

1. Students should recognise that $Y|X$ is Normally distributed with mean $X = x$ (and standard deviation 1). Therefore, we have:

```

1 question1 = function(n){
2   output = matrix(0, 2, n)
3   x = rnorm(n, mean = 100, sd = 5)
4   y = rnorm(n, mean = x, sd = 1)
5   output[1, ] = x
6   output[2, ] = y
7   return(output)}

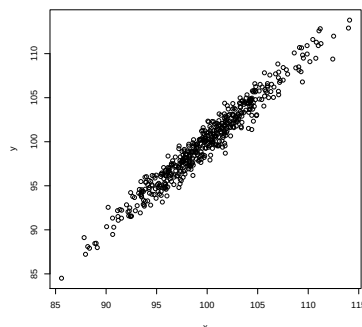
```

Giving:

```

1 test = question1(500)
2 plot(test[1, ], test[2, ], xlab='x', ylab='y')

```



2. (a) Students should notice that the PDF for $T|S$ is that of the Generalised Pareto Distribution (see question 1 in practical 2). Thus, a function could be written to sample from this distribution using the inverse CDF method (the CDF is given in practical 2):

```

1 inv.cdf = function(sigma, u){
2   ans = -sigma*log(1-u)
3   return(ans)}

```

Then we might have:

```

1 question2 = function(n){
2   output = matrix(0, 2, n)
3   s = rexp(n, 1)
4   t = inv.cdf(s, runif(n))
5   output[1, ] = s
6   output[2, ] = t
7   return(output)}

```

Alternatively, students might see this as an exponential distribution with rate $1/s$, giving:

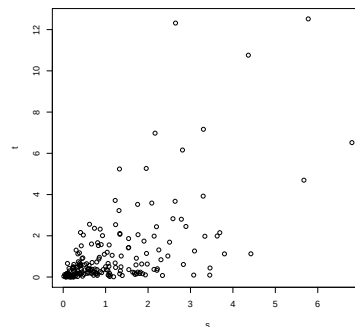
```
1 question2 = function(n){  
2   output = matrix(0, 2, n)  
3   s = rexp(n, 1)  
4   t = rexp(n, 1/s)  
5   output[1, ] = s  
6   output[2, ] = t  
7   return(output)}
```

Then we can put this function to good use:

```
1 test = question2(200)
```

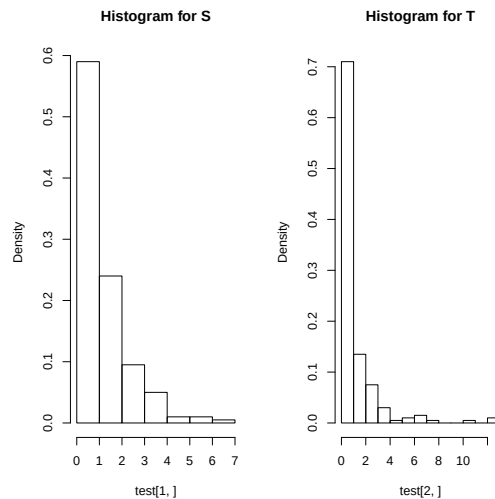
(b) Code:

```
1 plot(test[1,], test[2,], xlab = 's', ylab = 't')  
2 cor(test[,1], test[,2])  
3 [1] 0.6131116
```



(c) Code:

```
1 par(mfrow=c(1,2))  
2 hist(test[1,], probability=TRUE, main="Histogram for S")  
3 hist(test[2,], probability=TRUE, main="Histogram for T")
```



3. (a) Code:

```

1 disc = function(n){
2   r = runif(n, 0, 1)
3   theta = runif(n, 0, 2*pi)
4   output = matrix(ncol = 2, nrow = n)
5   x = r*cos(theta)
6   y = r*sin(theta)
7   output[,1] = x
8   output[,2] = y
9   return(output)}

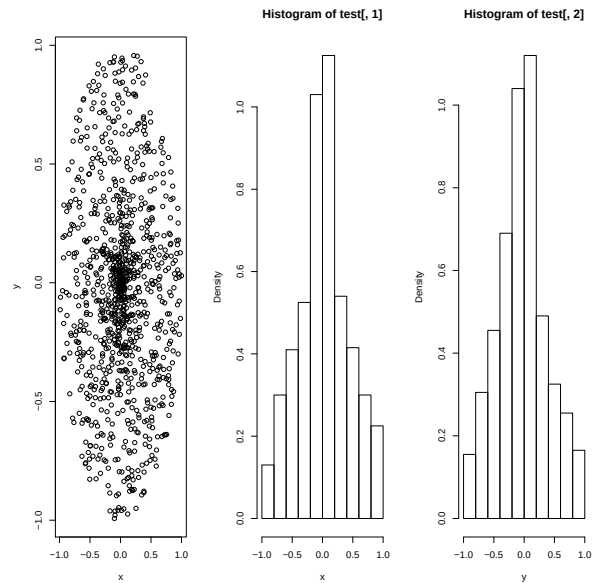
```

