MAS1802: Problem Solving II (Statistical Computing with R)

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School of Mathematics, Statistics & Physics

Semester 2, 2018/2019

Part V

Random Number Generation

Some thoughts...

- When writing big chunks of code, indent nicely! Generally, it is good practice for your code to be structured (bug-spotting, typos etc.). Certainly don't type everything onto a single line.
- `if/else` statements – feel free to put each `if` on a single line.
- Plus symbol in the console window – R is expecting something else – most likely a close bracket!
- Running functions – e.g. `IsEven(5)`
- Commenting in code is very important! Use the # symbol to let us know what you're doing!

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- Practical 3 due in **Friday, 11am**
- This Friday's practial: Group project time! Up to you who you work with – let me know by attaching all the NESS cover sheets for members of your group to the same report
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5.1 Randomness: quantifying uncertainty

- The concepts of uncertainty and randomness have intrigued humanity for a long time:
  - Chinese: formalised odds/chance some 3000 years ago
  - Greek philosophers discussed randomness at length
  - 16th Century: Italian mathematicians formalised odds associated with games of chance
  - 19th Century: Physics
  - Early 20th Century: Rapid growth in the formal analysis of randomness

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5.1 Randomness: quantifying uncertainty

- The world around us is not deterministic and we are faced continually with chance occurrences.
- Uncertainty is inherent in nature; for example, the behaviour of fundamental physical particles, genes and chromosomes in biology, and individuals in society under stress or strain.
- The methodology for exploring uncertainty involves the use of *random numbers*.

5.2 Pseudo–random numbers

Suppose we need to obtain a list of random digits $0, 1, 2, \ldots, 9$. How might we go about this? There are several options:

Fair ten–sided die

If the sides are labelled from 0 to 9 then tosses of this die will yield the required digits.

Decimal expansion of π

Irrational number – decimal expansion goes on forever, with no pattern!

$$\pi = 3.1415$$
$$92653589793$$
$$23846264338 3$$
$$2795028841971 69$$
$$3993751058209749 4$$
$$4592307816406286208998$$

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Tosses of a fair coin

Toss a fair coin four times. The following equally likely outcomes could correspond to the integers shown:

H H H H	0	H T H T	5
H H H T	1	H T T H	6
H H T H	2	H T T T	7
H H T T	3	T H H H	8
H T H H	4	T H H T	9

The coin has no ‘memory’, and so each block of 4 tosses is independent of any other. If any outcome other than those listed occurs, we can ignore and toss a new set of 4.

This method is rather inefficient as a lot of the time a combination of outcomes is rejected!

