# The Storm of the Century! (Statistics of Extremes)

**Lee Fawcett**

***School of Maths & Stats, Newcastle University***

lee.fawcett@ncl.ac.uk

# Background: Statistics of extremes

So many areas of Statistics are interested in what happens **on average**: Mean, median, mode...

One area that is different is the field of **extreme value theory**.

Why might we be interested in **extremes**, rather than averages?

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We can usually deal with averages here – it is the **extremes** that cause devastation!

Such scientists require the expertise of **mathematicians** to help with **predictions** and **calculations**.

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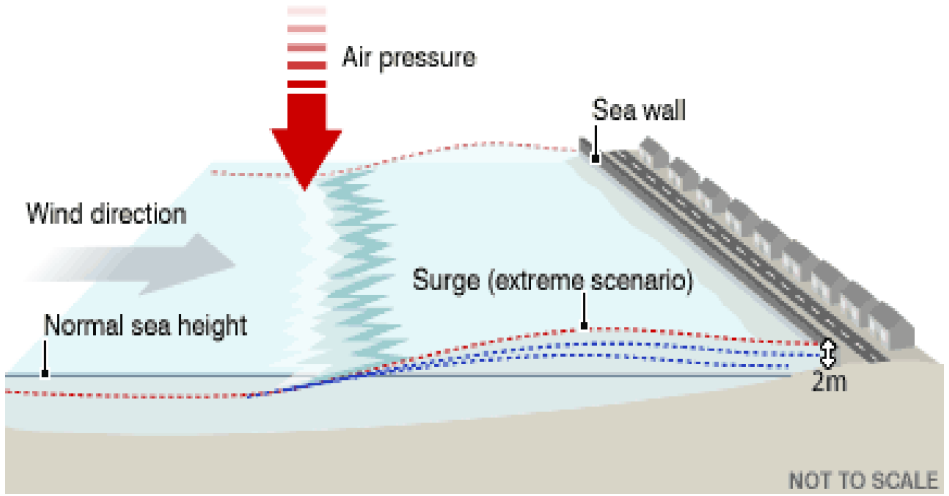
# Sea–surge


Air pressure
Sea wall
Wind direction
Surge (extreme scenario)
Normal sea height
2m
NOT TO SCALE

# Earthquakes/sea–surge: Japan, 2011

# Hurricane Sandy: USA, 2012

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# Hurricane ????: USA, 2018?

# Sea–surge: The Great North Sea Flood, 1953

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# Sea–surge in Newlyn, Cornwall

# Stormy weather in the UK!

- 5 December 2015: **Storm Desmond** – more than a month's rain in parts of Cumbria, flooding in Carlisle
- 22 December 2015: Further flooding in Cumbria
- Christmas Day: **Storm Eva** – 106 flood alerts in UK
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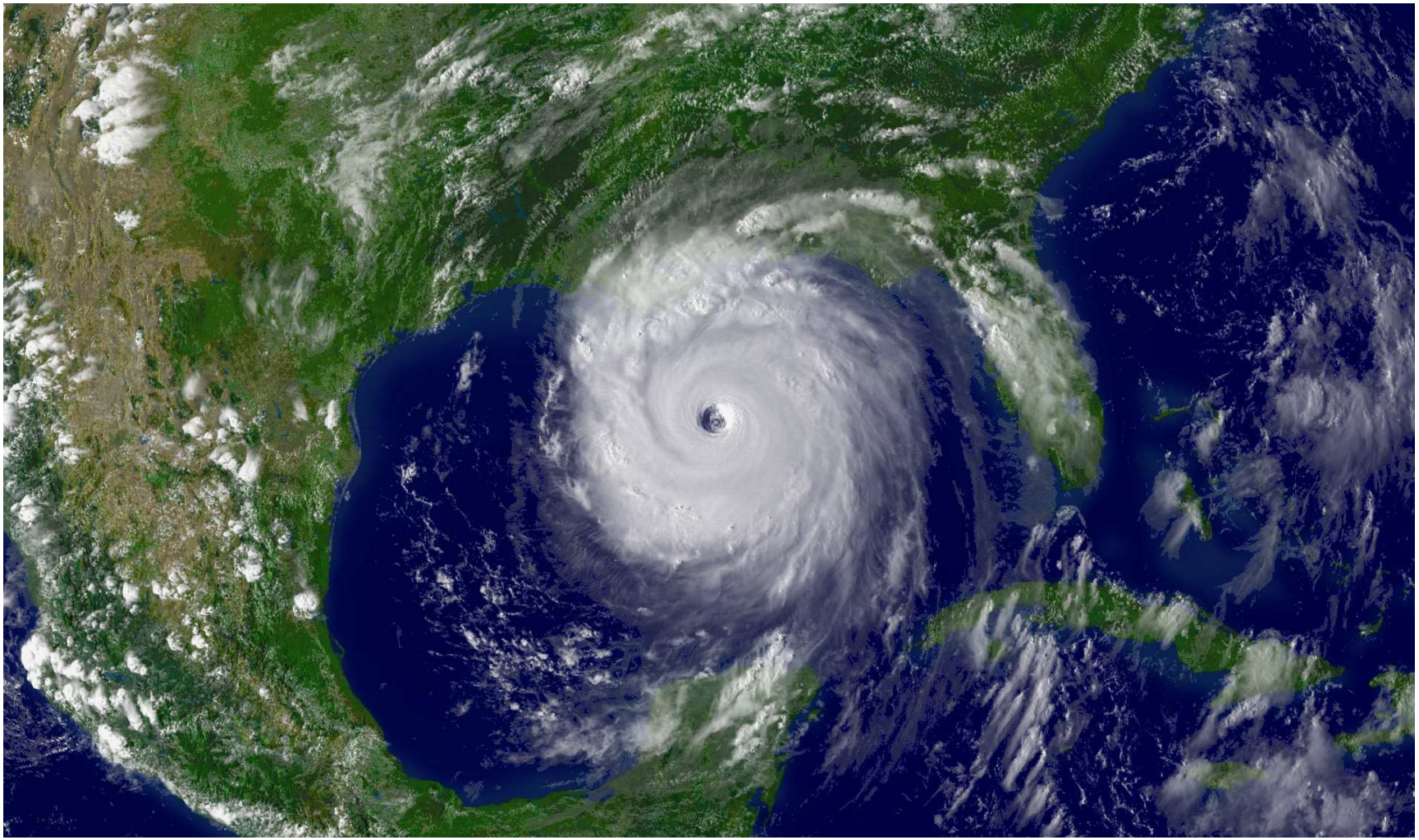

# Hurricane Katrina: New Orleans, August 2005

## Some facts

- Category 5 Hurricane (Category 3 when it made landfall)
- Killed 1833 people
- Caused $108 bn worth of damage
- Most damage/loss of life caused by **storm surge**
- Storm surge reached 14.4 feet above sea level
- Lowest air pressure 902 mb
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# Data application: Sea surges at New Orleans

Below are the annual maximum sea–surge levels observed at New Orleans over a 50 year period *before Hurricane Katrina*:

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 8.5 | 8.9 | 9.1 | 8.9 | 8.4 | 9.7 | 9.1 | 9.6 | 8.7 | 9.3 |
| 9.6 | 9.3 | 8.7 | 9.0 | 8.8 | 8.9 | 8.9 | 12.2 | 7.8 | 7.7 |
| 8.3 | 8.1 | 7.3 | 6.8 | 6.7 | 7.3 | 7.6 | 8.2 | 8.6 | 9.8 |
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Suppose you are the mathematician on the team of scientists working on a new section of flood defence system in New Orleans.

One of the civil engineers asks you to work out some exceedance probabilities for her.

In particular, she wants to know the probability that, this year, the annual maximum sea surge at New Orleans will

1. exceed **8.75 feet**;
2. exceed **11.25 feet**;
3. exceed **14 feet**.

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# Part A: The relative frequency approach

You will have used this approach to probability in your maths at School.

It helps if the data are re–written in ascending order:

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Using the relative frequency approach, work out the other two exceedance probabilities:

- $\boldsymbol{P}$(sea–surge exceeds 11.25 feet)
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$$P(\text{sea–surge exceeds 11.25 feet}) =$$

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| 6.7 | 6.8 | 7.3 | 7.3 | 7.3 | 7.4 | 7.6 | 7.7 | 7.8 | 8.1 |
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$$\boldsymbol{P}(\text{sea–surge exceeds 11.25 feet}) = \frac{6}{50} = 0.12$$

# Part A: The relative frequency approach

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 6.7 | 6.8 | 7.3 | 7.3 | 7.3 | 7.4 | 7.6 | 7.7 | 7.8 | 8.1 |
| 8.2 | 8.3 | 8.4 | 8.5 | 8.6 | 8.7 | 8.7 | 8.8 | 8.8 | 8.9 |
| 8.9 | 8.9 | 8.9 | 9.0 | 9.1 | 9.1 | 9.3 | 9.3 | 9.4 | 9.5 |
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$$\boldsymbol{P}(\text{sea–surge exceeds 14 feet}) =$$

