



# **MAS1403**

## **Quantitative Methods for Business Management**

Semester 1, 2017–2018

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# Announcements

- Back to standard classroom-based tutorials this week, with a twist...
- The class on Thursday at 11, with Ged, will be in **KGVI LT4** this week, not the usual LT1! (My classes at 1 and 2 remain unchanged)
- The **semester 1 written assignment** is now available to download from the course webpage – **deadline: 4pm, Thursday 14th December**

# **Lecture 8**

## **MORE DISCRETE PROBABILITY MODELS**

## 8.1 The Poisson distribution

The Poisson distribution is another very important discrete probability distribution.

- 1 It is often used to model **count data**
- 2 Unlike the binomial distribution, there is **no known fixed upper limit** to the number of events
- 3 The **rate** of occurrence,  $\lambda$ , is the parameter here – we assume events occur independently, with constant rate  $\lambda$

If these conditions are reasonable, then we say the number of events,  $X$ , occurring in a given interval, has a **Poisson distribution** with parameter  $\lambda$ .

## Example 1

Which of the following random variables could be modelled by a Poisson distribution? Suggest an alternative if the Poisson distribution is not appropriate, and state the values of any parameters.

- (a) Calls are received at a call centre at a constant rate of 3 per minute on average. Let  $X$  be the number of calls received in a 1 minute period.
- (b) An operator at a tele-sales marketing firm has 20 calls to make in an hour. History suggests that calls will be answered 55% of the time. Let  $Y$  be the number of answered calls in an hour.
- (c) Newcastle United score goals at a constant rate of 2.4 in 90 minutes, on average. Let  $Z$  be the number of goals scored in 45 minutes.

# Example 1

$X$ : Number of calls received in a 1 minute period

- Could be Poisson: we have count data, we have a fixed rate of occurrence (3 per minute) and we could assume independent events
- We have  $\lambda = 3$

$Y$ : Number of answered calls in an hour

- Cannot be Poisson: We have no rate of occurrence ( $\lambda$ ), and there is an upper limit to  $Y$  (20)
- The **binomial distribution** could be used: we have a fixed number of independent trials, each with two outcomes (“success” and “failure”), and we have a probability of success
- Specifically, we have  $Y \sim \text{Bin}(n = 20, p = 0.55)$ .

# Example 1

$Z$ : Number of goals scored in 45 minutes

- Could be Poisson: we have count data and we have a fixed rate of occurrence (2.4 per 90 minutes)
- We have  $\lambda = 1.2$

## 8.1.1 Probabilities, means and variances

If  $X$  follows a Poisson distribution we write  $X \sim \text{Po}(\lambda)$ , and

$$P(X = r) = \frac{\lambda^r e^{-\lambda}}{r!}, \quad r = 0, 1, \dots$$

If  $X \sim \text{Po}(\lambda)$ , then

$$\begin{aligned} E[X] &= \lambda \quad \text{and} \\ \text{Var}(X) &= \lambda. \end{aligned}$$

## Example 2

If  $X \sim \text{Po}(5)$  calculate:

- (a)  $P(X = 4)$
- (b)  $P(X \leq 1)$
- (c)  $P(X > 0)$
- (d)  $E[X]$
- (e)  $\text{SD}(X)$

## Example 2

We know that

$$P(X = r) = \frac{\lambda^r e^{-\lambda}}{r!},$$

and we have  $\lambda = 5$ .

(a)

$$P(X = 4) = \frac{5^4 e^{-5}}{4!} = 0.175.$$

(b)

$$\begin{aligned} P(X \leq 1) &= P(X = 0) + P(X = 1) \\ &= \frac{5^0 e^{-5}}{0!} + \frac{5^1 e^{-5}}{1!} \\ &= 0.0067 + 0.0337 = 0.0404. \end{aligned}$$

## Example 2

(c)

$$\begin{aligned}P(X > 0) &= 1 - P(X = 0) \\&= 1 - 0.0067 = 0.9933.\end{aligned}$$

(d)

$$E[X] = \lambda = 5.$$

(e)

$$\text{Var}(X) = \lambda = 5.$$

Therefore

$$\text{SD}(X) = \sqrt{5} = 2.236.$$

## Example 3

A new Mercedes-Benz car franchise forecasts that it will sell around three of its most expensive models each day.

- (a) What probability distribution might be reasonable to use to model the number of cars sold each day?
- Cannot be binomial - we don't have the probability of success or the number of trials (i.e. there is no known fixed upper limit to the number of cars sold each day)
  - Could be Poisson - we have a rate of occurrence

So  $X$ : Number of expensive models sold each day

$$X \sim \text{Po}(3)$$

## Example 3

- (b) What is the expected number and standard deviation of the number of cars sold each day?

$$E[X] = \lambda = 3 \text{ cars per day.}$$

Also

$$\text{Var}(X) = \lambda = 3,$$

and so

$$SD(X) = \sqrt{3} = 1.732 \text{ cars per day}$$

## Example 3

- (c) What is the probability that 3 cars are sold on a particular day?

$$P(X = 3) = \frac{3^3 e^{-3}}{3!} = 0.224.$$

- (d) What is the probability that no cars are sold on a particular day?

$$P(X = 0) = \frac{3^0 e^{-3}}{0!} = 0.0498.$$

- (e) What is the probability that at least one car is sold on a particular day?

$$\begin{aligned} P(X \geq 1) &= 1 - P(X = 0) \\ &= 1 - 0.0498 = 0.9502. \end{aligned}$$

## Example 3

- (f) Sales will be monitored over the next seven days and the sales team at the franchise will receive a warning if they make no sales on at least 1 of the 7 days. What is the probability that they receive a warning?

Let  $Y$ : Number of days on which zero sales are made

$$Y \sim \text{Bin}(7, 0.0498).$$

Then

$$\begin{aligned} P(Y \geq 1) &= 1 - P(Y = 0) \\ &= 1 - {}^7C_0 \times 0.0498^0 \times 0.9502^7 \\ &= 1 - 0.699 = 0.301. \end{aligned}$$

## Extra example

Recall the example at the start of the lecture last week, used to help motivate the study of probability models.

Let  $X$ : Number of cars observed every half an hour over a five hour period. We have

2   3   2   5   5   3   4   5   6   7

This gives

| $x$   | $P(X = x)$           | Poisson |
|-------|----------------------|---------|
| $< 2$ | $\frac{0}{10} = 0$   | 0.078   |
| 2     | $\frac{2}{10} = 0.2$ | 0.132   |
| 3     | $\frac{2}{10} = 0.2$ | 0.185   |
| 4     | $\frac{1}{10} = 0.1$ | 0.194   |
|       |                      |         |