



MAS1403

Quantitative Methods for Business Management

Semester 1, 2017–2018

Module leader: Dr. Lee Fawcett

Additional lecturers: Dr. Dave Walshaw and Dr. Ged Cowburn

Computer-based assessment

- CBA1 has now gone live in **assessed mode**
- At some point this week you must complete this CBA - don't forget, you can log-in/log-out, provided you save your work as you go
- You only get one shot as this CBA – multiple attempts are not allowed – and it counts! **3.33%**
- **Deadline for completion: 23:59 this coming Friday, 27th October**

Lecture 4

INTRODUCTION TO PROBABILITY

4. Introduction to Probability

Probability is the language we use to model uncertainty.

We all intuitively understand that few things in life are certain. There is usually an element of **uncertainty** or **randomness** around outcomes of our choices.

In business this uncertainty can make all the difference between a good investment and a poor one.

Hence, an understanding of probability and how we might incorporate this into our decision-making processes is important.

4.1 Definitions

We often use the letter P to represent a probability.

For example, $P(\text{Rain})$ would be the probability that it rains.

An **Experiment** is an activity where we do not know for certain what will happen, but we can *observe* what happens. **For example:**

- We will ask someone whether or not they have used our product.
- We will observe the temperature at midday tomorrow.
- We will toss a coin and observe whether it shows “heads” or “tails”.

4.1 Definitions

An **Outcome** is one of the possible things that can happen. For example, suppose that we are interested in the shoe size of the next customer to come into a shoe shop.

Possible outcomes include:

- “eight”
- “twelve”
- “nine and a half”

In any experiment, one, *and only one*, outcome occurs.

The **Sample space** is the set of all possible outcomes. For example, it could be the set of all shoe sizes.

An **Event** is a set of outcomes. For example “the shoe size of the next customer is less than 9” is an event. It is made up of all of the outcomes where the shoe size is less than 9.

4.1 Definitions

Probabilities are usually expressed in terms of **fractions**, **decimals** or **percentages**.

Therefore we could express the probability of it raining today as

$$P(Rain) = \frac{1}{20} = 0.05 = 5\%.$$

All probabilities are measured on a scale from zero to one.

- An **impossible** event has a probability of zero
- A **certain** event has a probability of one
- An **evens** event has a probability of 0.5
- Can you imagine whereabouts on this scale a **likely** event will lie? Or an **extremely unlikely event**?

4.1 Definitions

The collection of all possible outcomes – the **sample space** – has a probability of 1.

For example:

- Suppose an event has only two outcomes – *success* or *failure*
- Then $P(\text{success or failure}) = 1$

Another example:

- Suppose we have a fair six-sided die
- Then $P(1 \text{ or } 2 \text{ or } 3 \text{ or } 4 \text{ or } 5 \text{ or } 6) = 1$

4.1 Definitions

Two events are said to be **mutually exclusive** if both can not occur simultaneously. In the example above, the outcomes *success* and *failure* are mutually exclusive.

Two events are said to be **independent** if the occurrence of one does not affect the probability of the second occurring.

For example, if you toss a coin and look out of the window, the events “get heads” and “it is raining” would be independent.

4.1 Definitions

An example of non-independence

Imagine I go into the Student's Union and pick a student at random.

The events “**the student is female**” and “**the student is studying engineering**” are not independent.

there is a greater proportion of male students on engineering courses than on other courses at the University!

Example 1

Do you think the following pairs of events are independent?

- A : Molly plays table tennis, and B : Molly is good at maths
- C : Henry gets over 60 in MAS1403, and D : Henry gets under 40 in MAS1403

Example 1

Ask yourself this question:

Does knowing the likelihood of one event change my opinion about the likelihood of the other?

For example, if I know that Molly *does* play table tennis, does that change my opinion about whether she is good at maths?

- **Answer:** Probably not
- Therefore, events A and B are probably **independent**

Example 1

Suppose one of Henry's friends tells me that it is really likely that Henry will get over 60 in MAS1403. How likely is it that he will actually get under 40?

Suppose a group of Henry's friends now tells me that it is actually very unlikely that Henry will get over 60 in MAS1403. Does that now change my opinion about whether or not he will get under 40?

The two events C and D are **dependent**.

4.2 Measuring probability

There are **three** main ways in which we can measure probability:

- **Classical**
- **Frequentist**
- **Subjective/Bayesian**

All three obey the basic rules described so far.

Different people argue in favour of the different views of probability and some will argue that each kind has its uses depending on the circumstances.

Classical interpretation

This view is based on the concept of **equally likely events**.

