

Announcements

- Chapter 1 learning outcomes – will give out today – prize question!
- CBA1: Well done!

	n	n^*	Mean	Median	Max
2013	210	2	94	100	100
2014	210	4	92	100	100
2015	243	11	94	100	100

- Case studies: come to a close next week...
 - We will be asking you all an individual question in next week's workshop which counts towards your final mark
 - Individual written report (4 pages max) due in by **4pm next Friday**:
 - Must include a completed cover sheet
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Alcohol case study

"I heard that there were mistakes in the alcohol case study and that you were going to scrap it and give us all a new one"

NO!

- Some confusion over "elasticity"
- Different interpretations of this (Mathematics/Economics)
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Solving linear programming problems

In the previous section we discussed how to **formulate** a linear programming problem.

We now consider how to **solve** such problems.

Linear programming problems that involve only two decision variables, x and y , may be solved **graphically** – that is, by drawing lines associated with the constraints, identifying a region of possible solutions and then locating a point in this region that has a particular property.

We will now re-visit the two examples from the previous section.

Example 1.3 revisited (the chair manufacturer)

Recall that the problem posed in **example 1.3** could be summarised in the following way:

Maximise $Z = 10x + 12y$ subject to the following constraints:

$$3x + 3y \leq 120,$$

$$2x + 4y \leq 150,$$

$$x \geq 0 \quad \text{and}$$

$$y \geq 0.$$

Example 1.3 revisited (the chair manufacturer)

To find the **feasible region** for this problem (i.e. the region on a graph which satisfies all of our inequalities), we proceed by indicating, on a diagram, the region for which all of the inequalities hold.

Example 1.3 revisited (the chair manufacturer)

The first inequality is $3x + 3y \leq 120$. To show this on a graph, we first need to plot the line $3x + 3y = 120$.

■ When $x = 0$, we have

$$3 \times 0 + 3y = 120 \quad \text{i.e.}$$

$$3y = 120 \quad \text{i.e.}$$

$$y = 40.$$

■ When $y = 0$, we have

$$3x + 3 \times 0 = 120 \quad \text{i.e.}$$

$$3x = 120 \quad \text{i.e.}$$

$$x = 40.$$

The points $(0, 40)$ and $(40, 0)$ have been plotted on the graph on page 16, and shading used to illustrate the inequality.

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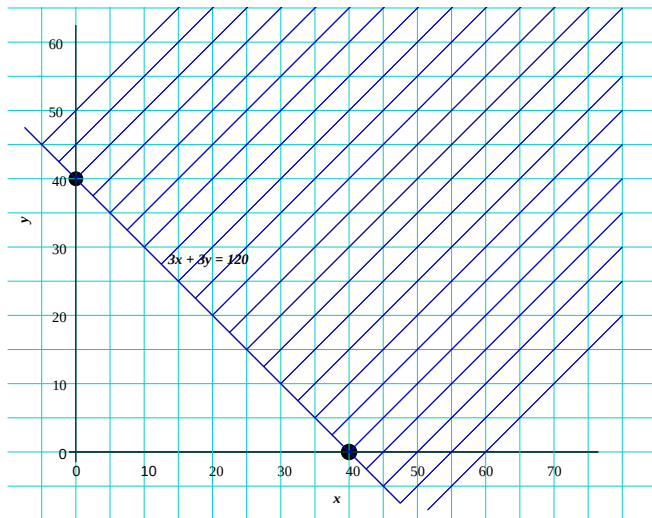
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Now consider the second inequality $2x + 4y \leq 150$. Again, to show this on a diagram, we first need to plot the line $2x + 4y = 150$.

■ When $x = 0$, we have

$$2 \times 0 + 4y = 150 \quad \text{i.e.}$$

$$4y = 150 \quad \text{i.e.}$$

$$y = 37.5.$$

■ When $y = 0$, we have

$$2x + 4 \times 0 = 150 \quad \text{i.e.}$$

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$$x = 75.$$

On the same graph, you should plot the points $(0, 37.5)$ and $(75, 0)$, and shade accordingly.