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| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
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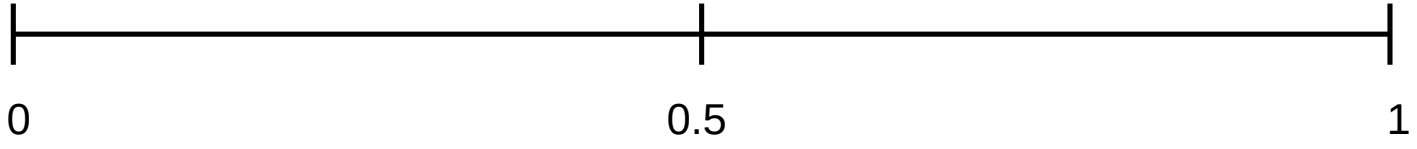
Think about the **probability scale**:



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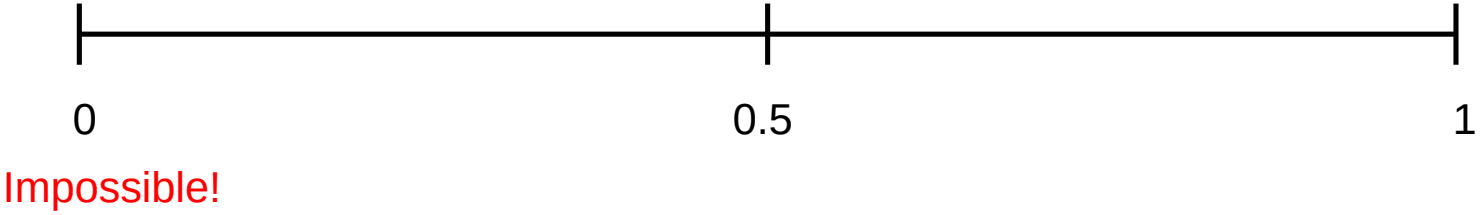

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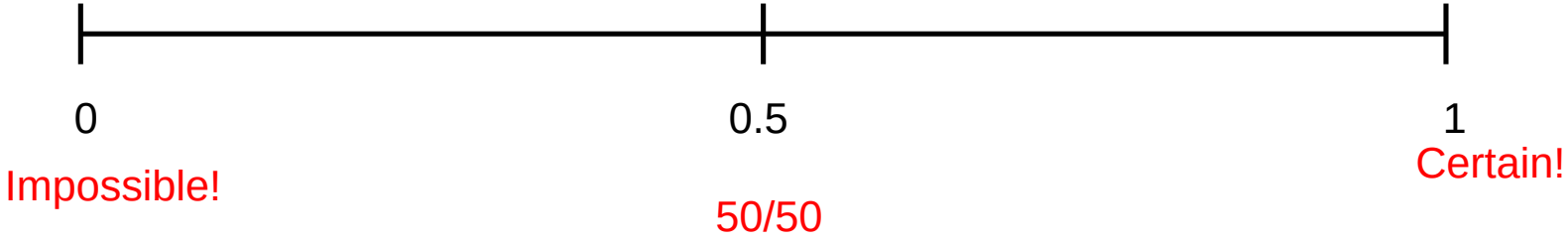

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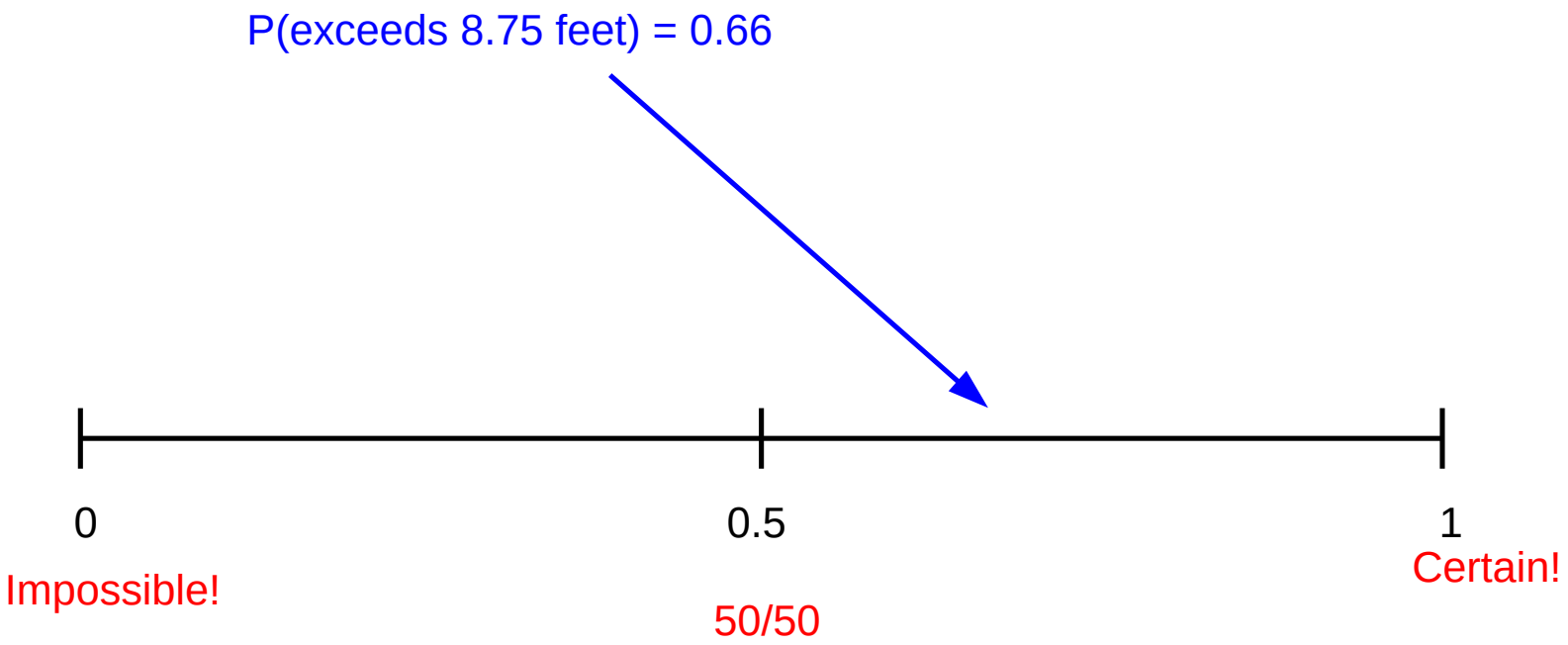

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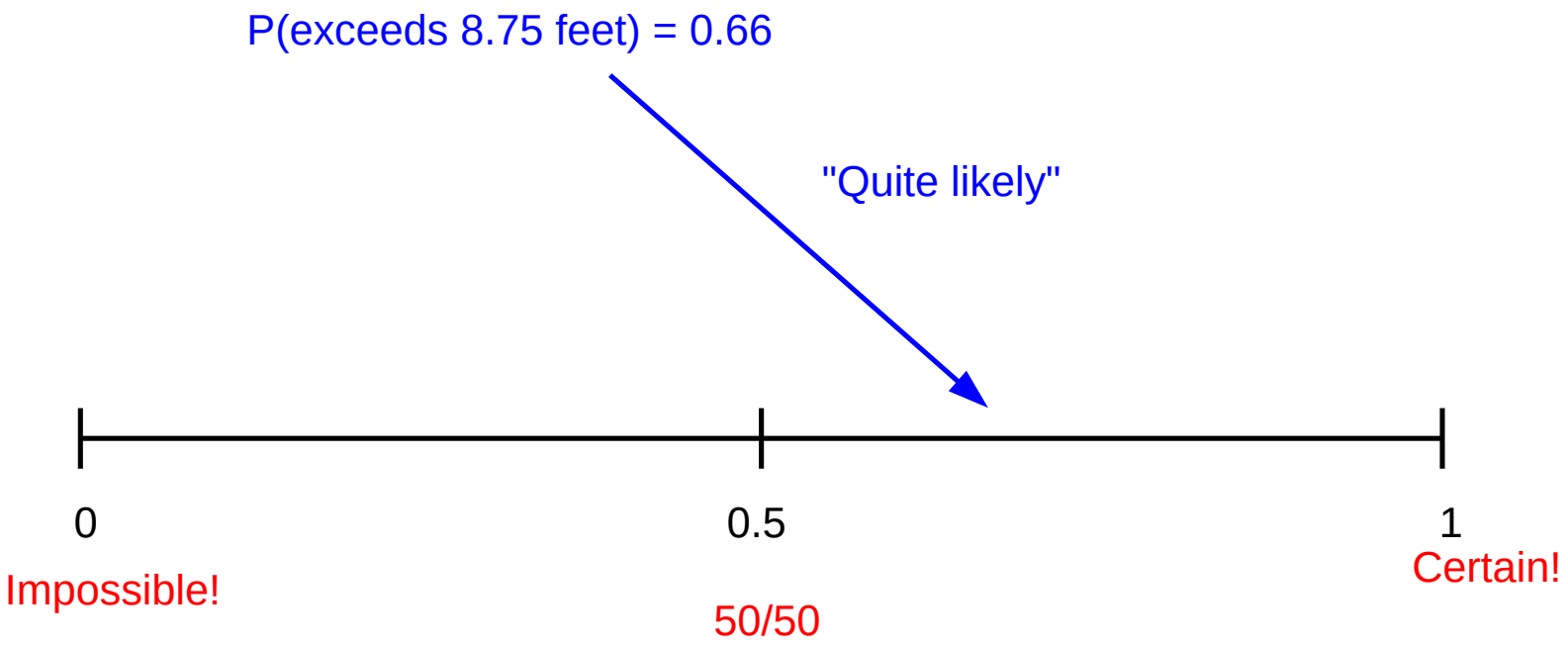