If we toss a fair coin, we have two possible outcomes – **Heads** or **Tails**. Both outcomes are **equally likely**. Thus

$$P(\text{Head}) = \frac{1}{2} \quad \text{and} \quad P(\text{Tail}) = \frac{1}{2}$$

Classical interpretation

The underlying idea behind this view of probability is **symmetry**.

In this example, there is no reason to think that the outcome *Head* and the outcome *Tail* have different probabilities.

Since there are two outcomes and one of them must occur, both outcomes must have probability $1/2$.

Classical interpretation

Another commonly used example is **rolling dice**.

There are **six** possible outcomes – (1, 2, 3, 4, 5, 6) – if the die is fair, each of them should have an equal chance of occurring.

Hence $P(1) = \frac{1}{6}$, $P(2) = \frac{1}{6}$, ...

What about other calculations, such as $P(\text{Even Number})$?

$$P(\text{Even Number}) = \frac{3}{6} = \frac{1}{2}.$$

This follows from the formula

$$P(\text{Event}) = \frac{\text{Total number of outcomes in which event occurs}}{\text{Total number of possible outcomes}}.$$

Frequentist interpretation

When the outcomes of an experiment are not equally likely, we can **conduct experiments** to give us an idea of how likely the different outcomes are.

Examples

- Probability of producing a defective item in a manufacturing process
 - We could monitor the process over a long period of time
 - The probability of a defective could be measured by the proportion of defectives in our sample
- Imagine we believed a coin was **unfair**
 - Toss the coin a large number of times
 - See how many heads you obtain, and express $P(\text{Head})$ as a proportion

Frequentist interpretation

By conducting experiments the probability of an event can easily be estimated using the following formula:

$$P(\text{Event}) = \frac{\text{Number of times an event occurs}}{\text{Total number of times experiment done}}.$$

Frequentist interpretation

The larger the experiment, the closer this probability is to the “**true**” probability.

The frequentist view of probability regards probability as the long run relative frequency (or proportion).

In the defects example, the “true” probability of getting a defective item is the proportion obtained in a very large experiment (strictly an **infinitely** long sequence of trials).

Frequentist interpretation

In the frequentist view, probability is a **property of nature**.

In practice we cannot conduct infinite sequences of trials, and so we never know the “true” values of probabilities.

We also have to be able to imagine a long sequence of **“identical”** trials.

This does not seem to be appropriate for **“one-off”** experiments like the launch of a new product.

Subjective/Bayesian interpretation

We are probably all intuitively familiar with this method of assigning probabilities:

- When we board an aeroplane, we judge the probability of it crashing to be sufficiently small that we are happy to undertake the journey
- The odds given by bookmakers on a horse race reflect people's beliefs about which horse will win

This view of probability does not fit within the frequentist definition as the race cannot be run a large number of times.

Subjective/Bayesian interpretation

Such ways of thinking about probabilities are **subjective**.

This in itself, some think, is a problem – probabilities which two people assign to the same event can be (very) different.

This becomes important if these probabilities are to be used in **decision making**.

- Suppose you need to decide whether to launch a new product
- You might get market researchers to investigate how likely it is that the product will be a success
- This would affect your decision about whether to launch or not
- We would need a way to reconcile any differences

4.3 Examples

Chicken King is a fast-food chain with 700 outlets in the UK. The table below describes the geographic location of its restaurants:

		Region				Total
		NE	SE	SW	NW	
Population	Under 10,000	35	42	21	70	168
	10,000–100,000	70	105	84	35	294
	Over 100,000	175	28	35	0	238
Total		28	175	140	105	700

A health and safety organisation selects a restaurant at random for a hygiene inspection.

4.3 Examples

Find:

(a) $P(\text{NE restaurant chosen})$

(b)

$P(\text{Restaurant chosen from a city with a population} > 100,000)$

(c) $P(\text{SW and city with a population} < 10,000)?$

4.3 Examples

We get:

$$\begin{aligned}P(\text{NE restaurant}) &= \frac{35 + 70 + 175}{700} \\&= \frac{280}{700} \\&= 0.4 \quad \text{and}\end{aligned}$$

$$\begin{aligned}P(\text{population} > 100,000) &= \frac{175 + 28 + 35}{700} \\&= \frac{238}{700} \\&= 0.34\end{aligned}$$

4.3 Examples

Similarly,

$$\begin{aligned}P(\text{SW and population} < 10,000) &= \frac{21}{700} \\ &= 0.03\end{aligned}$$

4.3 Examples

The spinner shown below is spun once.



4.3 Examples

Find

- (a) $P(\text{lands red})$,
- (b) $P(\text{lands triangle})$, and
- (c) $P(\text{lands quadrilateral})$?