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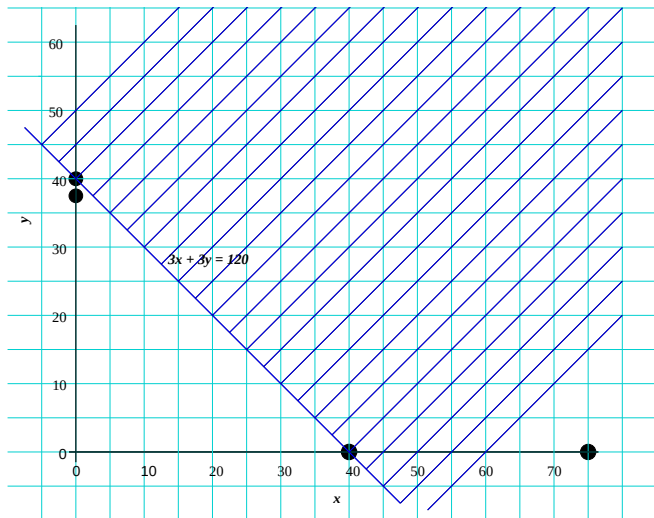
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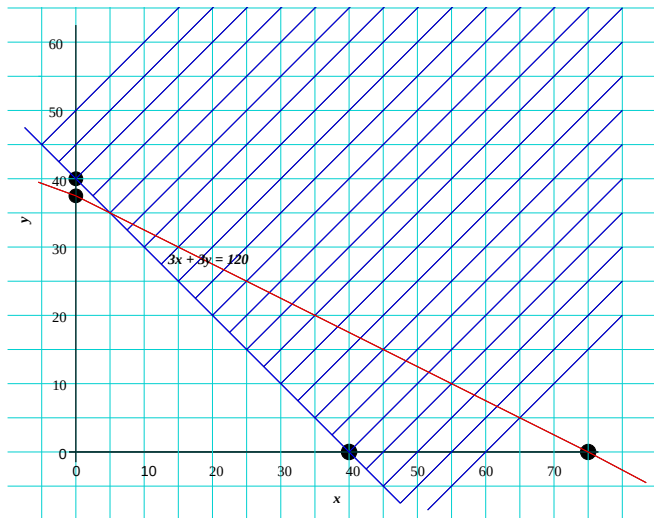
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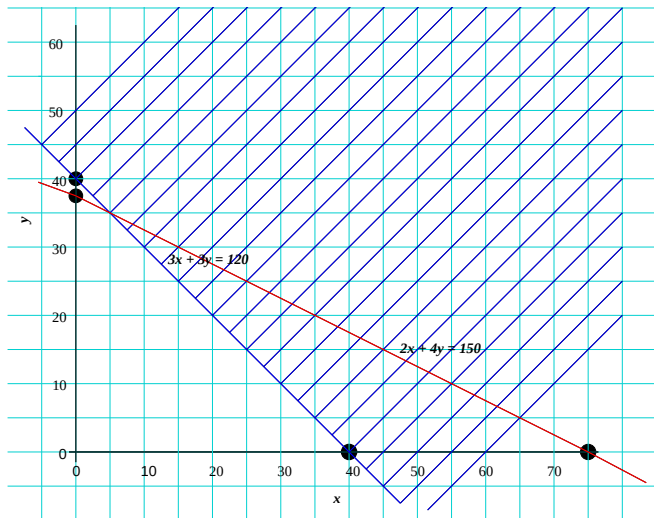
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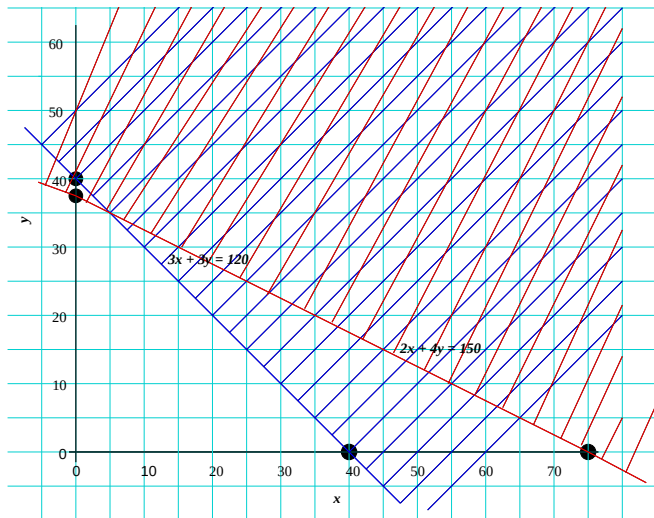
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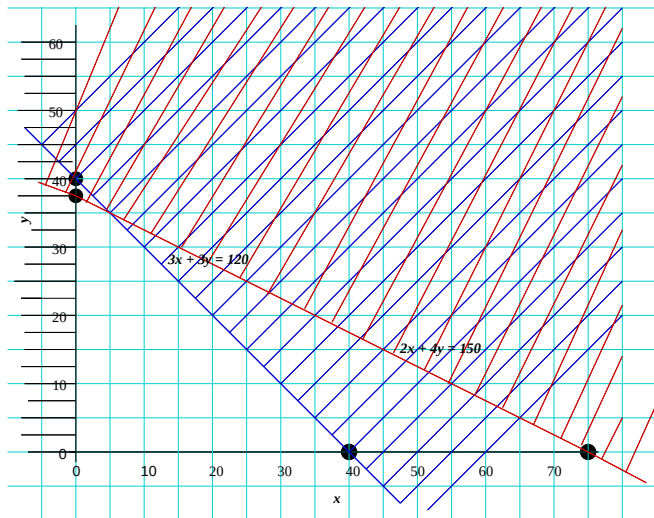
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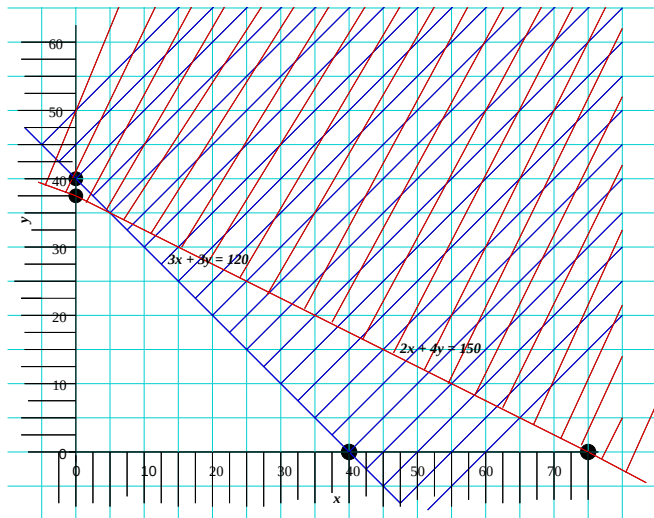
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We also need to respect the non-negativity constraints, and so we shade out the negative portion of the graph.

