

1. (a) Let $y = 1 - e^{-x/\sigma}$. Then

$$\begin{aligned} -x/\sigma &= \log(1 - y) \\ x &= -\sigma \log(1 - y), \end{aligned}$$

with $\hat{\sigma} = 12$.

Code:

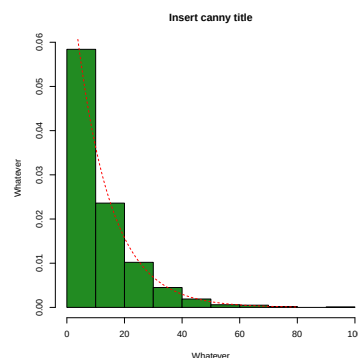
```
1 inv.cdf = function(y){
2   ans = -12*log(1-y)
3   return(ans)}
```

- (b) Code:

```
1 X = inv.cdf(runif(1000))
```

- (c) Code:

```
1 hist(X, probability=TRUE, col="ForestGreen", xlab="Whatever", ylab="
  Whatever", main="Insert canny title")
2 curve((1/12)*exp(-x/12), from=0, to=80, col="red", lty=2, add=TRUE)
```



- (d) Code:

```
1 summary(X)
2      Min.   1st Qu.   Median     Mean   3rd Qu.    Max.
3 0.00201  3.29576  7.96130 11.63675 16.07250 98.41827
```

The mean and quartiles match up very closely to those obtained from the wind speed data.

- (e) Code:

```

1 X1 = rexp(1000, 1/12)
2 summary(X1)
3      Min.   1st Qu.   Median     Mean   3rd Qu.     Max.
4  0.00696   3.20706   7.66012  11.89842  16.94143  76.07372

```

Summaries for X and $X1$ very similar - this form of the Generalised Pareto Distribution is equivalent to an exponential distribution with rate $1/\sigma$.

2. (a) Code:

```

1 Y1 = sqrt(runif(1000, 1, 5))
2 Y2 = log(runif(1000, 1, 5))

```

(b) For $Y1$:

$$F_{Y1}(y) = \Pr(Y_1 \leq y) = \Pr(\sqrt{X} \leq y) = \Pr(X \leq y^2) = F_X(y^2).$$

Now

$$\begin{aligned}
 f_{Y1}(y) &= \frac{d}{dy} F_X(y^2) \\
 &= f_X(y^2) \times 2y \quad \text{by chain rule} \\
 &= 0.25 \times 2y = 0.5y, \quad \sqrt{1} \leq y \leq \sqrt{5},
 \end{aligned}$$

since $f_X(x) = 0.25$ for all x (Uniform distribution with 'base length' 4, hence 'height' must be 0.25).

For $Y2$:

$$F_{Y2}(y) = \Pr(Y_2 \leq y) = \Pr(\log X \leq y) = \Pr(X \leq e^y) = F_X(e^y).$$

Now

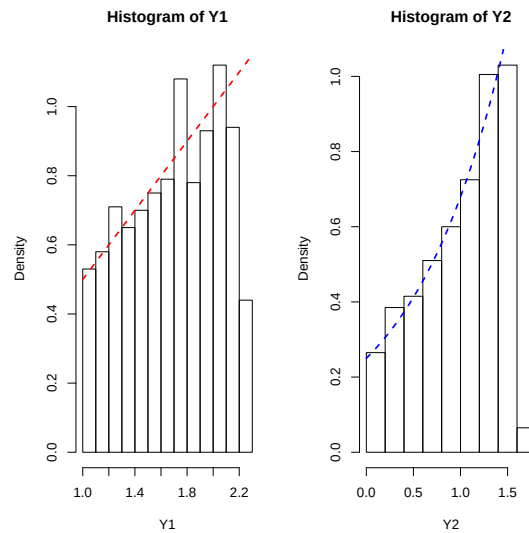
$$\begin{aligned}
 f_{Y2}(y) &= \frac{d}{dy} F_X(e^y) \\
 &= f_X(e^y) \times e^y \quad \text{by chain rule} \\
 &= \frac{e^y}{4}, \quad \log 1 \leq y \leq \log 5.
 \end{aligned}$$

(c) Code:

```

1 par(mfrow=c(1, 2))
2 hist(Y1, probability = TRUE)
3 curve(0.5*x, from=1, to=2.3, add=TRUE, col="red", lwd=2, lty=2)
4 hist(Y2, probability=TRUE)
5 curve(0.25*exp(x), from=0, to=1.6, add=TRUE, col="blue", lwd=2, lty
  =2)

```



3. (a) Code:

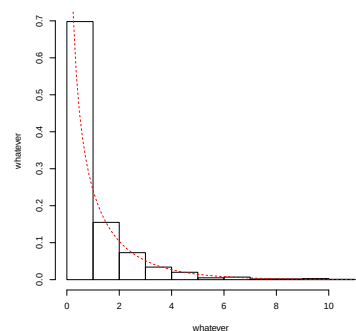
```
1 Y = (rnorm(1000))^2
```

(b) Code:

```
1 hist(Y, probability=TRUE, main="", xlab="whatever", ylab="whatever")
```

(c) Code:

```
1 curve((1/(2^(0.5))*gamma(0.5))*x^(-0.5)*exp(-x/2), from=0, to=12, col="red", lty=2, add=TRUE)
```



(d) Code:

```
1 length(Y[Y>3.84])/length(Y)
2 [1] 0.041
```

(e) Code:

```
1 Y1 = rchisq(1000,1)
2 1-pchisq(3.84,1)
3 [1] 0.05004352
```