4.3 Examples

We have:

$$P(\text{lands red}) = \frac{1}{6}$$

$$P(\text{lands triangle}) = \frac{1}{6}$$

$$P(\text{lands quadrilateral}) = \frac{4}{6} = \frac{2}{3}$$

4.3 Examples

On the probability scale, how likely do you think it is that Newcastle United will be **relegated** this season? Which approach to probability would you use to estimate this?

My answer: 0.00000000000001. This is a **subjective** interpretation.

4.4 The addition rule

The **addition rule** describes the probability of any of two or more events occurring.

The addition rule for two events A and B is

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B).$$

This describes the probability of *either* event A *or* event B (or both!) happening.

Example 2

Prospective interns at internet startup *BlueFox* face two aptitude tests.

If 35 percent of applicants pass the first test, 25 percent pass the second test, and 15 percent pass both tests, what percentage of applicants pass at least one test?

Example 2

$$P(\text{first}) = 0.35$$

$$P(\text{second}) = 0.25$$

$$P(\text{first and second}) = 0.15.$$

Therefore

$$P(\text{first or second}) = 0.35 + 0.25 - 0.15 = 0.45 = 45\%.$$

4.4 The addition rule

Note: if events A and B are **mutually exclusive** then

$$P(A \text{ and } B) = 0$$

since they cannot occur together. Therefore

$$P(A \text{ or } B) = P(A) + P(B).$$



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Lecture 5

CONDITIONAL PROBABILITY

5.1 The multiplication rule

The **multiplication rule** describes the probability of two (or more) events occurring.

The probability of two events A and B both occurring is

$$P(A \text{ and } B) = P(A) \times P(B|A),$$

where $P(B|A)$ is the **conditional probability** of B *given that* A has already happened.

Example 1

A small company has 10 employees: 4 male and 6 female. You, as the manager, select two employees at random to attend a training session. What is the probability that you select two female employees?

Let

A : First is female

B : Second is female

Then

$$P(A) = \frac{6}{10}.$$

Now

$$P(B|A) = \frac{5}{9},$$

and so

$$P(A \text{ and } B) = \frac{6}{10} \times \frac{5}{9} = \frac{30}{90} = \frac{1}{3}.$$

5.1 The multiplication rule

Rearranging the expression for the **multiplication rule** gives a formula for calculating conditional probabilities:

$$P(A \text{ and } B) = P(A) \times P(B|A)$$

$$\frac{P(A \text{ and } B)}{P(A)} = P(B|A).$$

Example 2

Recall that prospective interns at internet startup *BlueFox* face two aptitude tests.

If 35 percent of applicants pass the first test, 25 percent pass the second test, and 15 percent pass both tests, what percentage of applicants pass the second test given that they passed the first test?

Example 2

Let

A : $P(\text{Pass first test})$ and

B : $P(\text{Pass second test})$.

We want $P(B|A)$, which is found as

$$P(B|A) = \frac{P(A \text{ and } B)}{P(A)}.$$

Now

$$P(A) = 0.35 \quad \text{and} \quad P(A \text{ and } B) = 0.15.$$

Thus

$$P(B|A) = \frac{0.15}{0.35} = 0.429 = 42.9\%.$$

4.1 The multiplication rule

Two events A and B are independent if $P(B|A) = P(B)$, in which case

$$P(A \text{ and } B) = P(A) \times P(B).$$

In words,

*"It doesn't matter if A has happened or not,
our beliefs about B will not change"*

Example 3

Are the outcomes of the two aptitude tests at internet startup *BlueFox* independent? Justify your answer.

No, probably **not** independent.

Why? Surely, knowing that someone passes the first test will influence our belief about them passing the second test...

More mathematically, does $P(B|A) = P(B)$?

$$P(B|A) = 0.429 \quad (\text{from Example 2}).$$

However, from the information given,

$$P(B) = 0.25 \neq 0.429,$$

So A and B are **not** independent. In fact - as we might expect - knowing that someone passes the first test increases the probability that they will pass the second test (from 0.25 to 0.429).

Example 4

Employees at a Marketing firm are classified by age and sex as follows:

	under 30	30 to 50	over 50	
Male	0.275	0.125	0.025	0.425
Female	0.325	0.175	0.075	0.575
	0.6	0.3	0.1	

So, for example, 27.5% of employees are Male and under 30 years of age.

Example 4

From this table, calculate

- (a) $P(\text{Male})$
- (b) $P(30 \text{ to } 50)$
- (c) $P(\text{Male} | 30 \text{ to } 50)$
- (d) $P(30 \text{ to } 50 | \text{Male})$
- (e) Are the events “Male” and “30 to 50” independent?