Example 1.3 revisited (the chair manufacturer)



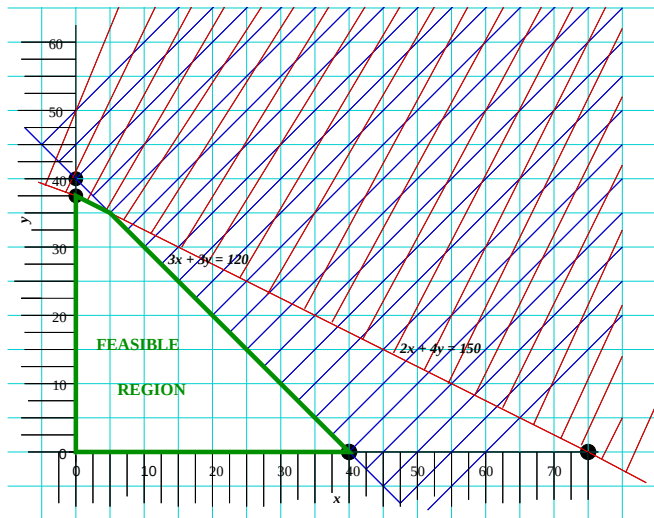
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The unshaded region in your graph shows the feasible region associated with our set of inequalities.

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What we must do now is find the point in that region which meets our objective – i.e. the point in that region which **maximises income**.

One way of doing this is to also plot the **objective function**.

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One way of doing this is to also plot the **objective function**.

Example 1.3 revisited (the chair manufacturer)

Now our objective function is

$$Z = 10x + 12y,$$

where Z is our income.

When Z takes different values we get a family of **parallel straight lines**.

We need to choose a starting value for Z in order to be able to plot the objective function.

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It's often a good idea to try a value which is a multiple of both the coefficients of x and y .

The coefficient of x is 10 and the coefficient of y is 12, so we could try a starting value of $Z = 120$. Thus, the objective function is now

$$10x + 12y = 120.$$

We can plot this line in the same way as before – i.e. consider what happens when x and y are zero.

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- When $x = 0$, we have

$$10 \times 0 + 12y = 120 \quad \text{i.e.}$$

$$12y = 120 \quad \text{i.e.}$$

$$y = 10.$$

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This line should also plotted on your graph.

- Notice that this line does not give the **optimal income**
- The origin represents zero income, and we want to move as far away from this as possible.