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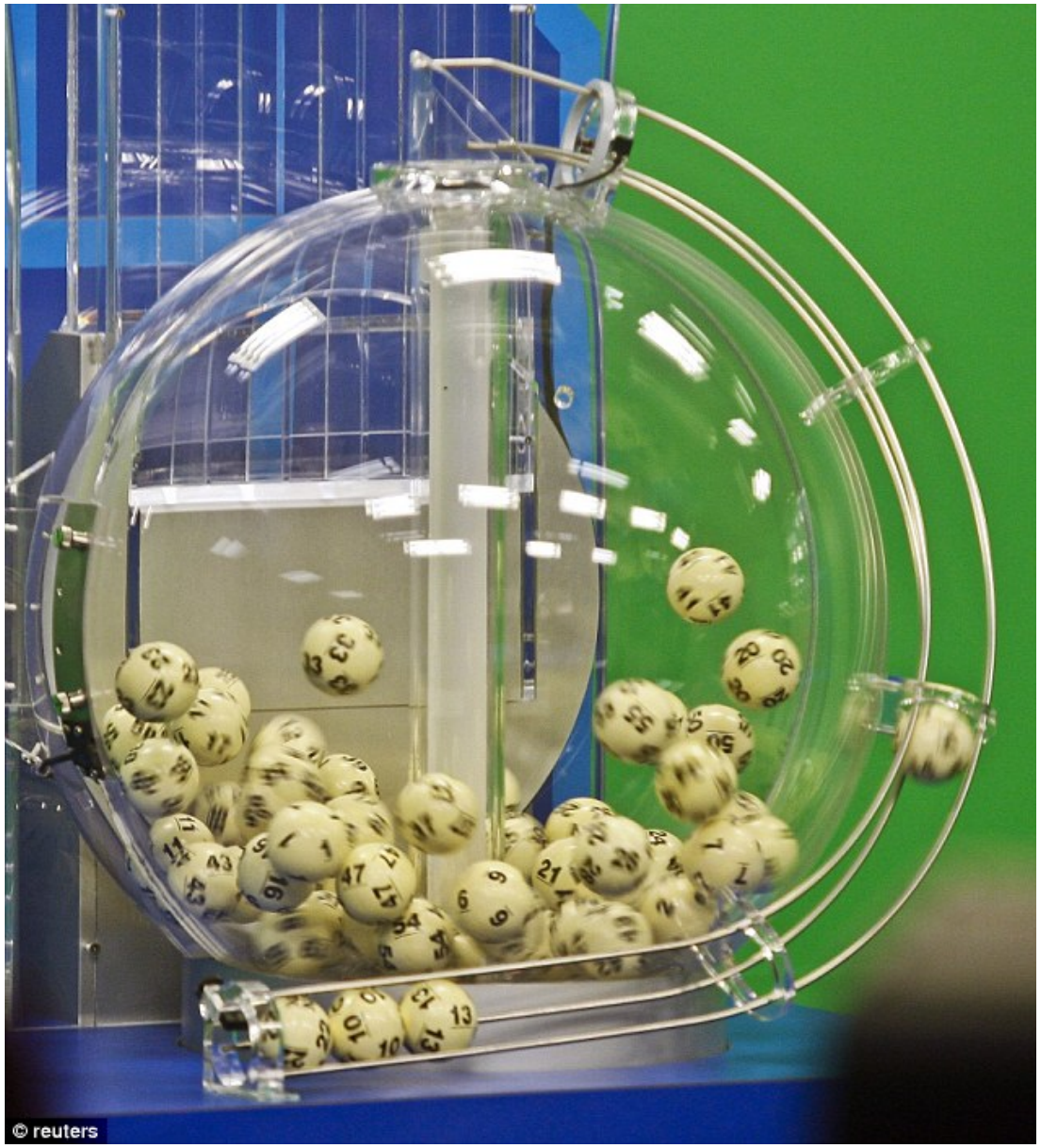
Other physical devices: Wheels of fortune

Other physical devices: Lottery machines



Other physical devices: Gamma ray counters

- Quantum mechanics predicts that the nuclear decay of atoms is random
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The German Federal Office for Information Security (*Bundesamt für Sicherheit in der Informationstechnik*, or BSI) has established criteria for quality RNG:

1. A sequence of random numbers has a high probability of containing no identical consecutive elements
2. A sequence of numbers which is indistinguishable from ‘true random’ numbers (tested using statistical tests)
3. It should be impossible to calculate – or guess – from any given sub-sequence, any previous or future values in the sequence
4. It should be impossible, for all practical purposes, for an attacker to calculate, or guess, the values used in the random number algorithm

Points 3 and 4 are crucial for many applications.

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5.2 Pseudo–random numbers

As we will see in this course, it is important to be able to reproduce a sequence of random numbers.

The advent of digital computers gave rise to a number of techniques that use a recursive relation, that is, the number r_i in a sequence is a function of the preceding numbers $r_{i-1}, r_{i-2}, \ldots$.

Some of these PRNG's have quite cool names: for example,

- the *Super-Duper* PRNG
- the *Blum-Blum-Shub* PRNG (after Lenore Blum, Manuel Blum and Michael Shub)
- the *Mersenne-Twister* PRNG (as used by R)

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5.3.1 The `runif` function

```
> runif(n, min=0, max=1)
```

- This function will generate n random numbers between the values of `min` and `max`.
- If the arguments `min` or `max` are omitted, then the default values are 0 and 1 respectively.