Example 4

(a) $P(\text{Male}) = 0.275 + 0.125 + 0.025 = 0.425.$

(b) $P(30 \text{ to } 50) = 0.125 + 0.175 = 0.3.$

(c)

$$\begin{aligned} P(\text{Male} | 30 \text{ to } 50) &= \frac{P(\text{Male and } 30 \text{ to } 50)}{P(30 \text{ to } 50)} \\ &= \frac{0.125}{0.3} = 0.417. \end{aligned}$$

Example 4

(d)

$$\begin{aligned}P(30 \text{ to } 50|\text{Male}) &= \frac{P(\text{Male and } 30 \text{ to } 50)}{P(\text{Male})} \\&= \frac{0.125}{0.425} = 0.294.\end{aligned}$$

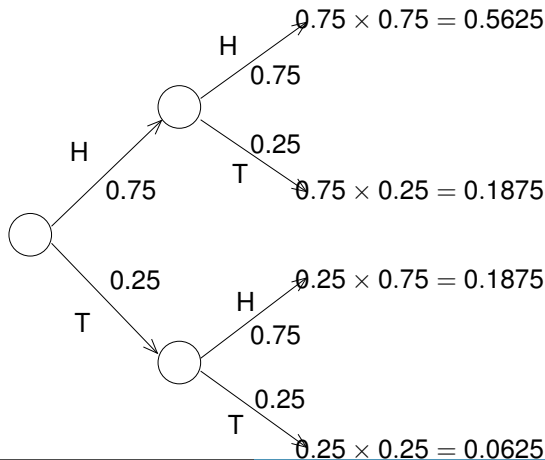
(e)

$$\begin{aligned}P(\text{Male}) &= 0.425 \quad \text{and} \\P(\text{Male}|30 \text{ to } 50) &= 0.417.\end{aligned}$$

Since $P(\text{Male}) \neq P(\text{Male}|30 \text{ to } 50)$, the two events are **not** independent.

5.2 Tree diagrams

Suppose we have a biased coin, with $P(\text{Head}) = 0.75$. Then the following tree diagram displays all outcomes, along with their associated probabilities, for two consecutive flips of the coin:



Example 6

A small company has 10 employees: 4 male and 6 female. You, as the manager, select two employees at random to attend a training session. What is the probability that you select one male and one female employee?



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Lecture 6

PROBABILISTIC DECISION-MAKING

Announcements

- **CBA2** is now live in **practice mode**
- Next week will be assessed mode
- **Deadline: 23:59 Friday 17th November**

6.1 Expected Monetary Value

Tree diagrams can be used to see the effect of making particular decisions in the face of uncertainty. This is achieved by weighting the probability of different outcomes by their value.

Often this value is **financial**.

The **Expected Monetary Value (EMV)** of a single event is the probability of that event multiplied by the monetary value of that outcome.

Example 1

For example, if you win £5 if you pulled an ace from a pack of cards, the EMV would be

$$EMV(Ace) = \frac{1}{13} \times 5 = 0.38.$$

In other words, if you repeated this bet a large number of times, overall you would come out, on average, 38 pence better off per bet.

6.1 Expected Monetary Value

Consider another bet. When rolling a die,

- if it's a six you have to *pay* £5
- if it's any other number you *receive* £2.50

Would you take on this bet?

6.1 Expected Monetary Value

Probability

Financial outcome

$$P(6) = 1/6$$

−£5

$$P(\text{Not a 6}) = 5/6$$

£2.50

Therefore

$$EMV(\text{Six}) = \frac{1}{6} \times -5.00 = -0.833$$

$$EMV(\text{Not a Six}) = \frac{5}{6} \times 2.50 = 2.0833$$

and hence the expected monetary value of the bet is

$$EMV(\text{Bet}) = -0.833 + 2.083 = 1.25.$$

6.1 Expected Monetary Value

Therefore, in the long run, this would be a bet to take on as it has a positive expected monetary value.

In general, the expected monetary value of a project or bet is given by the formula

$$EMV = \sum P(\text{Event}) \times \text{Monetary value of Event}$$

where the sum is over all possible events.

The *EMV* of a project can be used as a decision criterion for choosing between different projects and has applications in a large number of situations.

Example 2

Synaptec is a small technology company with a new product that they wish to launch on to the market.

It could go for a direct approach, launching onto the whole of the domestic market through traditional distribution channels, or it could launch only on the internet.

A third option exists where the product is licensed to a larger company through the payment of a licence fee irrespective of the success of the product.

How should the company launch the product?