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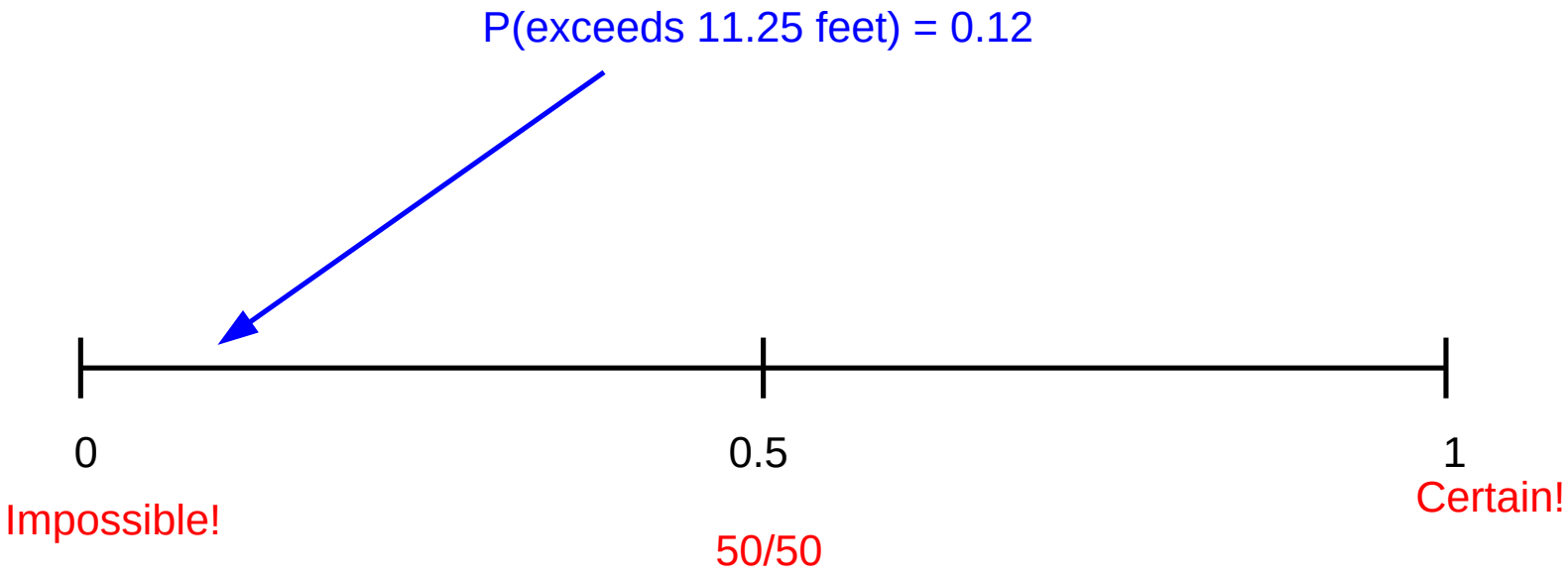

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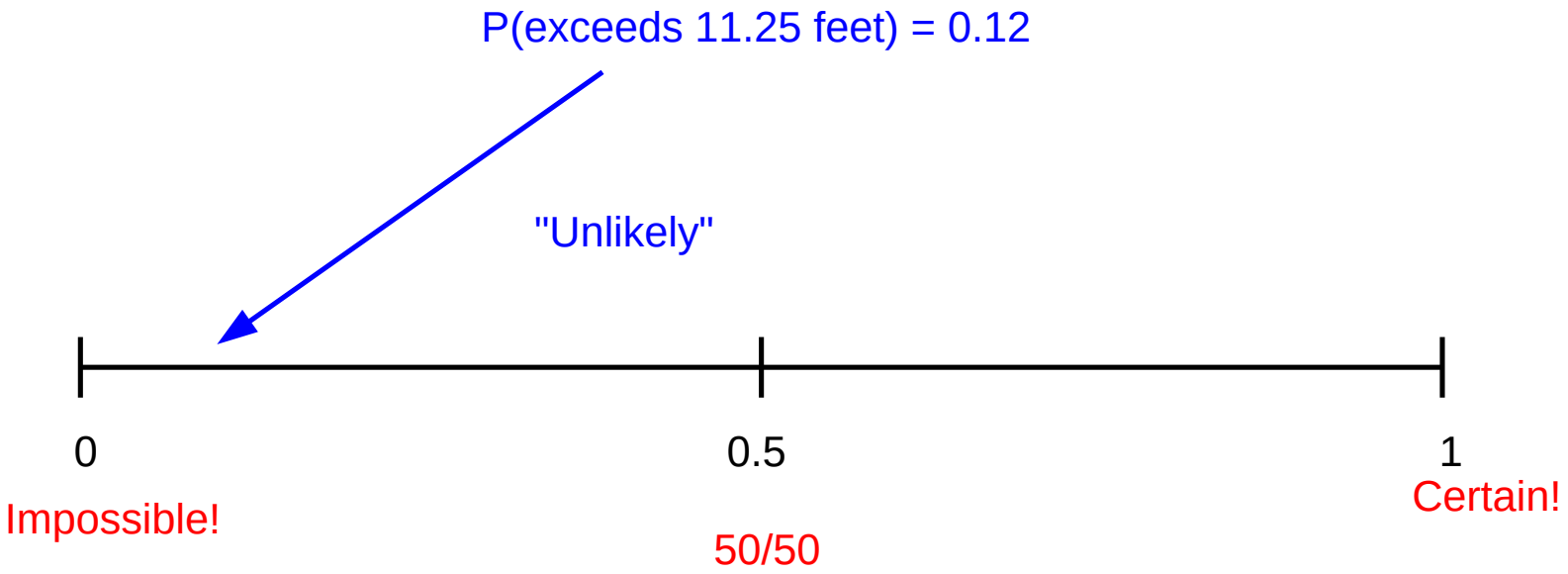

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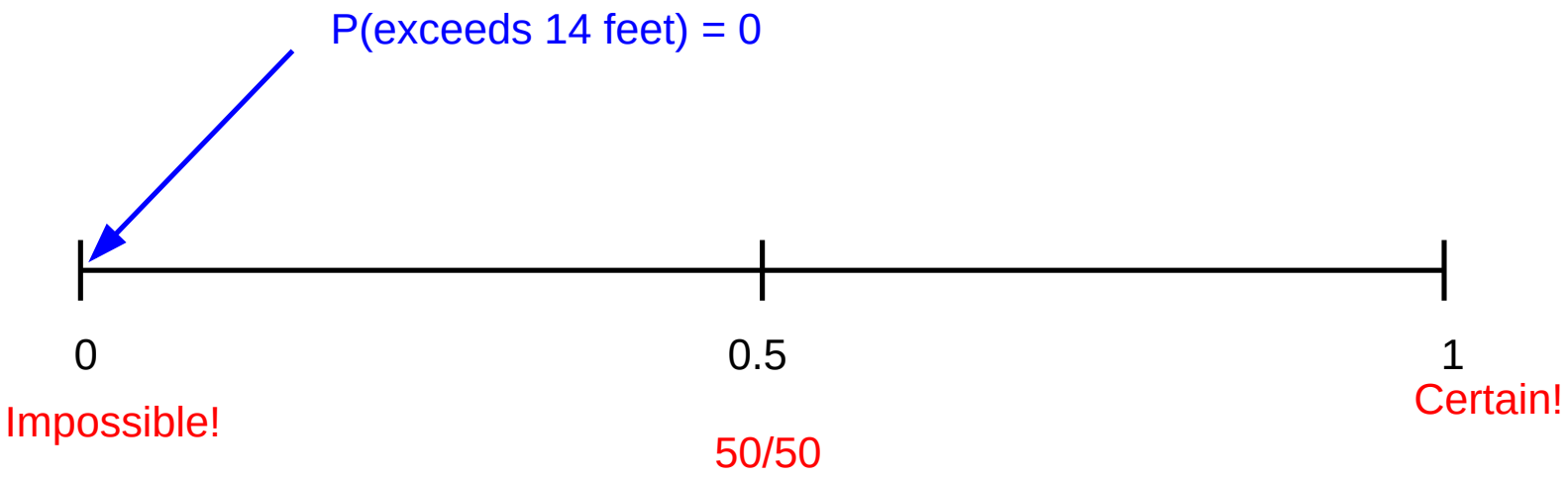