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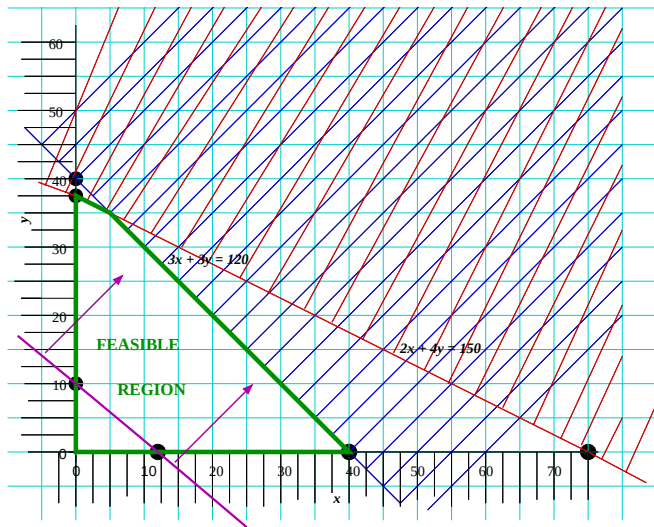
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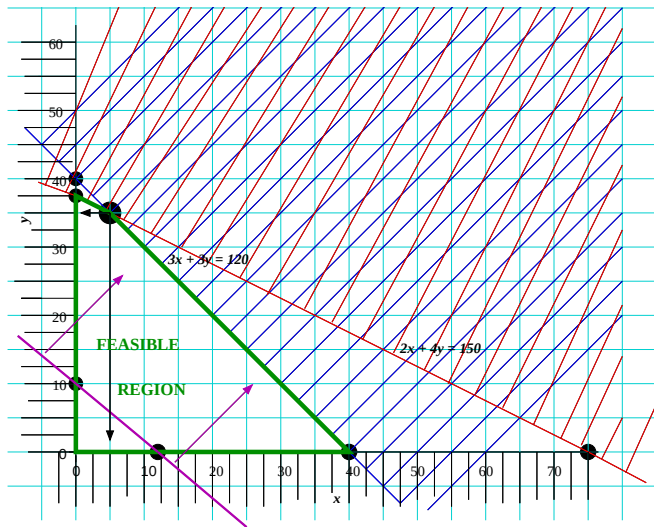
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You should see from your graph that the point within the feasible region that is furthest away from the origin, **but parallel to the objective function**, is $x = 5$ and $y = 35$.

Thus, to maximise income, we should make **5 type A chairs** and **35 type B chairs**.

This will give an income of

$$Z = 10x + 12y \quad \text{i.e.}$$

$$Z = 10 \times 5 + 12 \times 35 \quad \text{i.e.}$$

$$\begin{aligned} Z &= 50 + 420 \\ &= 470, \end{aligned}$$

i.e. £470.

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Example 1.4 revisited (*Sportizus*)

Recall the example about *Sportizus*, a company who produce replica football and rugby shirts. We summarised this problem as follows.

Maximise $P = 22x + 16y$ subject to the following constraints:

$$\begin{aligned}8x + 5y &\leq 600, \\9x + 15y &\leq 900, \\x &\geq 30 \quad \text{and} \\y &\geq 10,\end{aligned}$$

where x and y are the number of football and rugby shirts to make, respectively.

Let's now solve this problem graphically.

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Example 1.4 revisited (*Sportizus*)

Recall the example about *Sportizus*, a company who produce replica football and rugby shirts. We summarised this problem as follows.

Maximise $P = 22x + 16y$ subject to the following constraints:

$$\begin{aligned}8x + 5y &\leq 600, \\9x + 15y &\leq 900, \\x &\geq 30 \quad \text{and} \\y &\geq 10,\end{aligned}$$

where x and y are the number of football and rugby shirts to make, respectively.

Let's now solve this problem graphically.

Example 1.4 revisited (*Sportizus*)

The first inequality is $8x + 5y \leq 600$:

■ When $x = 0$, we have

$$5y = 600 \quad \text{i.e.}$$

$$y = 120.$$

■ When $y = 0$, we have

$$8x = 600 \quad \text{i.e.}$$

$$x = 75.$$

So plot $(0, 120)$, $(75, 0)$.