Then put the code to use:

```

1 test = disc(1000)
2 par(mfrow=c(1,3))
3 plot(test[,1], test[,2], xlab="x", ylab="y")
4 hist(test[,1], probability=TRUE, xlab="x")
5 hist(test[,2], probability=TRUE, xlab="y")

```



(b) (i) Code:

```

1 disc2 = function(n){
2   x = runif(n, -1, 1)
3   y = runif(n, -1, 1)
4   output = matrix(ncol = 2, nrow = n)
5   output[,1] = x
6   output[,2] = y
7   for(i in 1:n){
8     if((output[i, 1]^2 + output[i, 2]^2) > 1){output[i,] = c(NA, NA)}
9     return(output)}

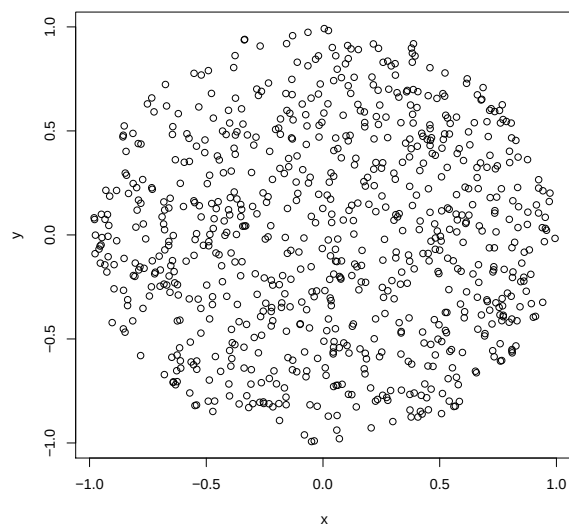
```

Now try the code:

```

1 test = disc2(1000)
2 test = test[complete.cases(test),]
3 plot(test[,1], test[,2], xlab="x", ylab="y")

```

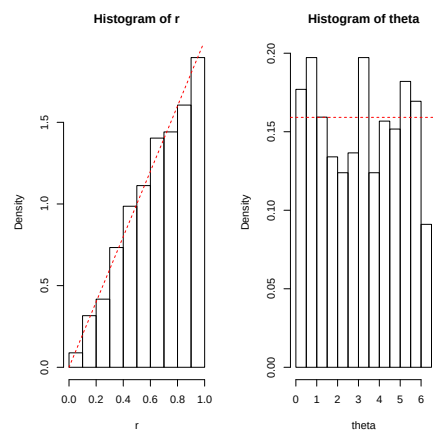


(ii) Code:

```
1 r = sqrt(test[,1]^2 + test[,2]^2)
2 theta = atan(test[,2]/test[,1])
3 for(i in 1:length(theta)){
4   if(test[i,1]>0 & test[i,2]<0){theta[i] = theta[i]+2*pi}
5   if(test[i,1]<0 & test[i,2]<0){theta[i] = theta[i]+pi}
6   if(test[i,1]<0 & test[i,2]>0){theta[i] = theta[i]+pi}
7 }
```

(iii) Code:

```
1 par(mfrow=c(1,2))
2 hist(r, probability = TRUE, xlab="r")
3 curve(2*x, from = 0, to = 1, col="red", lty=2, add=TRUE)
4 hist(theta, probability = TRUE, xlab="theta")
5 abline(h = 1/(2*pi), col="red", lty=2)
```



Mini test material:

1. We have

$$F_Y(y) = \Pr(Y \leq y) = \Pr(X \leq y^{1/3}) = F_X(y^{1/3}).$$

Therefore

$$\begin{aligned} f_Y(y) &= \frac{d}{dy} F_X(y^{1/3}) \\ &= f_X(y^{1/3}) \frac{1}{3} y^{-2/3} \\ &= \frac{1}{\sqrt{2\pi}} \exp\left\{-\frac{(y^{1/3})^2}{2}\right\} \times \frac{1}{3} y^{-2/3} \\ &= \frac{1}{\sqrt{18\pi} y^{2/3}} \exp\left\{-\frac{1}{2} y^{2/3}\right\}. \end{aligned}$$

So: Option **B**.

2. Notice that conditional on X , Y is exponentially distributed with rate $1/x$. Thus:

```
1 Q2 = function(n){
2   x = rexp(n, 2)
3   y = rexp(n, (1/x))
4   output = matrix(ncol = 2, nrow = n)
5   output[,1] = x
6   output[,2] = y
7   return(output)}
```

Then a scatterplot and correlation can be obtained using:

```
1 test = Q2(500)
2 plot(test[,1], test[,2])
3 cor(test[,1], test[,2])
4 [1] 0.5398825
```