5.3.1 The runif function

For example,

```
> runif(1)
[1] 0.2729965
> runif(1)
[1] 0.9990863
> runif(5)
[1] 0.8122783 0.9069950 0.1731072 0.3454292 0.6102412
> runif(1, 6, 7)
[1] 6.427512
```

5.3.1 The runif function

```
> set.seed(12345)
> runif(1)
[1] 0.7209039
> runif(1)
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5.3.2 The sample function

Another important R function that we will use is the sample function:

```
> sample(x, size, replace = FALSE, prob = NULL)
```

5.3.2 The `sample` function

This takes the following arguments

- `x`: a list of values
- `size`: non-negative integer giving the number of items to choose.
- `replace`: Should sampling be with replacement? Default: `FALSE`.
- `prob`: A vector of probability weights. Default: All values equally likely.

5.3.2 Example usage of `sample`

Suppose we wish to sample five numbers from $\{1, 2, 3, 4, 5, 6\}$, then

```
> set.seed(1)
> x = c(1, 2, 3, 4, 5, 6)
> sample(x, 5)
[1] 2 6 3 4 1
```

We can also sample with replacement:

```
> sample(x, 5, replace=TRUE)
[1] 6 6 4 4 1
```

This means that values **may** appear more than once.

5.3.3 Simulating the Captial One Cup draw

We are in the semi-finals of the *Capital One* Cup, and we need to organise the draw for the final stage. The remaining teams are:

Manchester Utd, Mancehster City, Sunderland, West Ham.

5.3.3 Simulating the Captial One Cup draw

Here's how we do this in R:

```
> set.seed(3)
> teams = c("Man Utd", "Man City", "Sunderland", "West
Ham")
> sample(teams, 4)
[1] "Man Utd"    "Sunderland" "West Ham"   "Man City"
```

So we have 'Man Utd vs Sunderland' and 'West Ham vs Man City'.

5.3.3 Simulating the Capital One Cup draw

However, if we think Sunderland are likely to get beaten by Man Utd, we can rig the voting:

