

Statistics Assignment

This assignment is worth 10% of the module marks. Solutions should be handed in to the office by 4pm on Friday 30 November. For information about the Late Work Policy, see blackboard or the preliminary module information given in the module notes.

A marking scheme is provided for each question. This is a list of items you must include in your answer. No other material need be included in order to obtain full marks.

Question 1. Suppose that X is a random variable with PDF

$$f_X(x) = \frac{x}{\sigma^2} e^{-x^2/(2\sigma^2)} \quad \text{for } x > 0$$

where σ is a fixed constant.

- (a) Find the CDF $F_X(x)$ of X .
- (b) Write a function to generate simulated values of X using the inverse CDF method.
- (c) Let d be the last digit of your student id. Let $\sigma = 1 + (d/2.5)$. Plot a histogram of a sample of X of size 1000 using your value of σ . Compute the sample mean.

Marking scheme:

- Write down the CDF together with brief justification.
- Include your R code for your function.
- Write down your student id and σ .
- Include the histogram and sample mean.

Question 2.

- (a) Write a R function to sample from the log-normal distribution $LN(\mu, \sigma^2)$ (see section 3.1 of the notes and example 3.3).
- (b) For $\sigma = 2$ generate samples from the log-normal distribution for a number of different values for μ in the range $\mu \in (0, 5)$. For each value of μ compute the log of the sample mean. Plot a graph of the log of the sample mean versus μ .
- (c) Repeat part (b), plotting the log of the sample median versus μ .
- (d) What can you conclude about the mean and median of the $LN(\mu, \sigma^2)$ distribution?

Marking scheme:

- Include your R code for your function.

- Include one plot for the log of the sample mean and one plot for the sample median.
- Write at most 2 sentences for part (d).

Question 3. Suppose $(X, Y)^T \sim N_2(\underline{\mu}, \Sigma)$ where

$$\underline{\mu} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \quad \Sigma = \begin{pmatrix} 1 & 0 \\ 0 & 2 \end{pmatrix}.$$

Suppose random variables (U, V) are defined by

$$\begin{pmatrix} U \\ V \end{pmatrix} = \frac{\sqrt{2}}{2} \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} X \\ Y \end{pmatrix}.$$

- Calculate the determinant J for the transformation from $(X, Y)^T$ to $(U, V)^T$.
- Generate a sample of pairs (U, V) , of size 500, and make a scatterplot.
- Assuming $(U, V)^T$ has a bivariate normal distribution, use your sample to estimate the parameters.

Marking scheme:

- Write down J together with a brief derivation.
- Include the scatterplot.
- Write down the estimates of $\mu_u, \mu_v, \sigma_u \sigma_v, \rho$.