Example 2

The company has done some initial market research and the managing director believes the probability of the product being successful can be classed into three categories:

High Medium Low

She thinks that these categories will occur with probabilities 0.2, 0.35 and 0.45 respectively and her thoughts on the likely profits (in £K) to be earned in each plan are

	High	Medium	Low
Direct	100	55	-25
Internet	46	25	15
Licence	20	20	20

Example 2

The EMV of each plan can be calculated as follows:

$$EMV(\text{Direct}) = 0.2 \times 100 + 0.35 \times 55 + 0.45 \times (-25) = \text{£}28\text{K}$$

$$EMV(\text{Internet}) = 0.2 \times 46 + 0.35 \times 25 + 0.45 \times 15 = \text{£}24.7\text{K}$$

$$EMV(\text{Licence}) = 0.2 \times 20 + 0.35 \times 20 + 0.45 \times 20 = \text{£}20\text{K}.$$

On the basis of expected monetary value, the best choice is the **direct approach**.

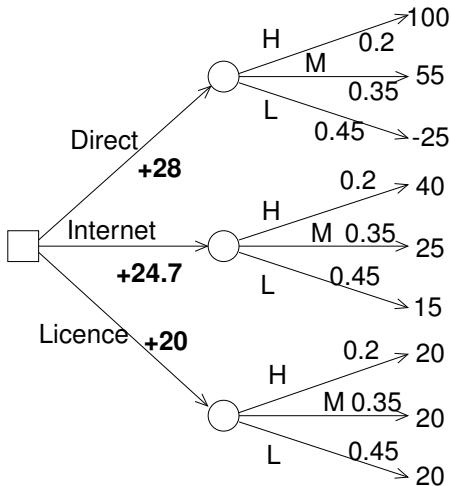
6.2 Decision trees

When we include a decision in a tree diagram (see Chapter 5) we use a rectangular node, called a **decision node** to represent the decision.

The diagram is then called a **decision tree**.

Example 3

The decision tree for the last example would look like this:



Example 3

Key points:

- There are no probabilities at a decision node but we evaluate the expected monetary values of the options.
- In a decision tree the first node (on the left) is always a decision node.
- There may also be other decision nodes.
- If there is another decision node then we evaluate the options there and choose the best one (based on EMV), and the expected monetary value of this option becomes the expected monetary value of the branch leading to the decision node.
- We work “backwards” through the tree (from right to left), evaluating EMVs and making decisions at each decision node.

Example 4

The manager of a small I.T. sales company has the opportunity to buy a fixed quantity of a new type of soundcard for home PCs which they can then offer for sale to clients.

The decision to buy the product and offer it for sale would involve a fixed cost of £200,000. The number of soundcards that would be sold is uncertain, but the manager's prior beliefs are expressed as follows.

- Sales will be “poor” with probability 0.2; this will result in an income of £100,000.
- Sales will be “moderate” with probability 0.5; this will result in an income of £220,000.
- Sales will be “good” with probability 0.3; this will result in an income of £350,000.

Example 4

For an additional fixed cost of £30,000, market research can be conducted to aid the decision-making process.

The outcome of the market research can be either positive or negative, with probabilities 0.58 and 0.42 respectively. Knowing the outcome of the market research changes the probabilities for the main sales project as follows:

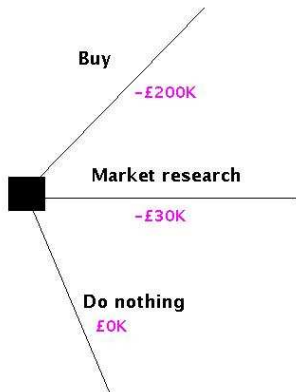
Market research	Main sales probabilities		
	Poor	Moderate	Good
Positive	0.15	0.45	0.4
Negative	0.6	0.35	0.05

Example 4

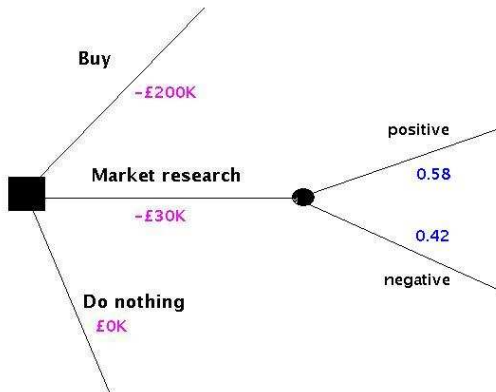
The manager will make decisions based on expected monetary value.

- (a) Draw a decision tree for this problem.
- (b) Use expected monetary value to determine the optimal course of action for the company.