```
> prob_weights = c(0.4, 0.4, 0.05, 0.2)
> sample(teams, 4, prob=prob_weights)
[1] "Man Utd"    "Man City"   "West Ham"
"Sunderland"
```

That's better!

5.3.3 Simulating the Capital One Cup draw

5.4 Simulating from discrete probability distributions

Suppose we want to simulate a discrete random variable X which has probability mass function

$$\Pr[X = x_j] = p_j \quad \text{for } j = 1, 2, \ldots,$$

and where $\sum_{\text{all}} p_j = 1$.

5.4 Simulating from discrete probability distributions

To accomplish this we first simulate a value u from a $U(0,1)$ distribution and set

$$x = \begin{cases} x_1 & \text{if } u < p_1 \\ x_2 & \text{if } p_1 \leq u < p_1 + p_2 \\ x_3 & \text{if } p_1 + p_2 \leq u < p_1 + p_2 + p_3 \\ \vdots & \vdots \\ x_j & \text{if } \sum_{i=1}^{j-1} p_i \leq u < \sum_{i=1}^{j} p_i \\ \vdots & \vdots \end{cases}$$

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5.4.1 Example

Simulate a random variable with the following probability mass function:

x	1	2	3	4
$\Pr[X = x]$	0.2	0.15	0.25	0.4

5.4.1 Solution

We calculate the CDF of the distribution:

x	1	2	3	4
$\Pr[X = x]$	0.2	0.15	0.25	0.4
$\Pr[X \leq x]$	0.2	0.35	0.60	1.0

Then we generate an observation u from $U \sim \mathrm{Uniform}(0, 1)$, so

- if $u < 0.2$, set $X = 1$;
- if $0.2 \leq u < (0.2 + 0.15) = 0.35$, set $X = 2$;
- if $0.35 \leq u < (0.2 + 0.15 + 0.25) = 0.6$, set $X = 3$;
- if $u \geq 0.6$, set $X = 4$.

Suppose $u = 0.821$. Then $X = 4$.

5.4.1 Solution

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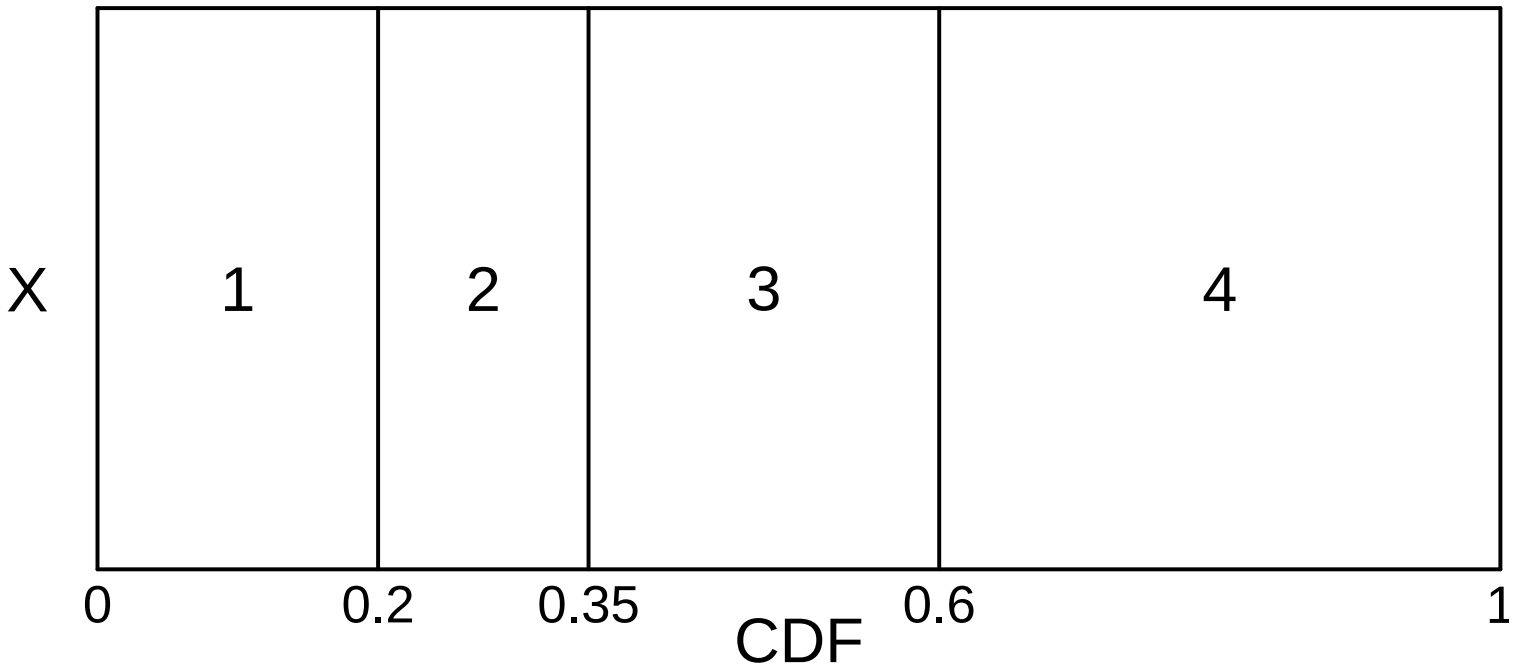

Figure: Simulating discrete random numbers.

5.4.1 Using R

To simulate from the above distribution in R, there are two (obvious) methods that we can use. We could use the `sample` command...

