

Predicting Halloween- and Carrington-type solar flares

Lee Fawcett, Thomai Tsiftsi, Daniel Juan Pineda

May 2018

Structure of this talk

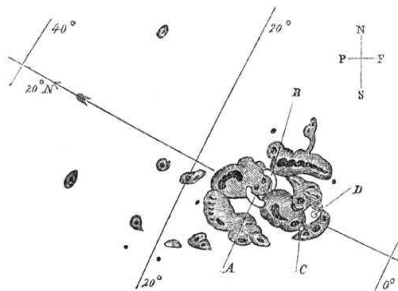
Carrington solar storm, 1859

- Powerful **geomagnetic solar storm** during solar cycle 10 (1855-1867)
- Solar **coronal mass ejection** (CME) hit Earth's magnetosphere inducing one of the largest geomagnetic storms on record
- Observed and recorded by **Richard Carrington**
- Aurorae visible in the northern hemisphere as far south as Monterrey, Cuba and southern Japan
- **Telegraph systems** failed across Europe and North America
- Peak fluxes classified as A, B, C, M, X: each letter indicates a flare an order of magnitude larger than the one preceding
 - A1 flare has a peak X-ray flux of $1 \times 10^{-8} \text{ Wm}^{-2}$
 - X1 flare has a peak X-ray flux of $1 \times 10^{-4} \text{ Wm}^{-2}$
 - Carrington event: estimated as X45, i.e. $45 \times 10^{-4} \text{ Wm}^{-2}$

Carrington solar storm, 1859

- Powerful **geomagnetic solar storm** during solar cycle 10 (1855-1867)
- Solar **coronal mass ejection** (CME) hit Earth's magnetosphere inducing one of the largest geomagnetic storms on record
- Observed and recorded by **Richard Carrington**
- Aurorae visible in the northern hemisphere as far south as Monterrey, Cuba and southern Japan
- **Telegraph systems** failed across Europe and North America
- Peak fluxes classified as A, B, C, M, X: each letter indicates a flare an order of magnitude larger than the one preceding
 - A1 flare has a peak X-ray flux of $1 \times 10^{-8} \text{ Wm}^{-2}$
 - X1 flare has a peak X-ray flux of $1 \times 10^{-4} \text{ Wm}^{-2}$
 - Carrington event: estimated as X45, i.e. $45 \times 10^{-4} \text{ Wm}^{-2}$

Carrington solar storm, 1859



The Auroral Display in Boston.

Boston, Friday, Sept. 2.

There was another display of the Aurora last night, so brilliant that at about one o'clock ordinary print could be read by the light. The effect continued through this forenoon, considerably affecting the working of the telegraph lines. The auroral currents from east to west were so regular that the operators on the Eastern lines were able to hold communication and transmit messages over the line between this city and Portland, the usual batteries being discontinued from the wire. The same effects were exhibited upon the Cape Cod and other lines.

The New York Times

Published: September 3, 1859

Copyright © The New York Times

A. STORM OF ELECTRICITY

TELEGRAPH WIRES USELESS FOR SEVERAL HOURS.

ONE OF THE MOST SEVERE DISTURBANCES FOR MANY YEARS, EXTENDING EVEN TO EUROPE—TELEPHONE WIRES ALSO OBSTRUCTED—BUSINESS DELAYED A GOOD PART OF THE DAY.

