MAS1403

Quantitative Methods for Business Management

Semester 1, 2017–2018

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Lecture 5

CONDITIONAL PROBABILITY

5.1 The multiplication rule

The **multiplication rule** describes the probability of two (or more) events occurring.

The probability of two events A and B both occurring is

$$P(A \text{ and } B) = P(A) \times P(B|A),$$

where $P(B|A)$ is the **conditional probability** of B *given that* A has already happened.

Example 1

A small company has 10 employees: 4 male and 6 female. You, as the manager, select two employees at random to attend a training session. What is the probability that you select two female employees?

Let

$$\begin{aligned} A &: \text{First is female} \\ B &: \text{Second is female} \end{aligned}$$

Then

$$P(A) = \frac{6}{10}.$$

Now

$$P(B|A) = \frac{5}{9},$$

and so

$$P(A \text{ and } B) = \frac{6}{10} \times \frac{5}{9} = \frac{30}{90} = \frac{1}{3}.$$

5.1 The multiplication rule

Rearranging the expression for the **multiplication rule** gives a formula for calculating conditional probabilities:

$$\begin{aligned} P(A \text{ and } B) &= P(A) \times P(B|A) \\ \frac{P(A \text{ and } B)}{P(A)} &= P(B|A). \end{aligned}$$

Example 2

Recall that prospective interns at internet startup *BlueFox* face two aptitude tests.

If 35 percent of applicants pass the first test, 25 percent pass the second test, and 15 percent pass both tests, what percentage of applicants pass the second test given that they passed the first test?

Example 2

Let

$$\begin{aligned} A &: P(\text{Pass first test}) \quad \text{and} \\ B &: P(\text{Pass second test}). \end{aligned}$$

We want $P(B|A)$, which is found as

$$P(B|A) = \frac{P(A \text{ and } B)}{P(A)}.$$

Now

$$P(A) = 0.35 \quad \text{and} \quad P(A \text{ and } B) = 0.15.$$

Thus

$$P(B|A) = \frac{0.15}{0.35} = 0.429 \quad = 42.9\%.$$

4.1 The multiplication rule

Two events A and B are independent if $P(B|A) = P(B)$, in which case

$$P(A \text{ and } B) = P(A) \times P(B).$$

In words,

"It doesn't matter if A has happened or not, our beliefs about B will not change"

Example 3

Are the outcomes of the two aptitude tests at internet startup *BlueFox* independent? Justify your answer.

No, probably **not** independent.

Why? Surely, knowing that someone passes the first test will influence our belief about them passing the second test...

More mathematically, does $P(B|A) = P(B)$?

$$P(B|A) = 0.429 \quad \text{(from Example 2).}$$

However, from the information given,

$$P(B) = 0.25 \neq 0.429,$$

So A and B are **not** independent. In fact - as we might expect - knowing that someone passes the first test increases the probability that they will pass the second test (from 0.25 to 0.429).

Example 4

Employees at a Marketing firm are classified by age and sex as follows:

	under 30	30 to 50	over 50	
Male	0.275	0.125	0.025	**0.425**
Female	0.325	0.175	0.075	**0.575**
	0.6	**0.3**	**0.1**	

So, for example, 27.5% of employees are Male and under 30 years of age.

Example 4

From this table, calculate

(a) $P(\text{Male})$
(b) $P(30 \text{ to } 50)$
(c) $P(\text{Male}|30 \text{ to } 50)$
(d) $P(30 \text{ to } 50|\text{Male})$
(e) Are the events “Male” and “30 to 50” independent?

Example 4

(a) $P(\text{Male}) = 0.275 + 0.125 + 0.025 = 0.425.$

(b) $P(30 \text{ to } 50) = 0.125 + 0.175 = 0.3.$

(c)

$$\begin{aligned} P(\text{Male}|30 \text{ to } 50) &= \frac{P(\text{Male and } 30 \text{ to } 50)}{P(30 \text{ to } 50)} \\ &= \frac{0.125}{0.3} = 0.417. \end{aligned}$$

Example 4

(d)

$$\begin{aligned} P(30 \text{ to } 50|\text{Male}) &= \frac{P(\text{Male and } 30 \text{ to } 50)}{P(\text{Male})} \\ &= \frac{0.125}{0.425} = 0.294. \end{aligned}$$