```
> x = c(1, 2, 3, 4)
> prob = c(0.2, 0.15, 0.25, 0.4)
> sum(prob)
[1] 1
> sample(x, 1, prob, replace=TRUE)
[1] 4
```

5.4.1 Using R

... or we could use a bunch of `if` statements (Monopoly!):

```
> u = runif(1)
> if(u <= 0.2) {
+   X = 1
+ } else if(u <= (0.2+0.15)) {
+   X = 2
+ } else if(u <= (0.2+0.15+0.25)) {
+   X = 3
+ } else {
+   X = 4
+ }
```

5.4.2 Simulating a Poisson random variable

For the uniform random numbers

$$0.253,\ 0.588,\ 0.789,$$

simulate three random numbers from a Poisson distribution with mean $\lambda = 2$.

5.4.2 Solution

The pdf of the Poisson distribution is

$$Pr[X = x] = \frac{e^{-\lambda}\lambda^x}{x!}$$

where $\lambda > 0$. So we have

x	0	1	2	3	4
$\Pr[X = x]$	0.135	0.271	0.271	0.180	0.090
$\Pr[X \leq x]$	0.135	0.406	0.677	0.857	0.947

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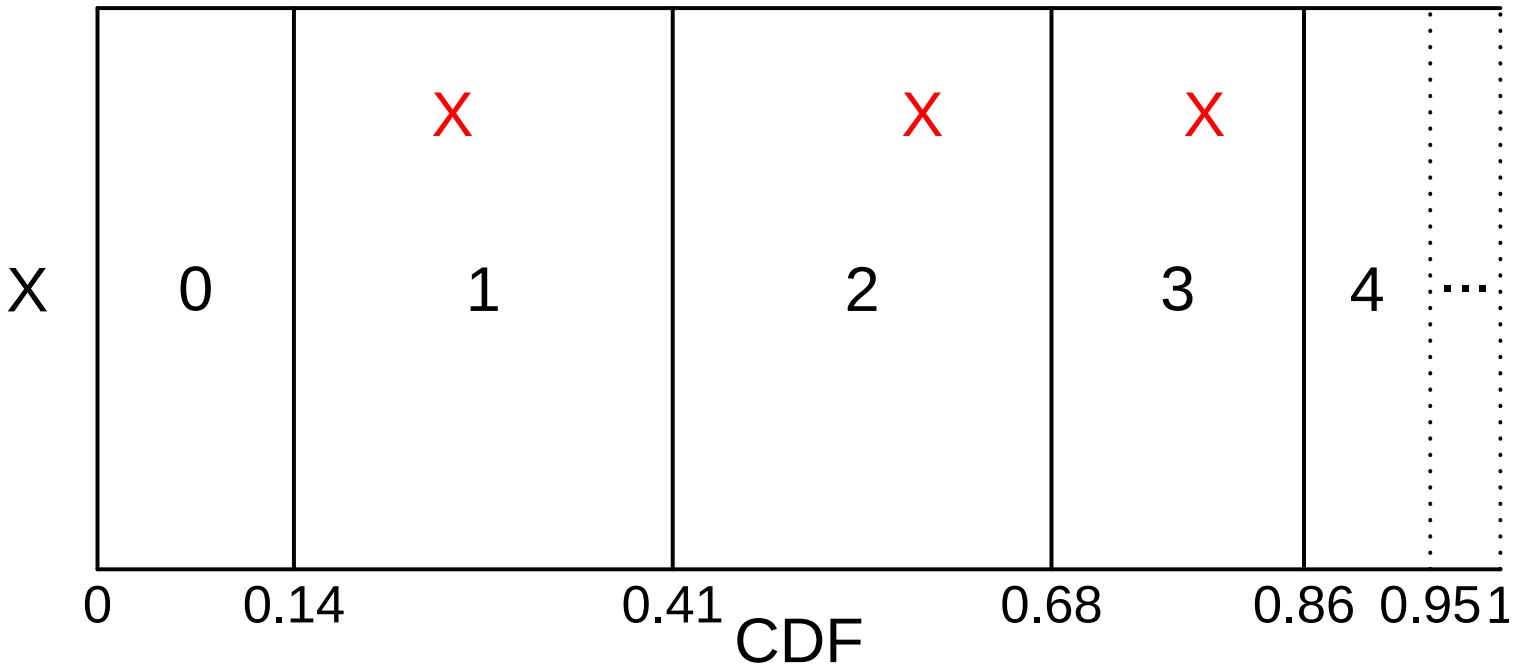

Figure: Diagram illustrating on simulating discrete random numbers from the Poisson distribution.

Hence our random numbers are 1, 2 and 3.

5.4.2 Using R