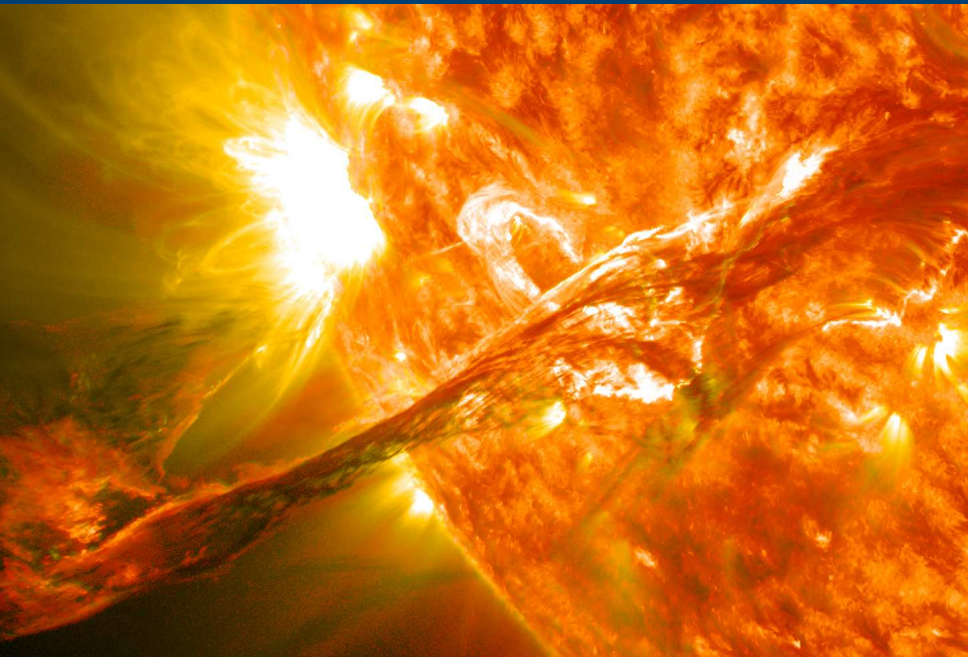
Yesterday's storm was accompanied by a more serious electrical disturbance than has been known for years. It very seriously affected the workings of the telegraph lines both on the land and in the sea, and for three hours—from 9 A. M. until noon—telegraph business east of the Mississippi and north of Washington was at a stand-still.



Halloween solar storms, 2003

- Series of solar flares and CMEs occurring in October/November 2003 during solar cycle 23 (1996-2008)
- Generated the largest solar flare ever recorded by the **GOES** system, estimated at X28
- **Satellite systems** and **communications** affected, numerous **spacecraft damaged**, **power outages** in Sweden and Finland
- Aurorae observed in Texas and southeast Mediterranean

Near miss: Carrington-like solar storm, 2012



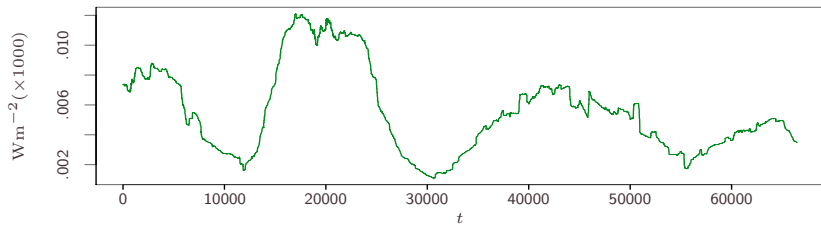
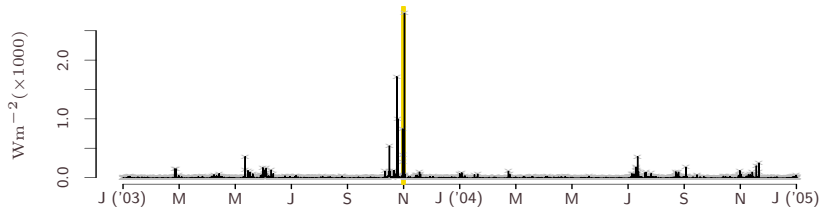
What's at stake?

- Disruption to **global positioning systems**. GPS industry estimated to be a \$1 trillion business
- **Satellite communications** at high risk
- Huge disruption to the **electrical grid**: areas such as the eastern US particularly vulnerable as power infrastructure highly interconnected, large cities without power for weeks
- Key questions: When might we see the next Carrington-like solar storm? What is the **return period** of a Carrington-like event? What is the strength of solar flare we might expect to see once per century?

