

Practical 5

Question 1 (3 marks)

$$\begin{aligned}\sum_{i=1}^n (x_i - \bar{x})^2 &= \sum_{i=1}^n (x_i^2 - 2x_i\bar{x} + \bar{x}^2) \\ &= \sum_{i=1}^n x_i^2 - 2\bar{x} \sum_{i=1}^n x_i + \sum_{i=1}^n \bar{x}^2 \\ &= \sum_{i=1}^n x_i^2 - 2\bar{x}n\bar{x} + n\bar{x}^2 \\ &= \sum_{i=1}^n x_i^2 - n\bar{x}^2\end{aligned}$$

Question 2 (3, 1, 1) marks

1. We have the congruential generator

$$r_i = (17r_{i-1} + 5) \mod 16,$$

with $r_0 = 8$. So

$$r_1 = (17 \times 8 + 5) \mod 16 = 13$$

$$r_2 = (17 \times 13 + 5) \mod 16 = 2$$

The total sequence is: [13,2,7,12,1]

2. Full period of 16 is achieved.
3. Yes, this example!

Question 3 (2, 2, 3, 3) marks

In each question we will use the following $U(0, 1)$ random numbers:

0.4129, 0.7642, 0.9251

Question 3a

If $U < 0.4$ we simulate a 0, else we simulate a 1. Hence, we obtain 1,1,1.

Equally valid is: if $U < 0.6$ we simulate a 1, else we simulate a 0. With this scheme we obtain: 1, 0, 0.

Question 3b

Use the formula from the notes:

$$X = 1 + \left\lfloor \frac{\ln(1 - U)}{\ln(1 - p)} \right\rfloor$$

Substitute in $p = 0.5$ and $u = 0.4129, 0.7642, 0.9251$, to get:

$$\begin{aligned} X &= 1 + \left\lfloor \frac{\ln(1 - 0.4129)}{\ln(0.5)} \right\rfloor \\ &= 1 + \lfloor 0.7683 \rfloor \\ &= 1 \end{aligned}$$

So values are: 1, 3,4

Question 3c

We first need to work out the CDF of the $Pois(5)$ distribution:

x	0	1	2	3	4	5	6	7	8	9	10
$\Pr[X \geq x]$	0.01	0.04	0.12	0.27	0.44	0.62	0.76	0.87	0.93	0.97	0.99

Hence, our Poisson random numbers are:

4, 7, 8

Question 3d

We first need to work out the CDF of the $Bin(10, 0.5)$ distribution:

x	0	1	2	3	4	5	6	7	8	9	10
$\Pr[X \geq x]$	0	0.01	0.05	0.17	0.38	0.62	0.83	0.95	0.99	1	1

Hence, our Binomial random numbers are:

5, 6, 7