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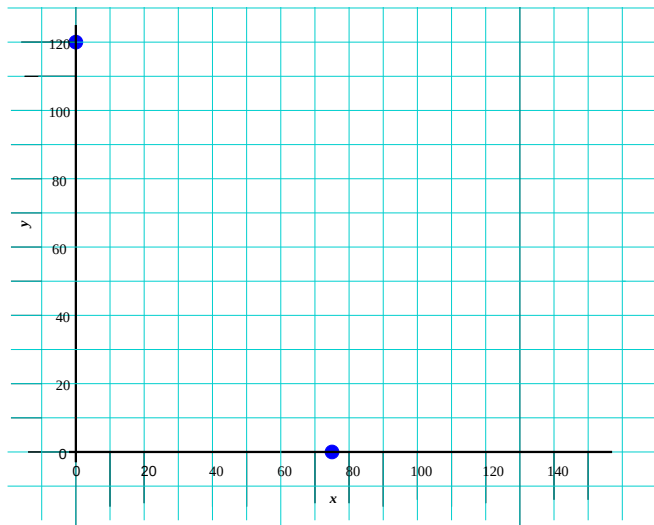
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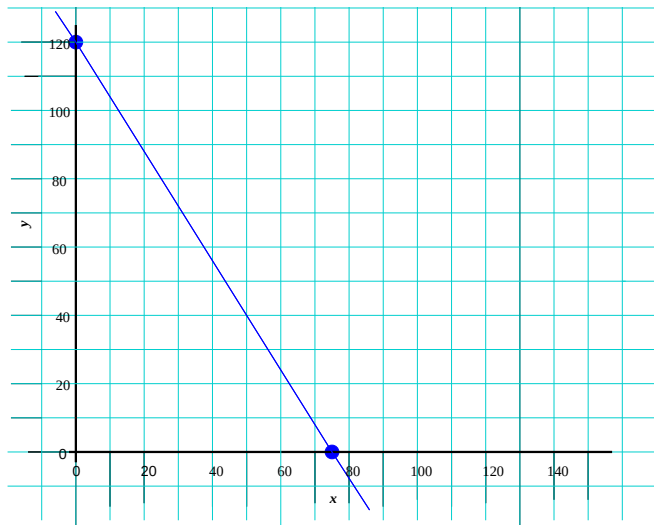
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So plot $(0, 120)$, $(75, 0)$.

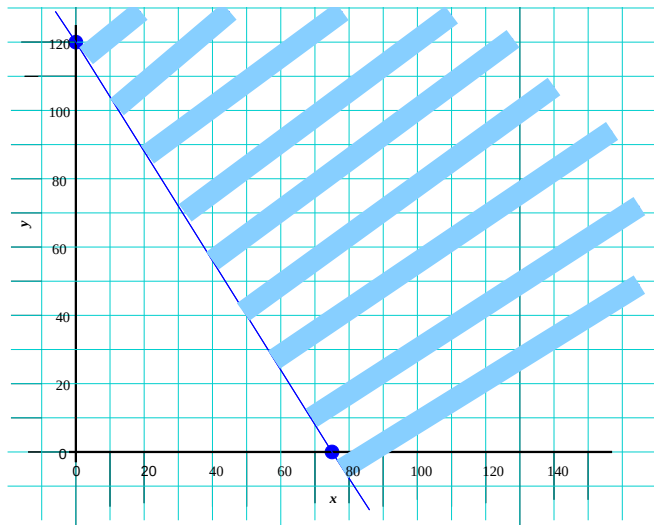
Example 1.4 revisited (*Sportizus*)



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Example 1.4 revisited (*Sportizus*)

The second inequality is $9x + 15y \leq 900$:

■ When $x = 0$, we have

$$\begin{aligned}15y &= 900 && \text{i.e.} \\ y &= 60.\end{aligned}$$

■ When $y = 0$, we have

$$\begin{aligned}9x &= 900 && \text{i.e.} \\ x &= 100.\end{aligned}$$

So plot $(0, 60)$ and $(100, 0)$.