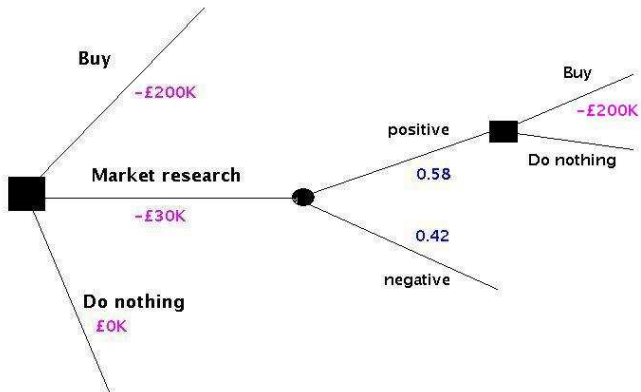
Example 4



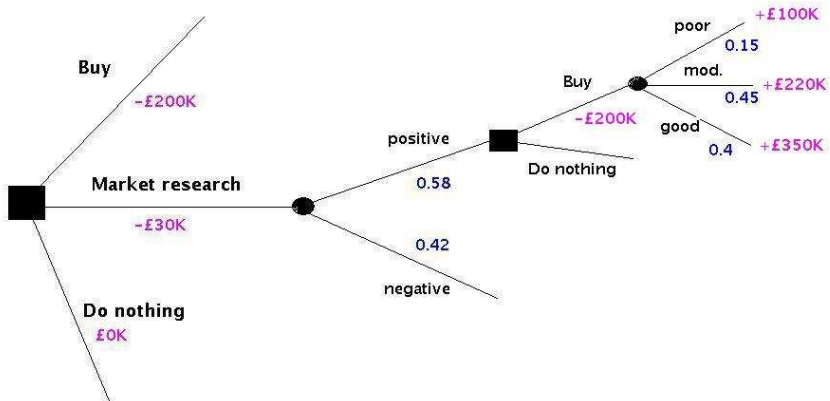
Example 4



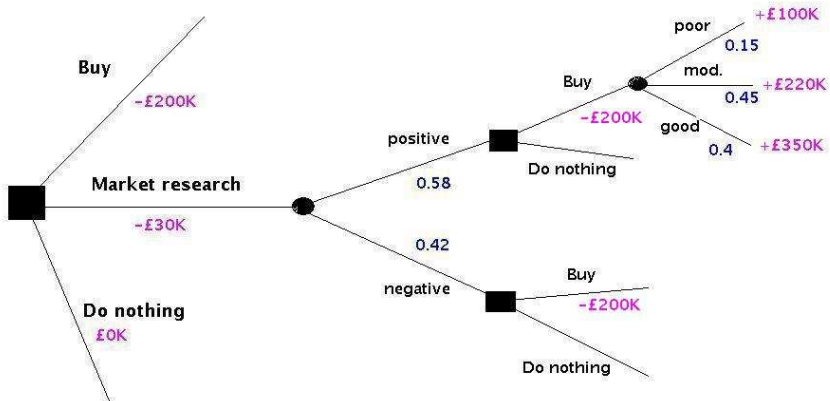
Example 4



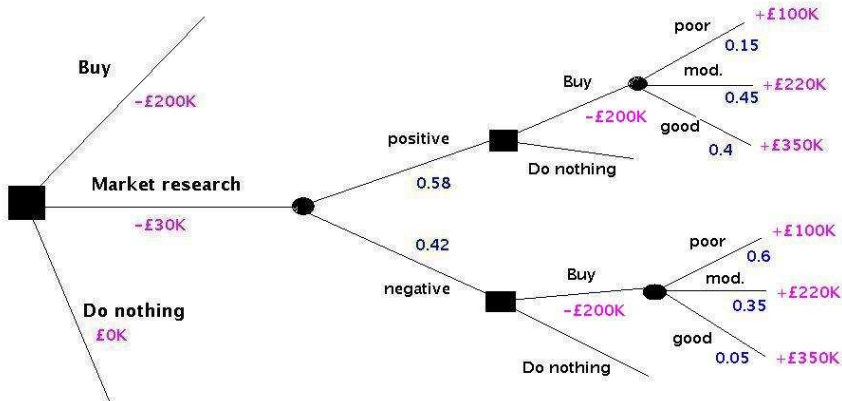
Example 4



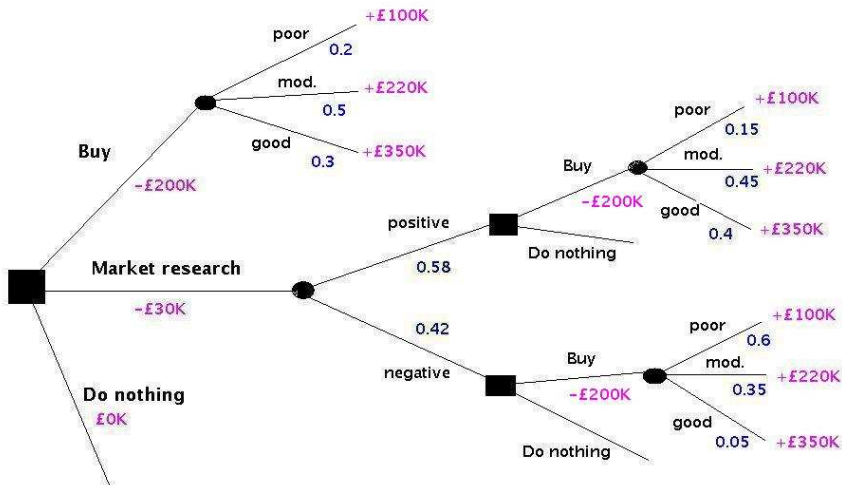
Example 4