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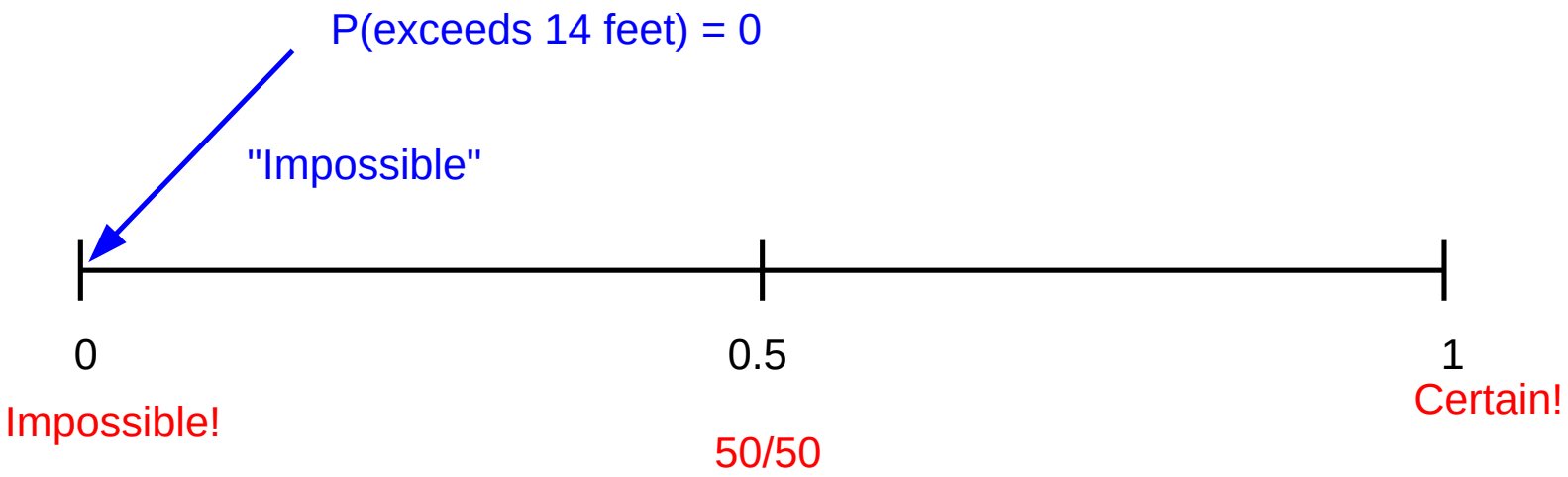

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What is wrong with the exceedance probability associated with a sea–surge of 14 feet?

Our answer suggests a sea–surge of more than 14 feet is impossible – however, this *did* happen during Hurricane Katrina (sea–surges reached 14.4 feet!).

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| **10.0** | **13** | $\mathbf{13/50 = 0.26}$ |
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| **10.0** | 13 | $13/50 =$ **0.26** |
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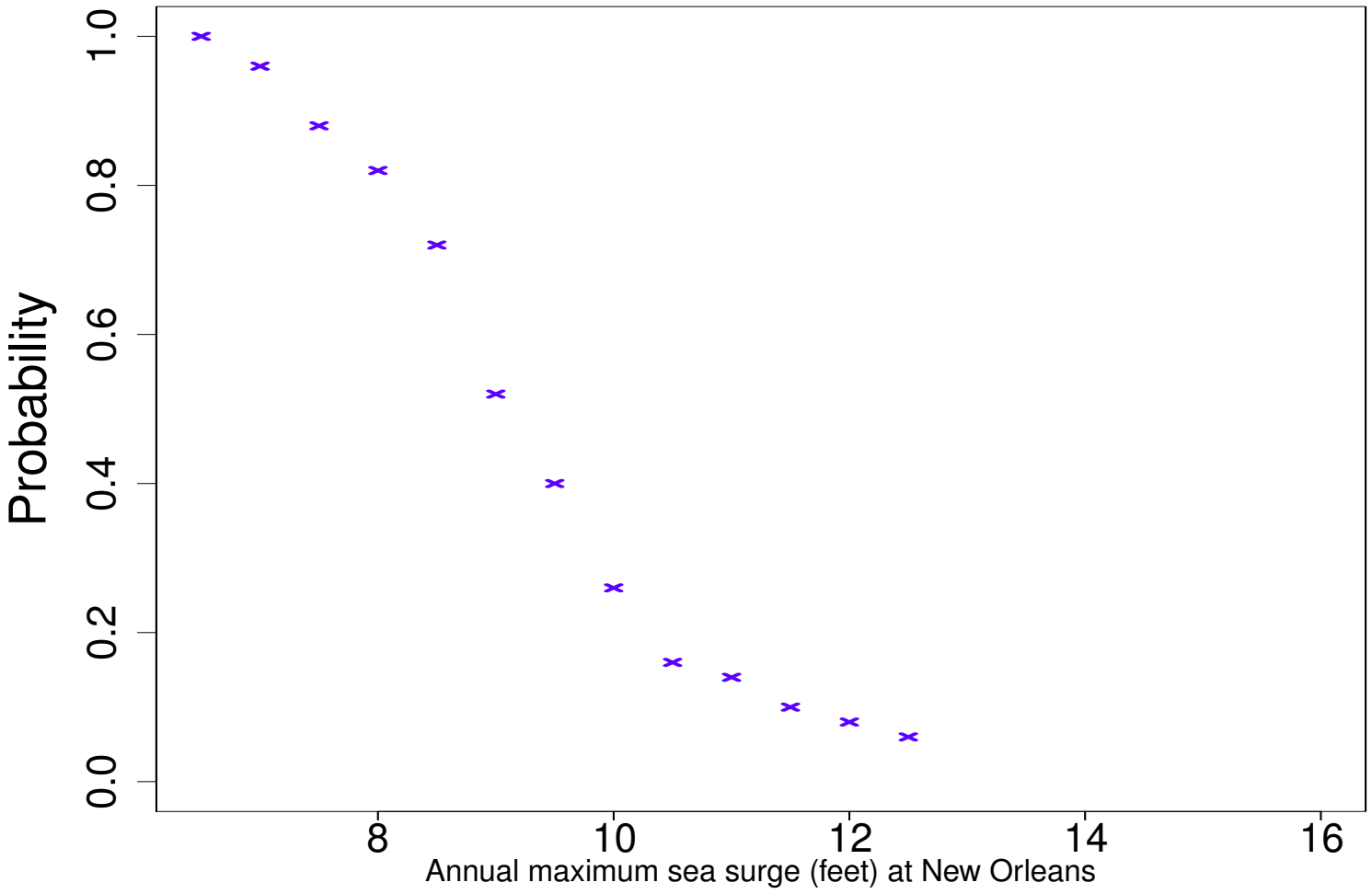
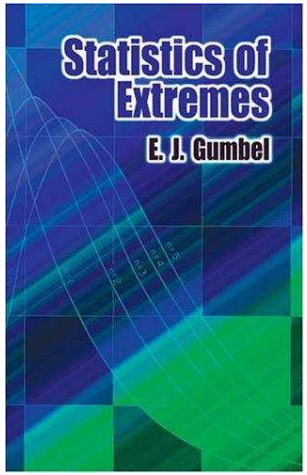


# Part B: A probability model for extremes

Between 1920 and the mid–1950s, some eminent mathematicians developed probability models for extremes.

The aim was to provide a **mathematical formula** that could "predict" the exceedance probabilities of real–life extremes, as in the table we've just drawn up for the New Orleans data.

One of these mathematicians was **Emil Julius Gumbel**.



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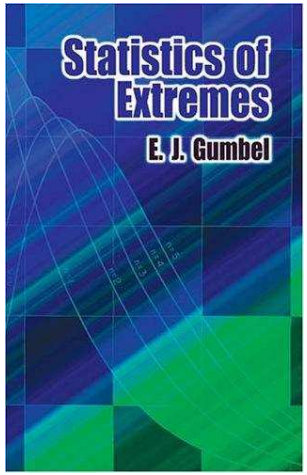

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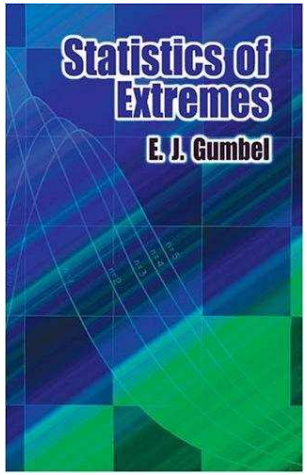

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Using **maximum likelihood estimation**, we can use the data to estimate the values of the location and scale parameters.

Doing so, gives

$$\hat{\mu} = 8.536 \qquad \text{and} \qquad \hat{\sigma} = 1.2$$

for our dataset. Plugging these into Gumbel's formula gives

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Then we can work out the exceedance probabilities for particular values $x$. For example, for $x = 7.5$, we get:

$$\boldsymbol{P}(X > \mathbf{7.5}) = 1 - \exp\left[-\exp\left\{-\left(\frac{\mathbf{7.5} - 8.536}{1.2}\right)\right\}\right] = \mathbf{0.907}.$$

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We could use Gumbel's formula for lots of different values of $x$ and then plot them on the same graph as the real data.

What do you notice when we do this?