Power law stuff - see (for eg) Riley (2012) Space Weather: On the probability of occurrence of extreme space weather events

- Extracted from the **Space Weather Prediction Center** (SWPC) online portal
- Recorded by the **Geostationary Operational Environmental Satellite** (GOES) system, operated by the US National Oceanic and Atmospheric Administration
- Data represent solar flares normalised from H_{α} spectral observations and X-ray fluxes spanning a period of 43 years: November 1975-October 2017
- Data exhibit substantial short-term temporal dependence and cyclic variability

Data



Modelling extremes: Starting point

Let $\{X_n\}$ denote a stationary sequence of random variables with common distribution function F , and let $M_n = \max\{X_1, \dots, X_n\}$.

It is typically the case that, as $n \rightarrow \infty$,

$$\Pr(M_n \leq x) \approx F^{n\theta}(x), \quad (1)$$

where $\theta \in (0, 1)$ is known as the **extremal index**; see, for e.g., [Leadbetter & Rootzén \(1988\)](#).

As $\theta \rightarrow 0$ there is increasing dependence in the extremes of the process; for an independent process, $\theta = 1$.

Modelling extremes: Block maxima

The **Generalised Extreme Value** (GEV) distribution is the limiting model for F^n , with distribution function (d.f.)

$$\mathcal{G}(y; \mu, \varsigma, \xi) = \begin{cases} \exp \left[- (1 + \xi(y - \mu)/\varsigma)^{-1/\xi} \right], & \xi \neq 0 \\ \exp \left[-\exp \left(-(y - \mu)/\varsigma \right) \right] & \xi = 0, \end{cases}$$

defined on $\{y : 1 + \xi(y - \mu)/\varsigma > 0\}$, where $-\infty < \mu < \infty$, $\varsigma > 0$ and $-\infty < \xi < \infty$ are parameters of location, scale and shape.

- It can be shown that $F^{n\theta}$ is also GEV with d.f. $\mathcal{G}(y; \mu^*, \varsigma^*, \xi)$, provided long-range dependence is negligible¹
- In practice, the GEV is used to model maxima over some convenient calendar unit – usually years

¹Technically, this requires **Leadbetter's $D(u_n)$ condition** to hold; see **Leadbetter et al. (1983)**.

Modelling extremes: Threshold excesses

Pickands (1975) showed that for $\theta = 1$ and u large, $(X - u | X > u)$ follows a **Generalised Pareto** (GP) distribution with d.f.

$$\mathcal{H}(y) = \begin{cases} 1 - (1 + \xi y / \sigma)^{-1/\xi}, & \xi \neq 0 \\ 1 - \exp[-y/\sigma] & \xi = 0, \end{cases}$$

defined on $\{y : y > 0 \text{ and } (1 + \xi y / \sigma) > 0\}$, where $\sigma = \varsigma + \xi(u - \mu) > 0$ and ξ are the scale and shape.

- Simple graphical diagnostics (not discussed here) available for selection of threshold u
- Extremal index θ can't be absorbed into the parameters of the GPD; careful consideration of extremal dependence required

Modelling extremes: Return level estimation

The **r -year return level z_r** can then be obtained by inversion of $\mathcal{G}(z_r)$ or $\mathcal{H}^\theta(z_r)$.

For threshold excesses, on equating to $1 - r^{-1}$ this gives

$$z_r = \begin{cases} u + \sigma \xi^{-1} \left[(\lambda_u^{-1} w_r)^{-\xi} - 1 \right] & \xi \neq 0 \\ u - \sigma \log (\lambda_u^{-1} w_r) & \xi = 0, \end{cases}$$

where $w_r = 1 - [1 - (rn_y)^{-1}]^{1/\theta}$, λ_u is the rate of threshold excess and n_y is the (average) number of observations per year.

For seasonally varying extremes, solve

$$\prod_{m=1}^M \mathcal{H}_m^{\theta_m}(z_r) = 1 - r^{-1}, \quad (2)$$

where $\mathcal{H}_m$ is the GP distribution function in season m with parameter set $(\lambda_{u_m}, \sigma_m, \xi_m)$, and θ_m is the extremal index in season m .