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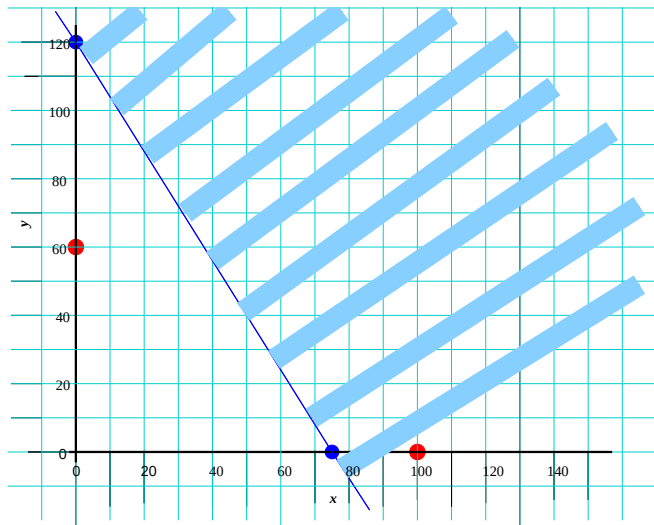
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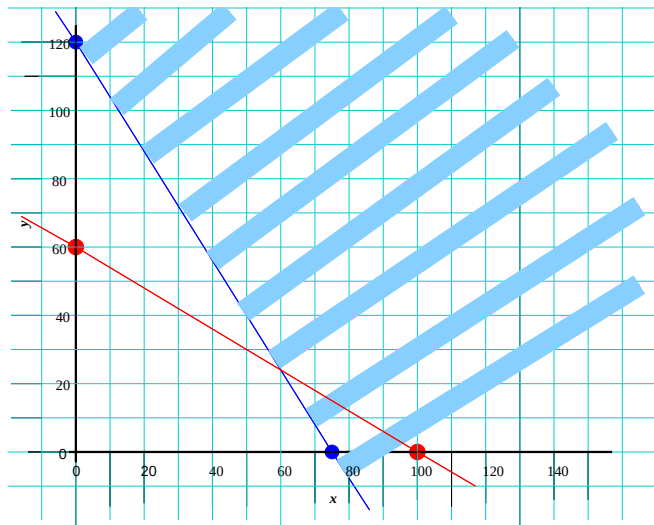
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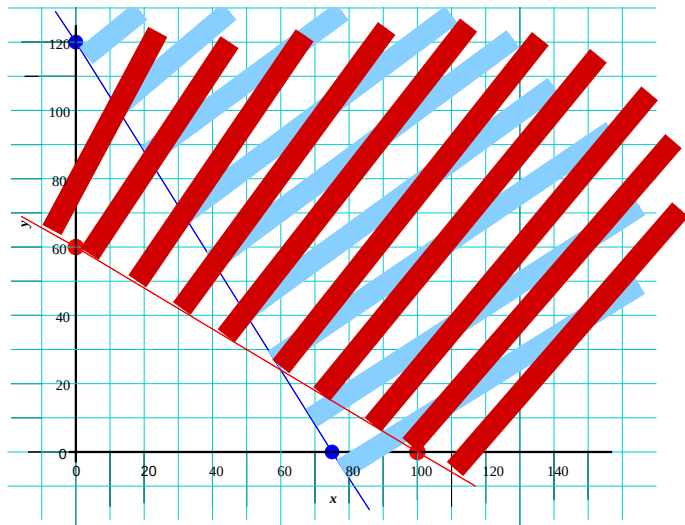
Example 1.4 revisited (*Sportizus*)



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We also require $x \geq 30$ and $y \geq 10$.

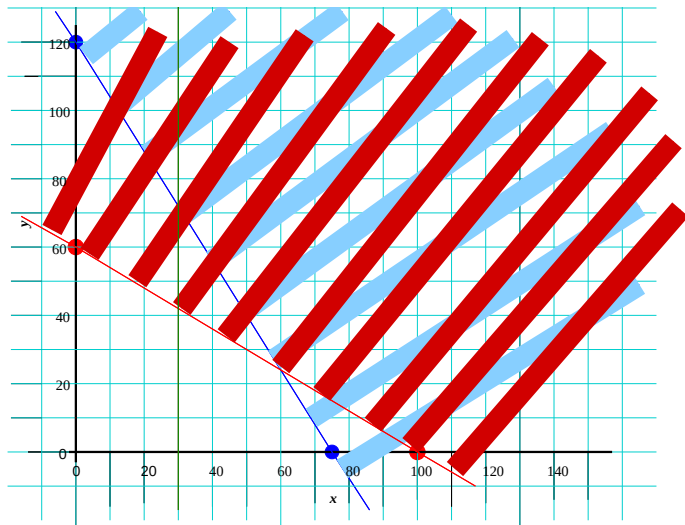
Thus, on the graph, you should also shade out all x values less than 30 and all y values less than 10, to leave $x \geq 30$ and $y \geq 10$ *unshaded*.