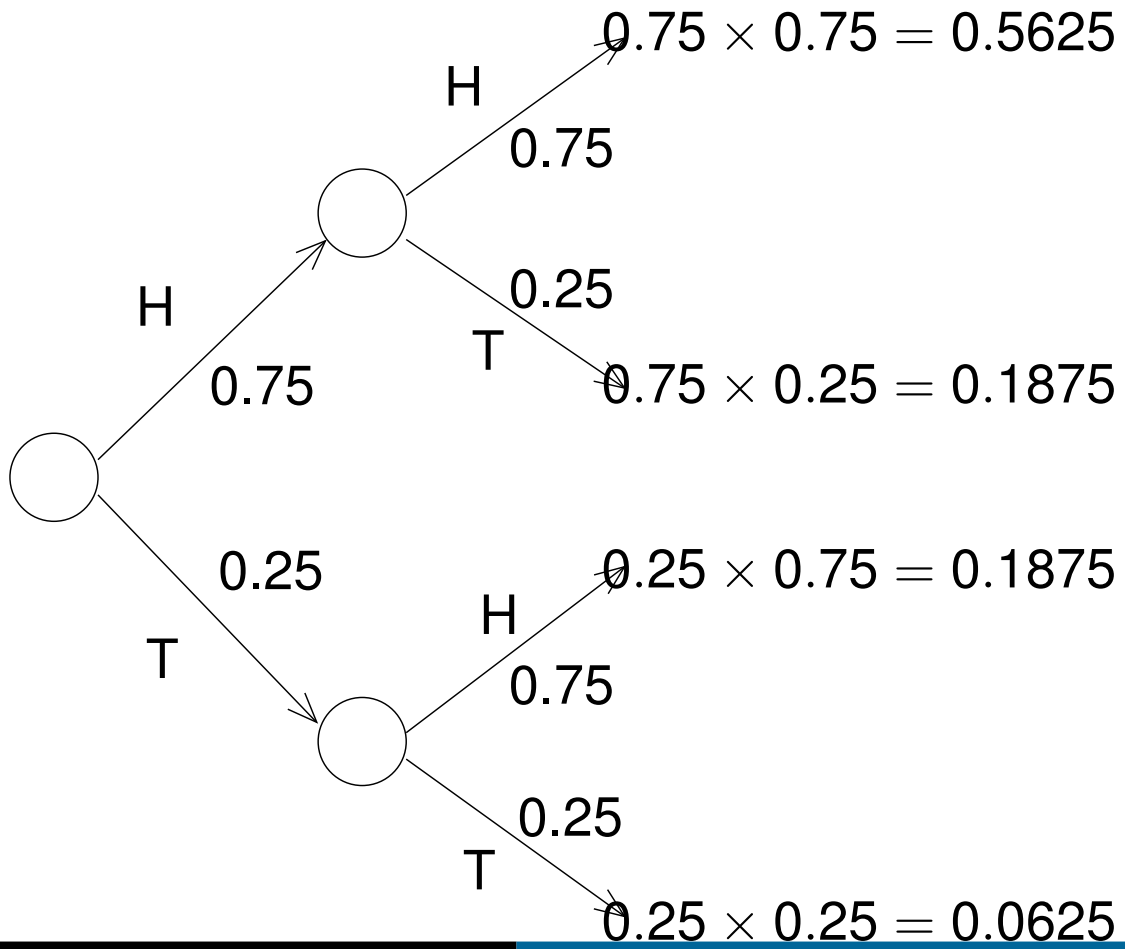
(e)

$$\begin{aligned} P(\text{Male}) &= 0.425 \quad \text{and} \\ P(\text{Male}|30 \text{ to } 50) &= 0.417. \end{aligned}$$

Since $P(\text{Male}) \neq P(\text{Male}|30 \text{ to } 50)$, the two events are **not** independent.

5.2 Tree diagrams

Suppose we have a biased coin, with $P(\text{Head}) = 0.75$. Then the following tree diagram displays all outcomes, along with their associated probabilities, for two consecutive flips of the coin:



Example 6

A small company has 10 employees: 4 male and 6 female. You, as the manager, select two employees at random to attend a training session. What is the probability that you select one male and one female employee?