

Practical 3: Thursday 22nd November, 4–6pm

1. Suppose that $X \sim N(100, 25)$ and that

$$f_{Y|X}(Y|X = x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}(y-x)^2}.$$

Write an R function to sample realizations from (X, Y) , and explore the dependence between X and Y .

2. Suppose $S \sim \text{Exp}(1)$ and that

$$f_{T|S}(T|S = s) = \begin{cases} s^{-1}e^{-t/s}, & \text{when } t \geq 0; \\ 0, & \text{otherwise.} \end{cases}$$

- (a) Write an R function to generate a 2×200 matrix of samples from (S, T) . [Hint: Refer to question 1 in last week's practical session.]
- (b) Explore the dependence between S and T .
- (c) Produce histograms of the marginal distributions for S and T .
3. Consider distributions of points that lie on a disc of radius 1. Suppose (R, Θ) are the polar coordinates of a random point, and suppose that $R \sim U(0, 1)$ and $\Theta \sim U(0, 2\pi)$. Use the code on page 23 of the lecture notes to investigate the distribution of points that lie on this disc; specifically, reproduce the plot at the foot of page 23, and obtain histograms of the marginal distributions for the cartesian coordinates of points on the disc.
4. You should see from your work in question 3 that the cartesian coordinates of the points on this disc are *not* scattered uniformly across the disc.
- (a) In R, write a function to obtain points with cartesian coordinates that *are* scattered uniformly across the unit disc. Produce a plot to check that your points are indeed scattered uniformly. [Hint: The aim here is to produce a plot such as that at the foot of page 23 of the lecture notes, but with a random scatter of points across the disc; you could make use of `runif(n, -1, 1)` for cartesian coordinates x and y , but you will want to discard any pairs of points such that $x^2 + y^2 > 1$.]
- (b) Transform the coordinates of your simulated points in part (a) to polar form.
- (c) Produce histograms of the marginal distributions for the polar coordinates of your simulated points, and verify your solutions to Example 4.3 in the lecture notes by superimposing the marginal PDFs you obtained here.

Class test next week: Some information

In the test next week you will be expected to upload a single PDF file to NESS, generated within MS WORD. This PDF will contain your typed solutions to:

- (i) some multiple choice questions based on the more theoretical parts of the course;
- (ii) some computing questions that will require the inclusion of R code.

You can practice the file upload process now through the ‘mini test’ below. The real test next week will be a bit longer than this. An example PDF solution file will be available on Blackboard/the course webpage, as will a full specimen test/solution file.

Mini test

Open a blank MS WORD document. On the top line of this document write your name in size 12 font. On the second line, write your student number. Your solutions to the following questions should be typed into this WORD document, with any necessary R code copied-and-pasted into the document too. Make sure you clearly indicate the question number for each solution you type, including any sub-question numbering (e.g. **1.**, **2(a).**, etc.).

1. Suppose $X \sim N(0, 1)$ and that $Y = X^3$. Which option A–D correctly specifies the probability density function of Y ? In your solutions, make sure you type **one** choice (A, B,...) next to the question number (1).

$$\text{A. } f_Y(y) = \frac{1}{(2\pi)^{3/2}} e^{-\frac{3}{2}y^2}, 0 \leq y < \infty;$$

$$\text{B. } f_Y(y) = \frac{1}{\sqrt{18\pi}y^{2/3}} e^{-\frac{1}{2}y^{2/3}}, 0 \leq y < \infty;$$

$$\text{C. } f_Y(y) = \frac{1}{\sqrt{8\pi}y} e^{-\frac{1}{2}y}, 0 \leq y < \infty;$$

$$\text{D. } f_Y(y) = \lambda e^{(y-\lambda e^y)}, -\infty < y < \infty.$$

2. Suppose $X \sim \text{Exp}(2)$ and that

$$f_{Y|X}(Y|X = x) = \begin{cases} \frac{e^{-y/x}}{x}, & \text{when } y \geq 0, \\ 0, & \text{otherwise.} \end{cases}$$

Write an R function to generate 500 pairs of realizations of (X, Y) . Produce a scatter-plot of your realizations of X against those for Y , and obtain the correlation between X and Y . In your solutions, include (i) your main function; (ii) your scatterplot; (iii) the value of the correlation. No other code or solutions are needed.

Save your file as a PDF document in WORD by: clicking ‘File’ in the top -left corner of the WORD window; selcting ‘Save As’; clicking ‘PDF’ or ‘XPS’. A dialogue box should appear - make sure it says ‘PDF’ in the ‘Save as Type’ box. Give your document a sensible name, and then click ‘Save’. In NESS, go to MAS2602; then under the ‘Coursework’ option down the left-hand-side click ‘Submit’ and select ‘Practice NESS upload’. Enter your email address to receive a receipt, select your PDF file for upload and then click ‘Submit your work’.