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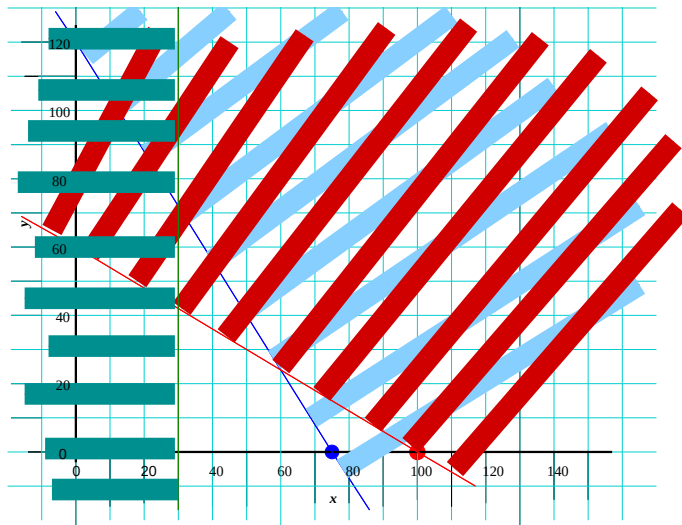
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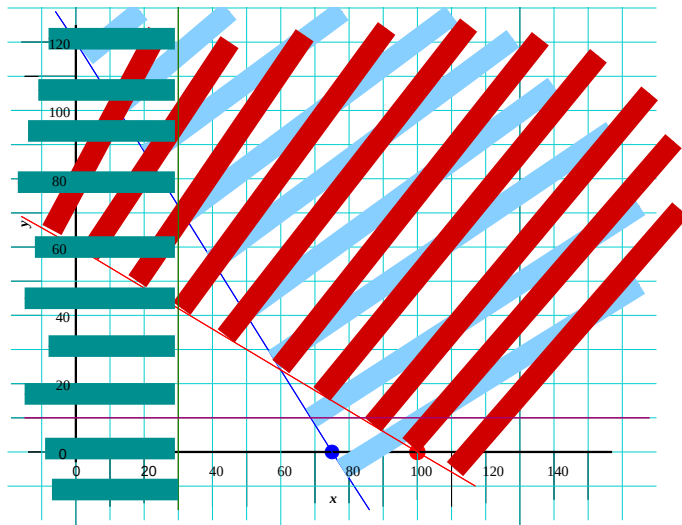
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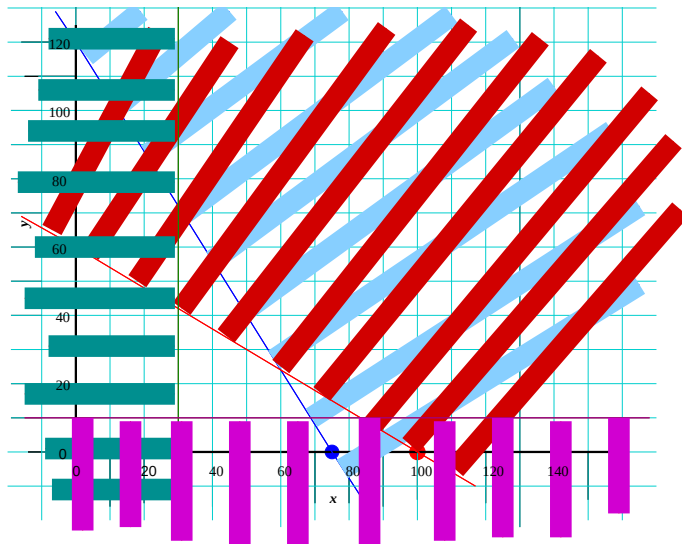
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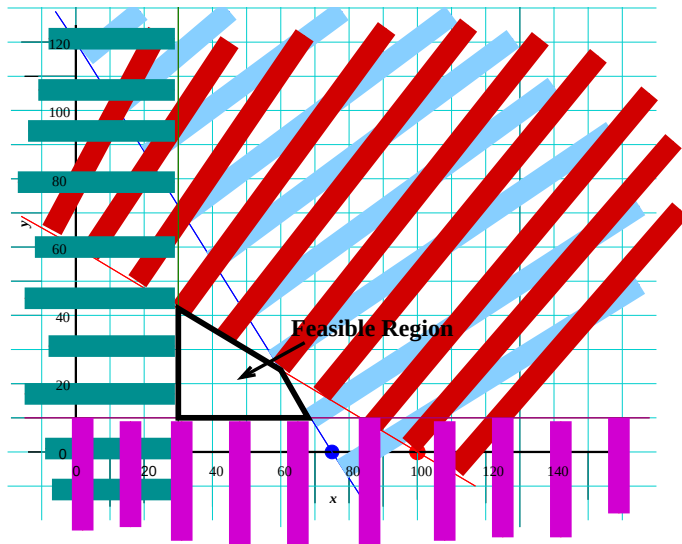
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Finally, we need to plot the **objective function**. Here, we have

$$P = 22x + 16y;$$

as with the previous example, we need a starting value for P in order to plot this line.