```
> rpois(1,2)
[1] 3
```

```
> rpois(10,2)
[1] 0 1 2 0 0 0 2 1 3 1
```

5.4.3 Simulating from other distributions

Simulating random values from other standard probability distributions is easy in R.

Just as we have seen the intrinsic functions `runif` and `rpois`, there are other functions such as `rgeom` and `rbinom`.

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5.4.3 Simulating from other distributions

For example, suppose $X \sim \mathrm{Bin}(6, 0.2)$ and $Y \sim \mathrm{Geom}(0.75)$, and we want to simulate 5 random values from X and 3 random values from Y:

```
> (X=rbinom(5,6,0.2))
[1] 0 1 1 3 0
```

```
> (Y=rgeom(3,0.75))
[1] 2 0 1
```

5.4.3 Variations on a theme

We have seen, for example, `runif`, `rpois`, `rbinom` and `rgeom`.

Replacing the `r` prefix with:

- d, calculates the density of the distribution at a particular value.
  For example, `dpois(1,2)` would find $Pr[X = 1]$ when $X \sim Po(2)$
- p, calculates the distribution function at a particular value.
  For example, `pbinom(2,6,0.2)` would find
  $Pr[X \leq 2] = Pr[X = 0] + Pr[X = 1] + Pr[X = 2]$ when
  $X \sim \mathrm{Bin}(6, 0.2)$
- q, finds quantiles of a distribution for a particular probability.
  For example, `qnorm(0.7,0,1)` finds x such that $Pr[X \leq x] = 0.7$, when
  $X \sim N(0, 1)$

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  For example, `qnorm(0.7,0,1)` finds x such that $Pr[X \leq x] = 0.7$, when
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5.4.3 Variations on a theme

We have seen, for example, `runif`, `rpois`, `rbinom` and `rgeom`.

Replacing the `r` prefix with:

- d, calculates the density of the distribution at a particular value.
  For example, `dpois(1,2)` would find $Pr[X = 1]$ when $X \sim Po(2)$
- p, calculates the distribution function at a particular value.
  For example, `pbinom(2,6,0.2)` would find
  $Pr[X \leq 2] = Pr[X = 0] + Pr[X = 1] + Pr[X = 2]$ when
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- **Debugging**
  - For question 2, do each part (a)–(h) one at a time, checking the graph at each stage...
  - ...then, if you get something weird, you'll have more idea where you went wrong
  - Each time you edit your code, do so in a different script, so you don't screw up the code that you know works!
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  - Initially, all probabilities should hover around 1/40
  - Once you start adding in extra layers (chance, community chest etc) this will change for some squares
  - Lots of things send you to Jail! So expect a spike here...
- **Rolling doubles – this bit is hard!**
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Rolling doubles

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> df = data.frame(d1=sample(1:6,3,replace=TRUE),d2=sample(1:6,3,replace=TRUE))
> df$Total = apply(df,1,sum)
> df$IsDouble = df$d1==df$d2
> df
  d1 d2 Total IsDouble
1  6  3     9    FALSE
2  4  3     7    FALSE
3  2  2     4     TRUE
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> if(df$IsDouble[1]==TRUE & df$IsDouble[2]==TRUE & df$IsDouble[3]==TRUE) {
      current = 11 #Go To Jail
  } else if (df$IsDouble[1]==TRUE & df$IsDouble[2]==TRUE & df$IsDouble[3]!=TRUE) {
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Ultimate goals:

- Find out how many packets of stickers are needed to fill the album!
- How much will this cost?
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