The **Gumbel** model is not the only model to choose from:

- The **Fréchet** model
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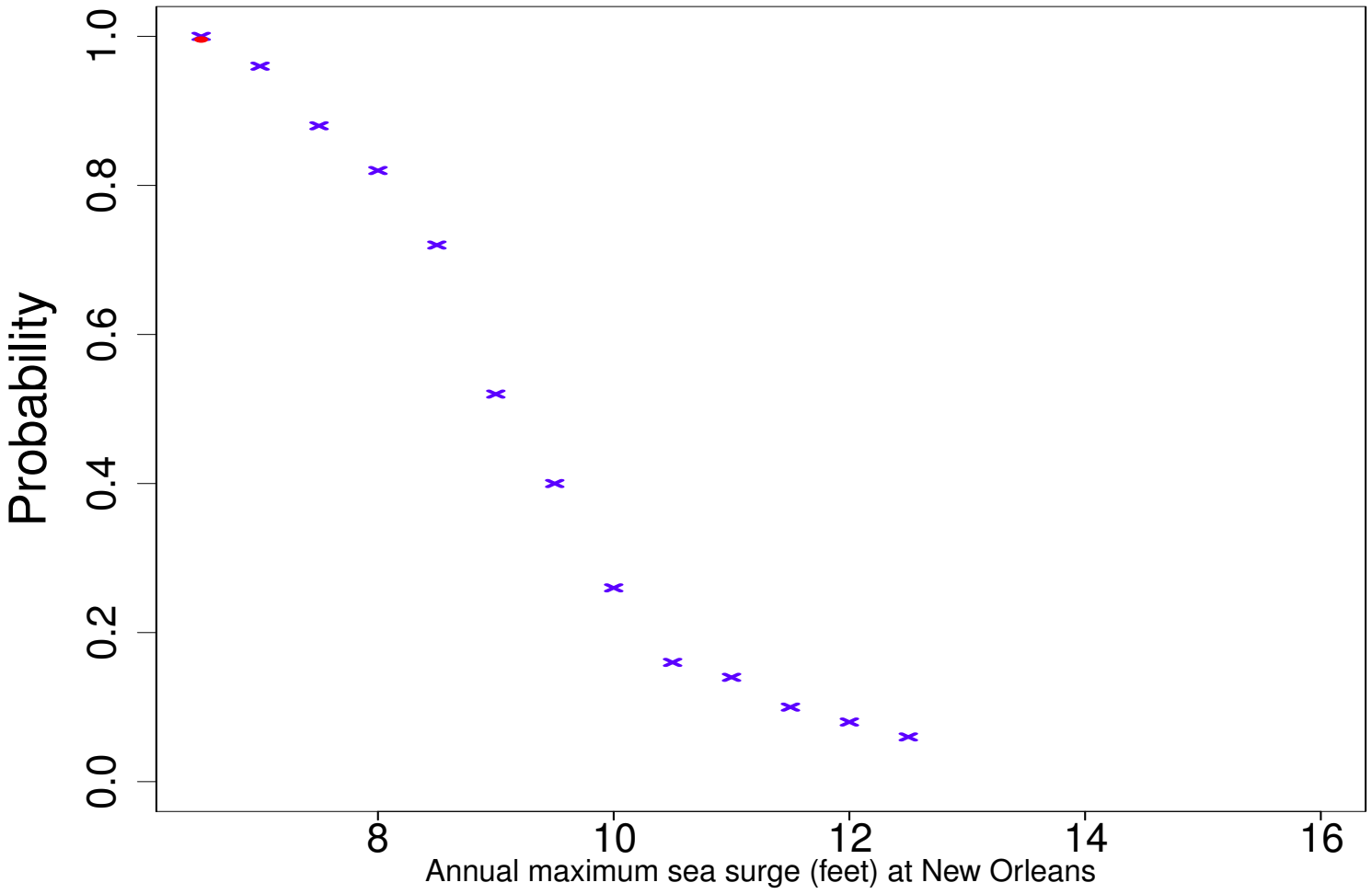

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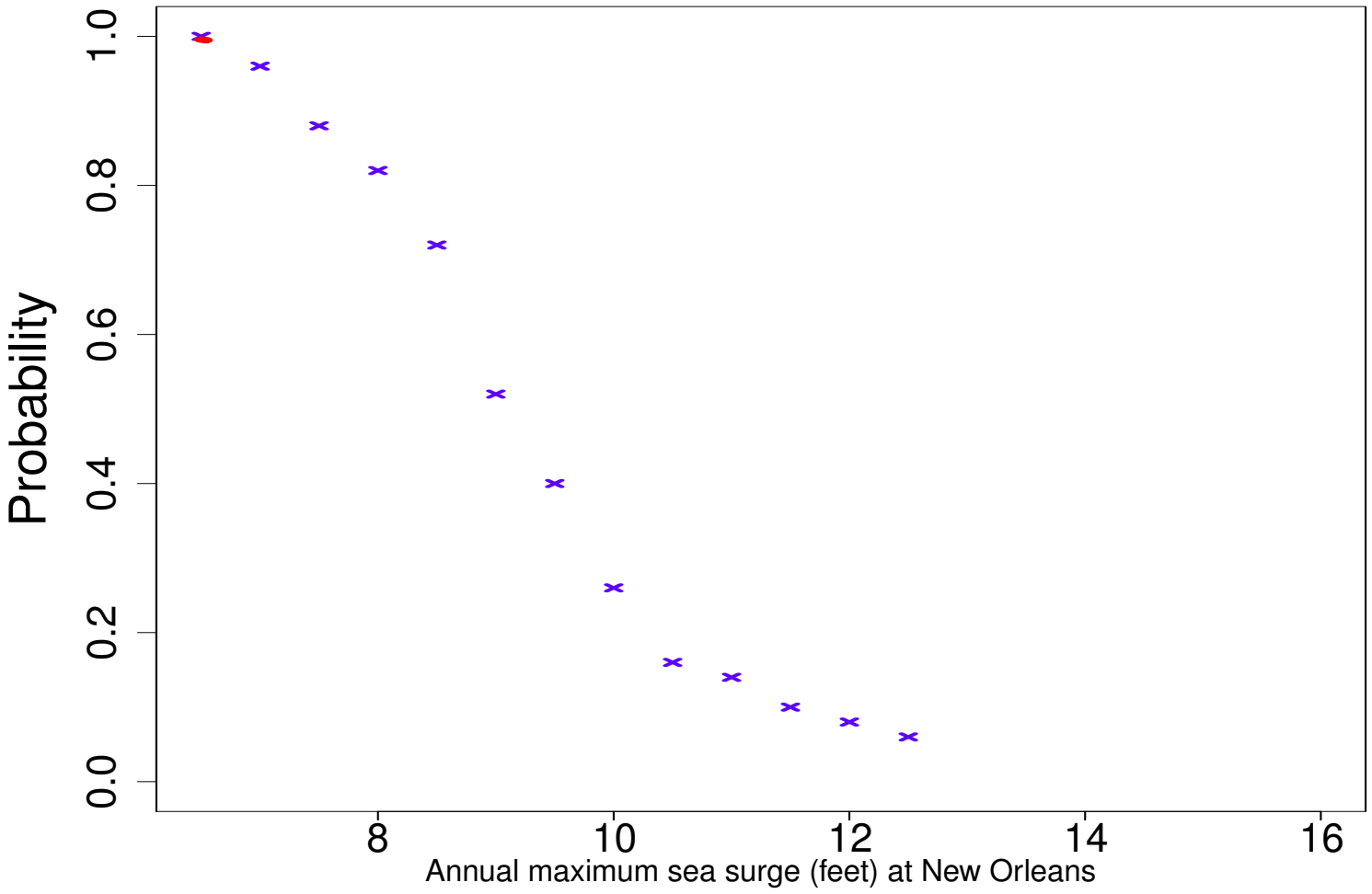

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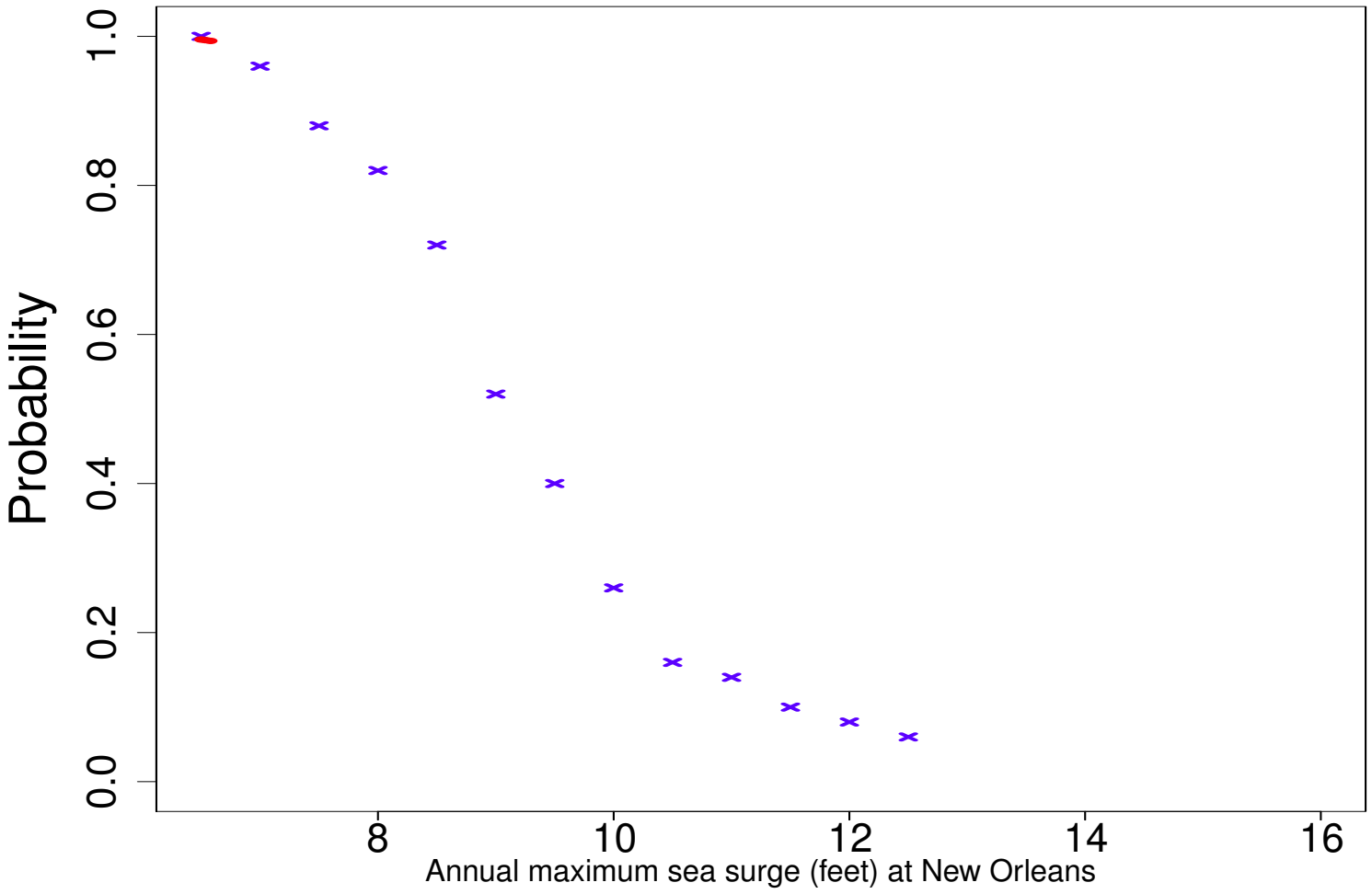