You should notice that starting with $P = 22 \times 16 = 352$ will give $(0, 22)$ and $(16, 0)$, and so we also plot this line on the graph.

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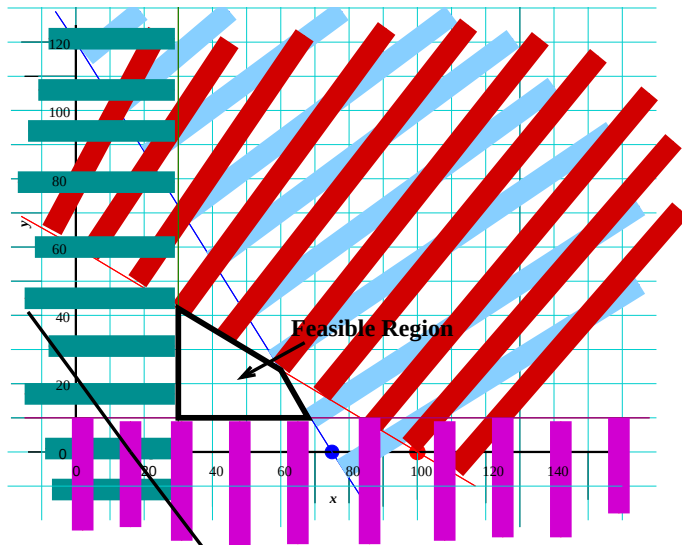
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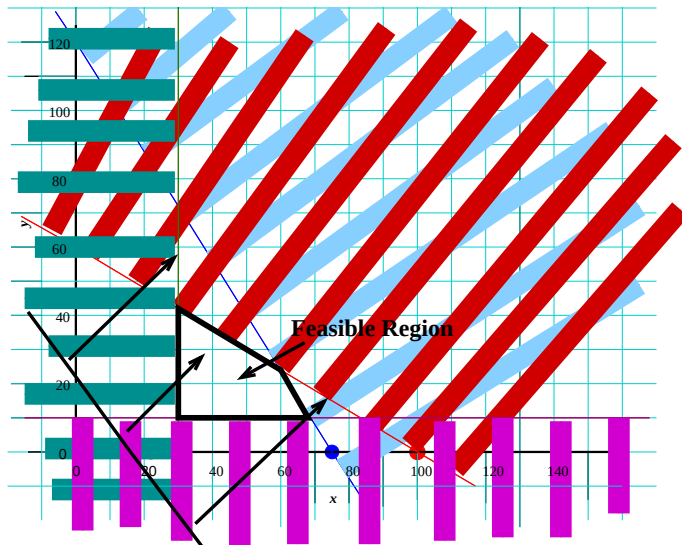
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To maximise profit, $x = 60$ and $y = 24$, i.e. we should make **60 football shirts** and **24 rugby shirts** each day.

This will give a maximum daily profit of

$$\begin{aligned} P &= 22x + 16y \\ &= 22 \times 60 + 16 \times 24 \\ &= \text{£}1704. \end{aligned}$$

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Algebraic solutions for linear programming problems

Notice that in both the examples we have looked at so far, the graphical representation indicates that the solution is at the intersection of two lines on the graph.

Thus, we could find also the solution **algebraically**, as the solution to a pair of **simultaneous equations**.

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Algebraic solutions for linear programming problems

For example, for *Sportizus*, the solution was at the intersection of the two lines

$$\begin{aligned}8x + 5y &= 600 \\9x + 15y &= 900.\end{aligned}$$

Multiplying the first equation by 3 will allow us to eliminate y on subtraction:

$$\begin{aligned}24x + 15y &= 1800 \\ \underline{9x + 15y} &= \underline{900} \\ 15x + 0y &= 900,\end{aligned}$$

and so $x = 900/15 = 60$. Substituting this into the first equation gives:

$$\begin{aligned}480 + 5y &= 600 \\ 5y &= 120 \\ y &= 24.\end{aligned}$$

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