Modelling extremes: Return level estimation

The **r -year return level z_r** can then be obtained by inversion of $\mathcal{G}(z_r)$ or $\mathcal{H}^\theta(z_r)$.

For threshold excesses, on equating to $1 - r^{-1}$ this gives

$$z_r = \begin{cases} u + \sigma \xi^{-1} \left[(\lambda_u^{-1} w_r)^{-\xi} - 1 \right] & \xi \neq 0 \\ u - \sigma \log(\lambda_u^{-1} w_r) & \xi = 0, \end{cases}$$

where $w_r = 1 - [1 - (rn_y)^{-1}]^{1/\theta}$, λ_u is the rate of threshold excess and n_y is the (average) number of observations per year.

For seasonally varying extremes, solve

$$\prod_{m=1}^M \mathcal{H}_m^{\theta_m}(z_r) = 1 - r^{-1}, \quad (2)$$

where $\mathcal{H}_m$ is the GP distribution function in season m with parameter set $(\lambda_{u_m}, \sigma_m, \xi_m)$, and θ_m is the extremal index in season m .

Modelling extremes: Return level estimation

The **r -year return level z_r** can then be obtained by inversion of $\mathcal{G}(z_r)$ or $\mathcal{H}^\theta(z_r)$.

For threshold excesses, on equating to $1 - r^{-1}$ this gives

$$z_r = \begin{cases} u + \sigma \xi^{-1} \left[(\lambda_u^{-1} w_r)^{-\xi} - 1 \right] & \xi \neq 0 \\ u - \sigma \log(\lambda_u^{-1} w_r) & \xi = 0, \end{cases}$$

where $w_r = 1 - [1 - (rn_y)^{-1}]^{1/\theta}$, λ_u is the rate of threshold excess and n_y is the (average) number of observations per year.

For seasonally varying extremes, solve

$$\prod_{m=1}^M \mathcal{H}_m^{\theta_m}(z_r) = 1 - r^{-1}, \quad (2)$$

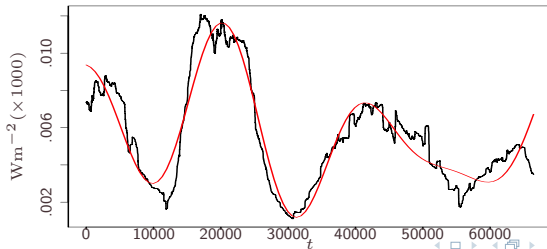
where $\mathcal{H}_m$ is the GP distribution function in season m with parameter set $(\lambda_{u_m}, \sigma_m, \xi_m)$, and θ_m is the extremal index in season m .

Cyclic variation

- Close inspection of the time series suggests non-stationarity **inherited from the solar cycle**, affecting both the frequency and the severity of extreme flares
- After trial-and-improvement, fit sinusoidal waves of the form

$$\sum_{i=1}^3 A_i \sin \omega_i t + B_i \cos \omega_i t + C$$

to the series of moving averages, subtracting the estimated function before performing extreme value analyses



Typical POT analysis

- Identify sets of **independent clusters** of solar flare extremes over some suitably high threshold u
- Runs declustering, with cluster termination interval κ , most commonly used
- Fit the GP distribution to the set of independent **cluster peak excesses**
- Substitute $(\hat{\lambda}_u, \hat{\sigma}, \hat{\xi}, \theta = 1)$ into Eq. (2) to obtain $\hat{z}_r / \hat{r}$:

(u, κ)		$\hat{z}_{50}$		$\hat{r}$ years	
		Post. mean	95% CI	Halloween	Carrington
$u = 90\%$ quantile	$\kappa = 10$	X34.5	(X30.5, X44.5)	38	110
	$\kappa = 15$	X35	(X31.5, X47)	37.5	100
	$\kappa = 30$	X35.5	(X32, X65)	33	96.5
$u = 95\%$ quantile	$\kappa = 10$	X35	(X30, X46)	38	100
	$\kappa = 15$	X36	(X31, X47.5)	37	97
	$\kappa = 30$	X37.5	(X30, X66.5)	30	79