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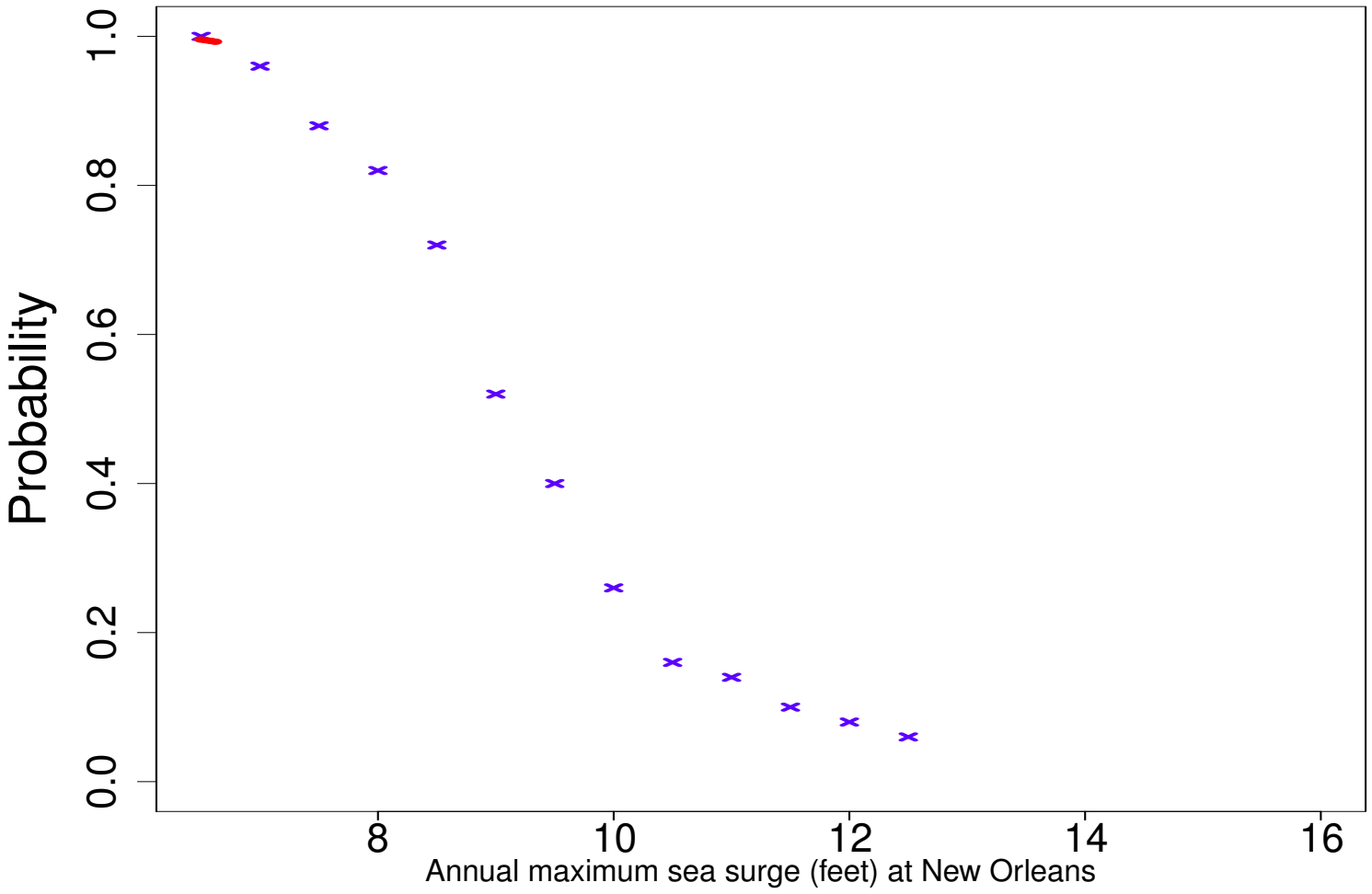

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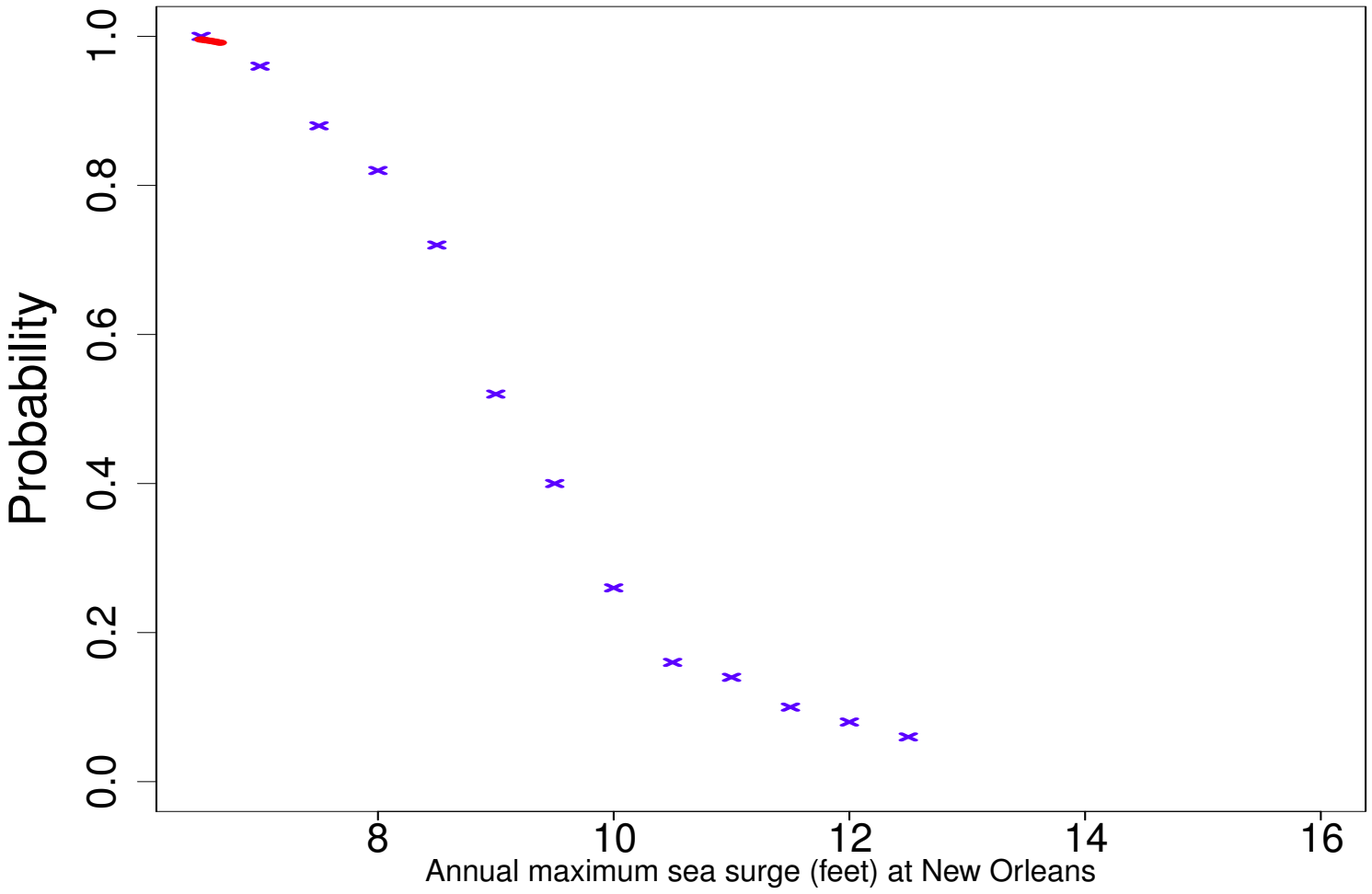

