

Practical 5: Statistical Process Control

What you need to do

Before starting the Minitab practical, it would be a good idea (if you've not already done so) to bookmark the webpage containing all of the information for Semester 2 of ACE2013:

www.mas.ncl.ac.uk/~nlf8/teaching/ace2013

To add this page to your favourites, select **Favourites** → **Add to Favourites** from the menu bar at the top. You will now be able to get to this page easily by selecting **Favourites** and selecting the page from the list.

In this practical you will be using **Minitab** to produce control charts similar to those you've seen in recent lectures. **It will really help if you have the lecture notes for Chapter 3 in front of you.**

Along with assessed work from the other practical sessions, solutions to the starred questions in this practical should be typed up and submitted (with a cover sheet – see the course website) to the homework submission postbox outside the Maths & Statistics General Office, on the third floor of the Herschel Building, by no later than **4pm, Monday 12th May 2014.**

Log into a PC and open **Minitab** by clicking on **Start** → **All programs** → **Statistical Software** → **Minitab 16 Statistical Software English.**

Go to the ACE2013 website and click on the **DuLux** link in the material for Practical 5. Once opened, you will see that there are data in the first 21 columns of this worksheet.

Recall from recent lectures that *Dulux* produce 5 litre tins of paint. The management realise that, owing to *chance variation*, not every tin will have exactly 5 litres of paint in it; when the tin-filling process is *under control*, we were told that $\mu = 5$ litres and $\sigma = 0.15$ litres.

For a period of 24 hours, the company samples 20 tins of paint every hour and records the amount of paint in each tin. The first column in your **Minitab** worksheet shows the time at which the sample was taken; the next twenty columns show the volume of paint in each of the twenty tins in that sample.

1. Find the sample means $\bar{x}_j$, $j = 1, \dots, 24$, for each sample taken by clicking on **Calc** → **Row Statistics**; select **Mean** as the **Statistic** and enter **C2-C21** as the **Input variables** (recall that column 1 just contains the times, not the actual sample data), and **Store result in column C23** before clicking **OK**. This will store the sample means in column C23. Make sure you have done this correctly by comparing these values to the values in Table 3.1 (page 87) of the lecture notes.
2. You could now produce an $\bar{x}$ -chart for the tin-filling process. Recall that this is simply a time series plot for the sample means, so you could now click on **Graph** → **Time Series Plot** →

Simple→OK, and in the **Series** box enter the column which has the sample means in it (C23); then click OK. This should produce a graph similar to that shown in figure 3.1 of your lecture notes. Note that this graph will not include the control limits calculated in lectures – these would have to be superimposed by hand.

- *3. A more convenient way of producing the $\bar{x}$ -chart, which would automatically include control limits, is to click on Stat→Control Charts→Variable Charts for Subgroups→Xbar. Using the first drop-down menu, select Observations for a subgroup are in one row of columns, and in the box underneath enter C2-C21, i.e. the columns in the worksheet that contain the data. Click Xbar options and enter the Mean (5) and the Standard deviation (0.15). Click OK and OK again. This should produce the plot you have in figure 3.1 of the lecture notes, with the control limits included. Check that the *LCL* and *UCL* match those calculated on page 88 of the lecture notes, and copy-and-paste this plot into your solutions. Comment.
- *4. The default settings for the control limits on an $\bar{x}$ -chart in Minitab are ± 3 standard errors from the mean: the so-called “3-sigma control limits”. Using the Xbar Options menu in question 3, you should see if you can work out how to produce an $\bar{x}$ -chart with “2-sigma control limits”. Copy-and-paste this plot into your solutions. How does this differ from the chart produced in question 3?
- 5. Recall from lectures that, due to the central limit theorem, we have

$$\bar{x} \sim N\left(\mu, \frac{\sigma^2}{n}\right) \quad \text{i.e.}$$

$$\bar{x} \sim N(5, 0.001125).$$

We used this result to find that

$$\Pr(\text{false alarm}) = 0.0026$$

when we use 3-sigma control limits; using 2-sigma control limits gives

$$\Pr(\text{false alarm}) = 0.046$$

(see pages 89 and 90 of the lecture notes). We can use Minitab to find these probabilities. Recall that we can use the *standard* Normal distribution $Z \sim N(0, 1)$ here directly.

- (i) Click on Calc→Probability distributions→Normal
- (ii) Select Cumulative probability
- (iii) Enter the Mean and Standard deviation of the *standard* Normal distribution
- (iv) Enter the Input constant as -3
- (v) Press OK

In the Minitab session window you should get:

Cumulative Distribution Function

Normal with mean = 0 and standard deviation = 1

```
x P( X<=x )  
-3      0.0013499
```

This is the probability that, if the process is under control, a sample mean falls below the lower control limit. The probability that a sample mean will fall *above* the *upper* control limit will be exactly the same, owing to the symmetry of the Normal distribution, and so the Probability of a *false alarm* is just

$$2 \times 0.0013499 = 0.0026998;$$

i.e. roughly the answer we got by hand in lectures (**Minitab** is more accurate).

***6.** Use the menu options given in question 5 to verify that the probability of a *false alarm* is

- (i) 0.0455002 when we use 2-sigma control limits;
- (ii) 0.0124194 when we use 2.5-sigma control limits.

Copy-and-paste any relevant **Minitab** output into your solutions.

7. As you will have seen in lectures this week, it is often the case that we do not know the true values of μ and σ (in this example we were told that $\mu = 5$ and $\sigma = 0.15$). In such circumstances, we need to *estimate* these parameters from the data. Suppose we did not know that $\mu = 5$ and $\sigma = 0.15$ in the *Dulux* example. Then we estimate these with

$$\bar{\bar{x}} = \frac{\sum_{j=1}^k \bar{x}_j}{k} \quad \text{and}$$

$$S = \sqrt{\frac{\sum_{j=1}^k s_j^2}{k}}$$

for μ and σ respectively. The first is known as “x bar bar”, or “x double bar”, and is just the mean of the sample means. The second is known as the “pooled standard deviation”.

Click the cursor anywhere in the top window (the white window above the spreadsheet) to activate the session window, and then select **Editor**→**Enable commands**. This will initiate the **Minitab** command prompt which looks like **MTB >**.

- (i) Find $\bar{\bar{x}}$ by typing **Mean C23** in front of the command prompt and hitting enter.
- (ii) Store the sample standard deviations in column C24 by repeating question 1 but selecting **Standard deviation** as the **Statistic** instead of **Mean** (and using C24 instead of C23). Now at the **Minitab** command prompt type

```
MTB > LET C25=C24*C24
```

This will store the sample *variances* in column 25. Now type

```
MTB > SUM C25
```

Now use your calculator to find S using the formula above.

***8.** In question 7 you should have found that

$$\begin{aligned}\bar{\bar{x}} &= 4.96125 & \text{and} \\ S &= 0.1795.\end{aligned}$$

You should now use Minitab to re-produce the $\bar{x}$ -chart using these estimates. Click on Stat → Control charts → Variables charts for subgroups → Xbar.

Click on Xbar options and now delete the Mean and Standard deviation values from before (we are no longer assuming that these are known). Now select the Estimate tab and select Pooled standard deviation as the Method for estimating standard deviation (also *un*-tick the Use unbiasing constant box). Go into the S Limits tab and make sure you are back to using 3-sigma control limits before selecting OK and OK again. Copy-and-paste the resulting plot into your solutions.

***9.** You should see from the plot in question 8 that Minitab has now used $\bar{\bar{x}}$ and S to construct the $\bar{x}$ -chart. How does this compare to the chart produced when we assumed μ and σ were *known*?