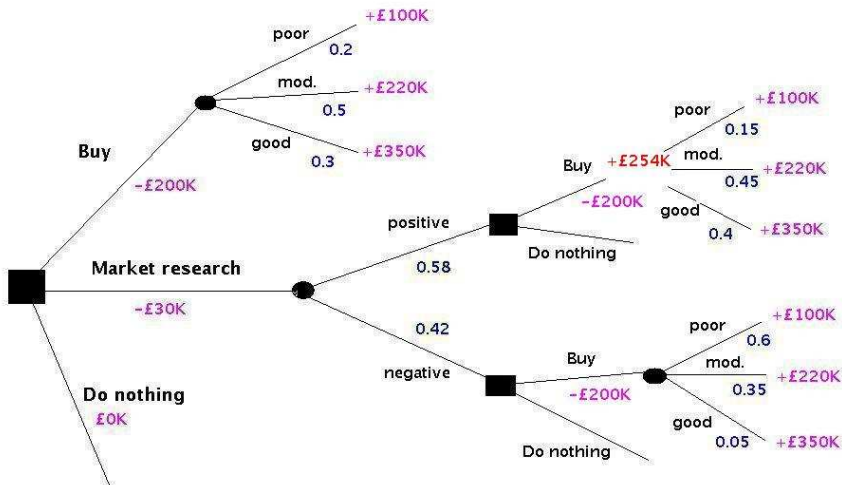
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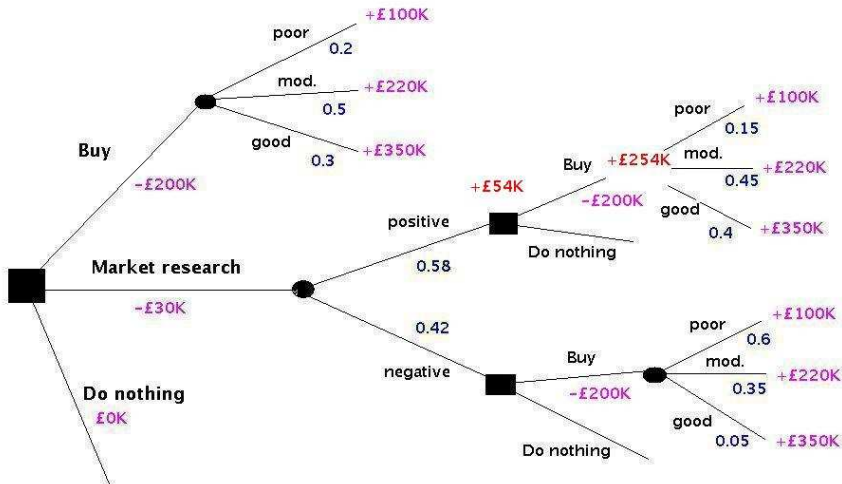
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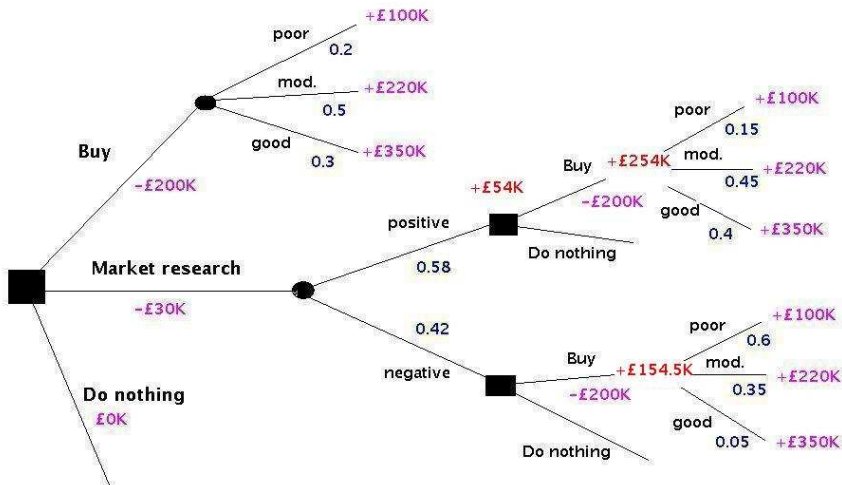
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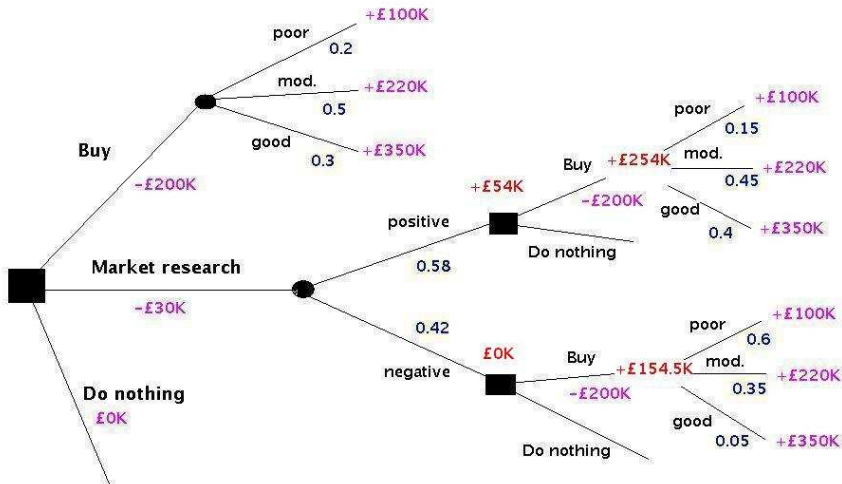