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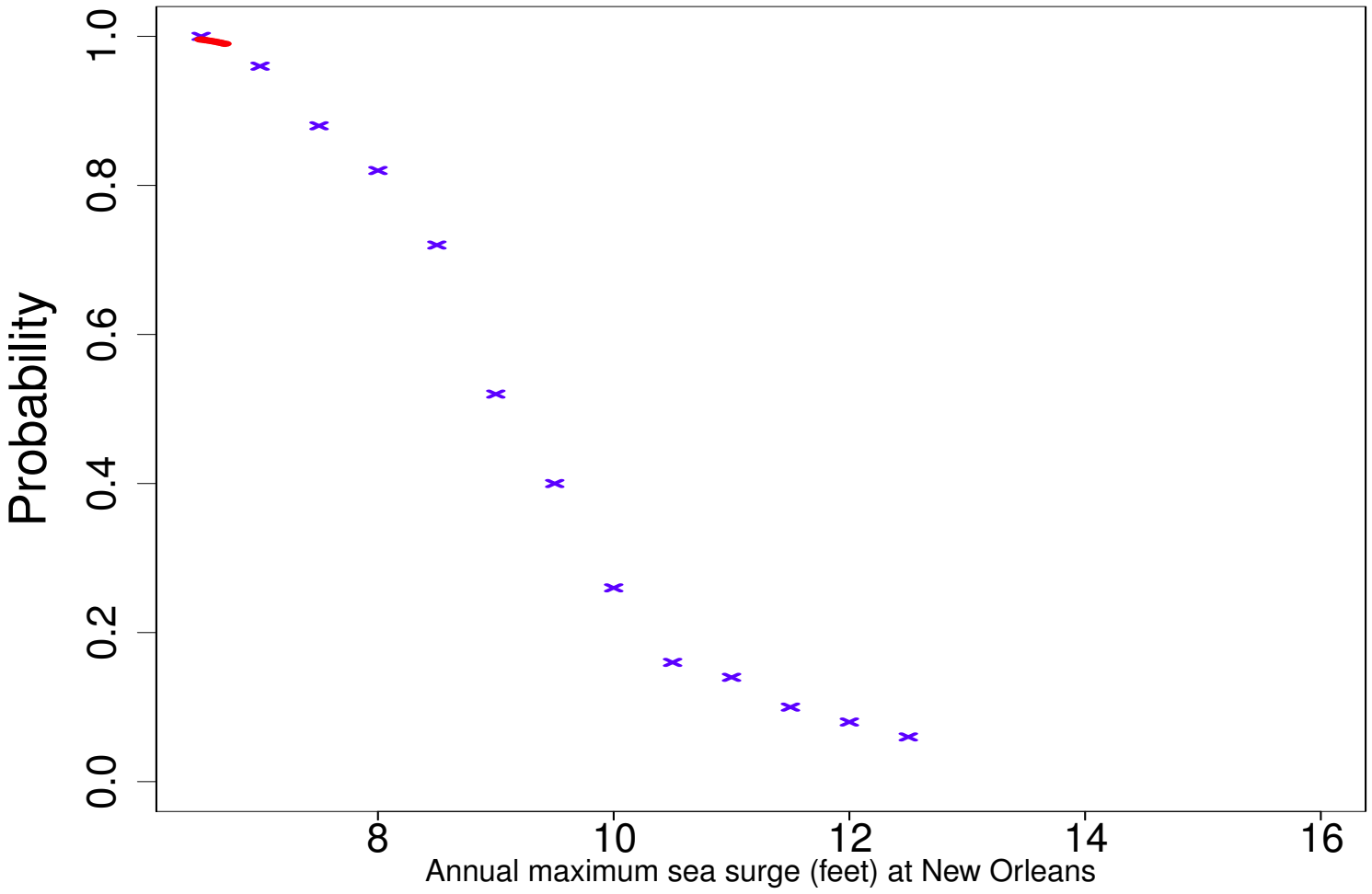

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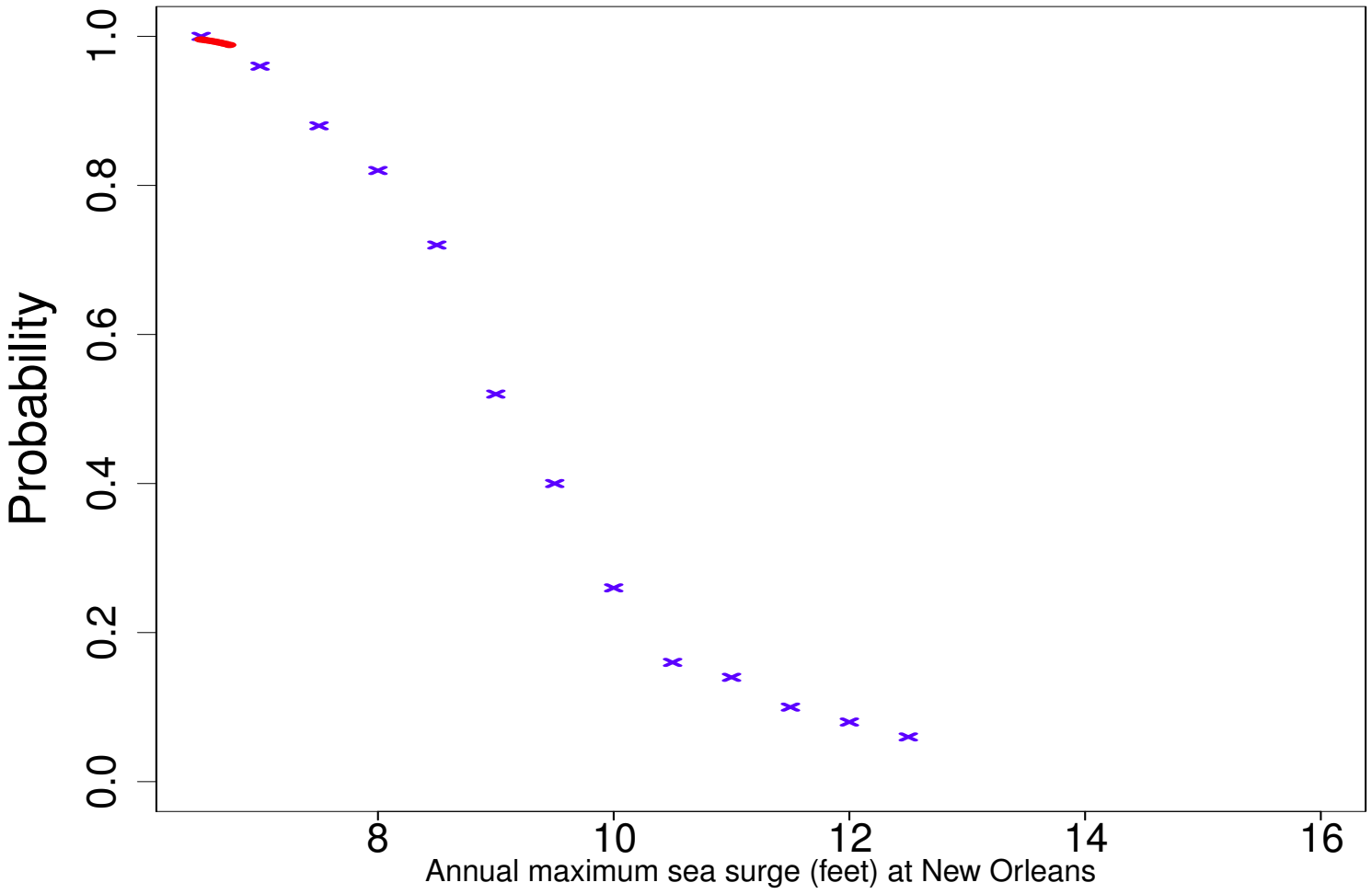

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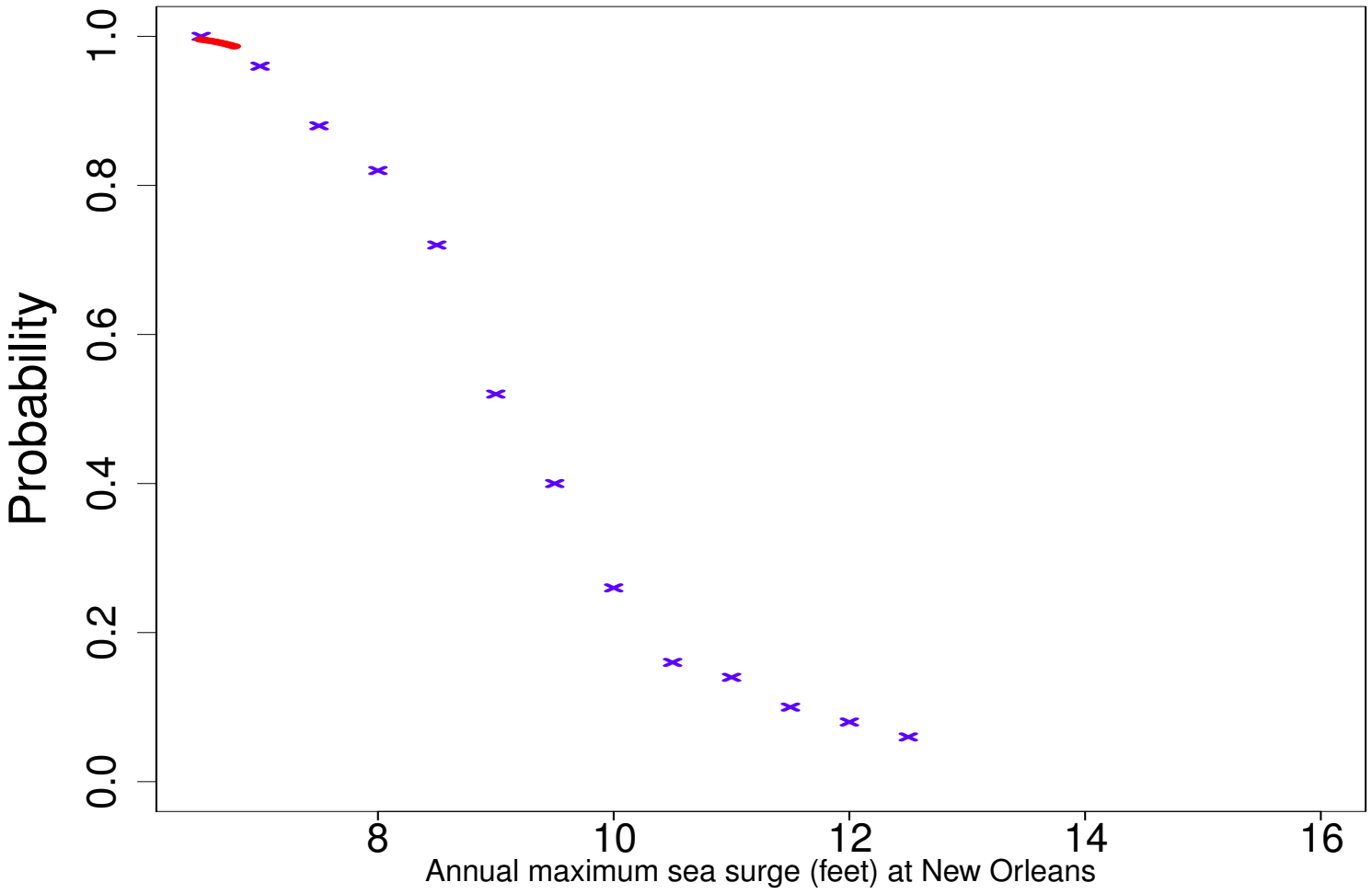

