2 = log(runif(1000,1,5))
```

- (b) For  $Y1$ :

$$F_{Y1}(y) = \Pr(Y_1 \leq y) = \Pr(\sqrt{X} \leq y) = \Pr(X \leq y^2) = F_X(y^2).$$

Now

$$\begin{aligned} f_{Y1}(y) &= \frac{d}{dy} F_X(y^2) \\ &= f_X(y^2) \times 2y \quad \text{by chain rule} \\ &= 0.25 \times 2y = 0.5y, \quad \sqrt{1} \leq y \leq \sqrt{5}, \end{aligned}$$

since  $f_X(x) = 0.25$  for all  $x$  (Uniform distribution with ‘base length’ 4, hence ‘height’ must be 0.25).

For  $Y2$ :

$$F_{Y2}(y) = \Pr(Y_2 \leq y) = \Pr(\log X \leq y) = \Pr(X \leq e^y) = F_X(e^y).$$

Now

$$\begin{aligned} f_{Y2}(y) &= \frac{d}{dy} F_X(e^y) \\ &= f_X(e^y) \times e^y \quad \text{by chain rule} \\ &= \frac{e^y}{4}, \quad \log 1 \leq y \leq \log 5. \end{aligned}$$

(c) Code:

```
par(mfrow=c(1,2))
hist(Y1, probability=TRUE)
curve(0.5*x, from=1, to=2.3, add=TRUE, col="red", lwd=2, lty=2)
hist(Y2, probability=TRUE)
curve(0.25*exp(x), from=0, to=1.6, add=TRUE, col="blue", lwd=2, lty=2)
```

3. (a) Code:

```
Y = (rnorm(1000))^2
```

(b) Code:

```
hist(Y, probability=TRUE, main="", xlab="whatever", ylab="whatever")
```

(c) Code:

```
curve((1/(2^(0.5)*gamma(0.5)))*x^(-0.5)*e^(-x/2), from=0, to=12, col="red", lwd=2, lty=2)
```

(d) Code:

```
length(Y[Y>3.84])/length(Y)
```

(e) Code:

```
rchisq(1000,1)
1-pchisq(3.84,1)
```