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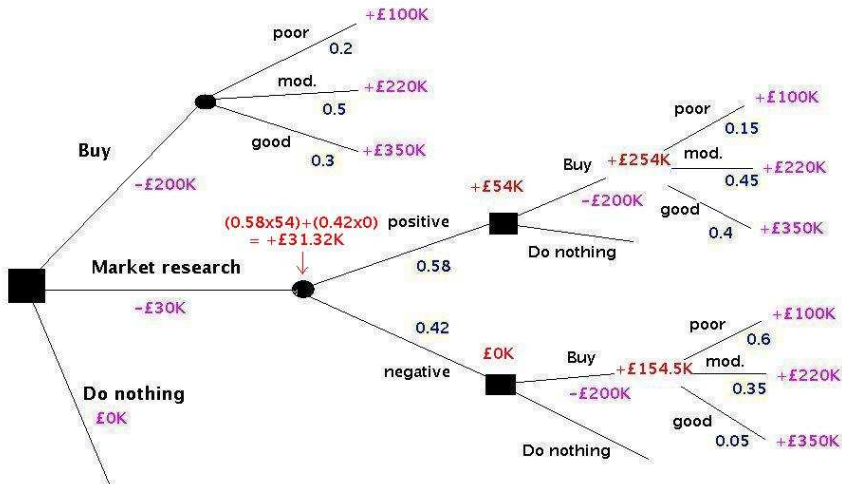
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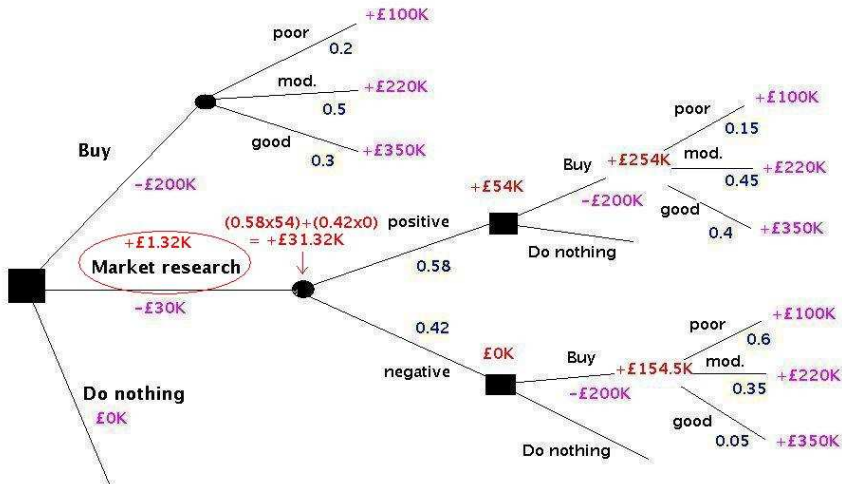
Example 4



Example 4



Example 4



Example 4