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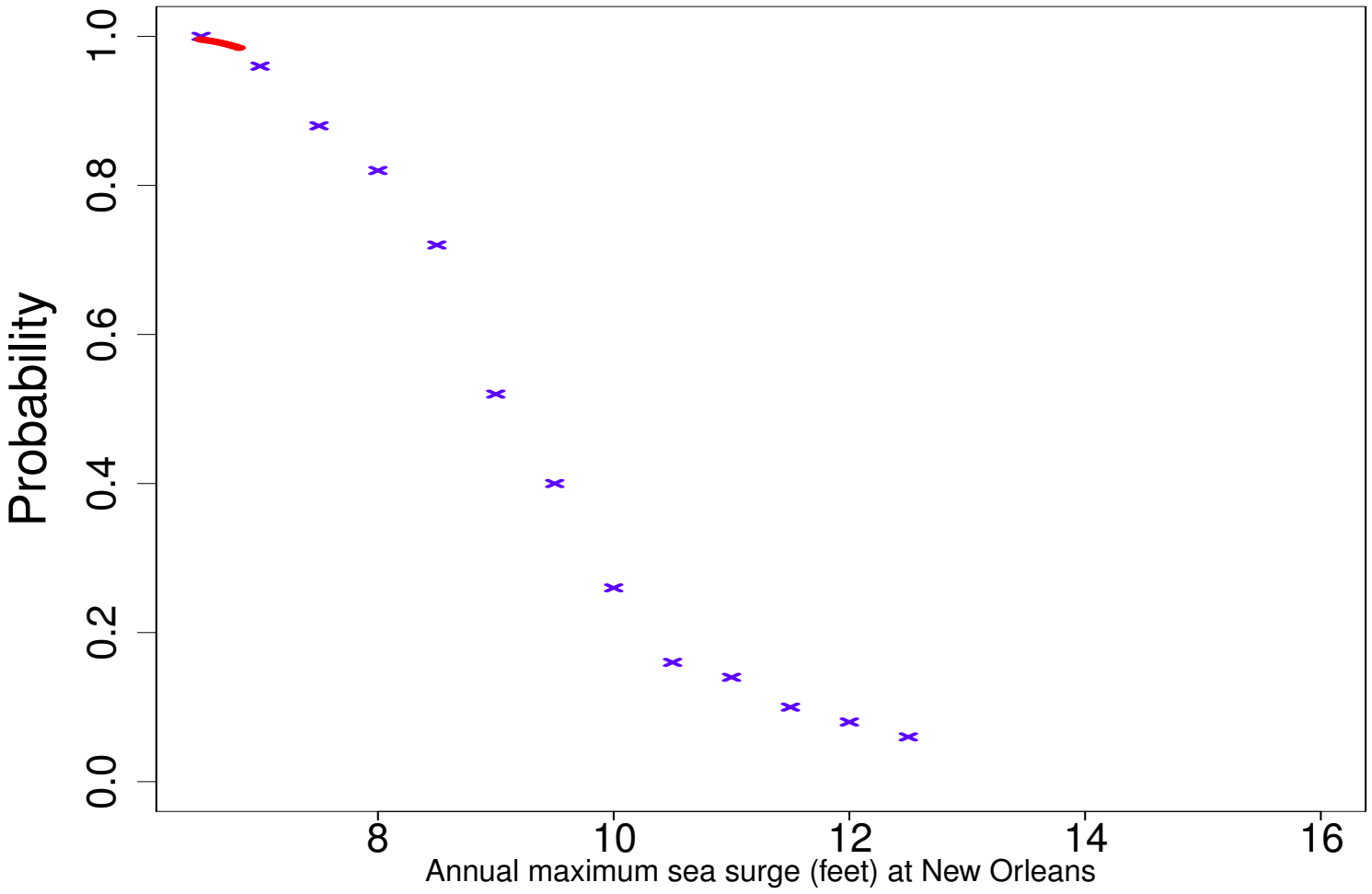

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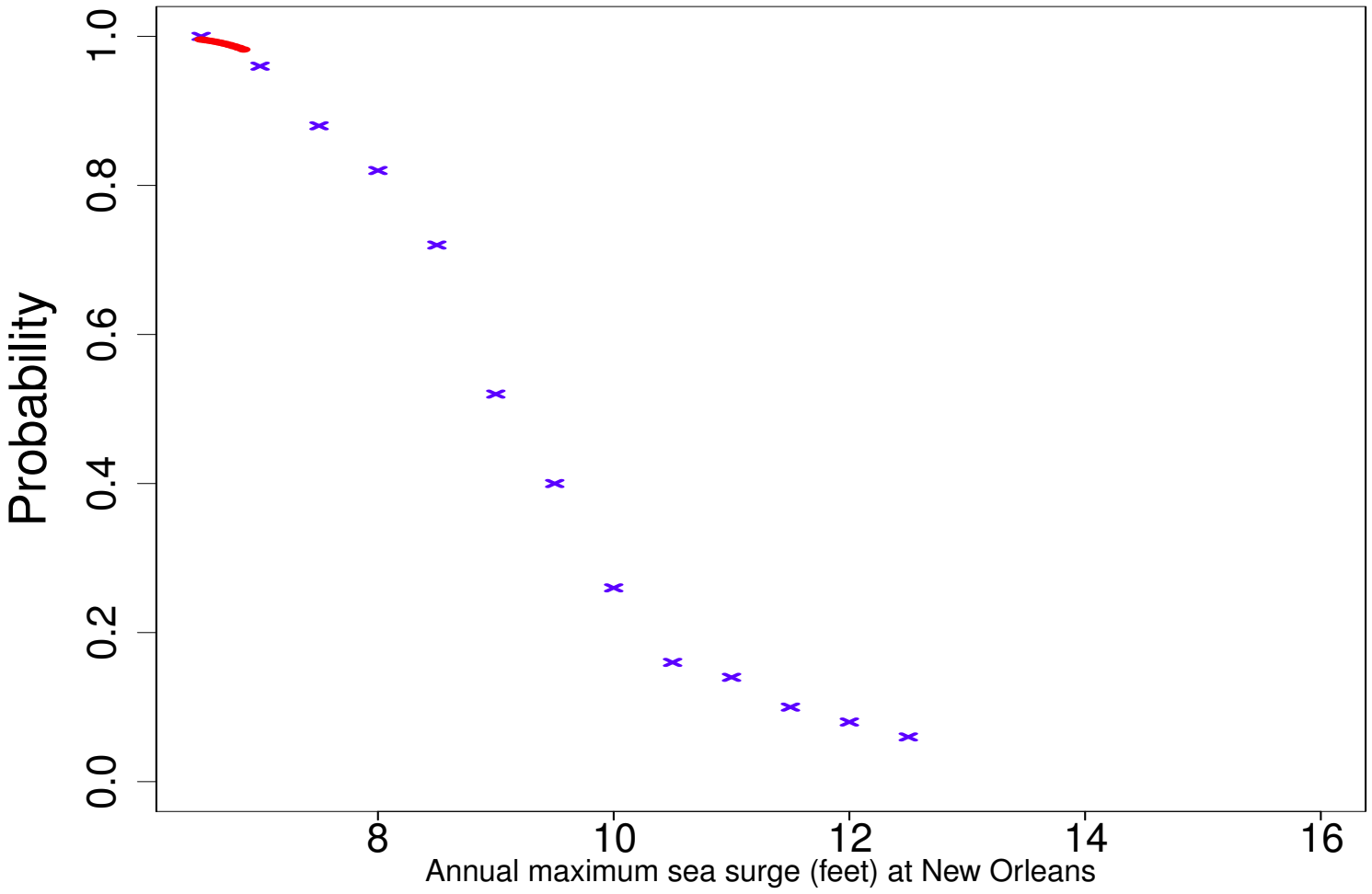

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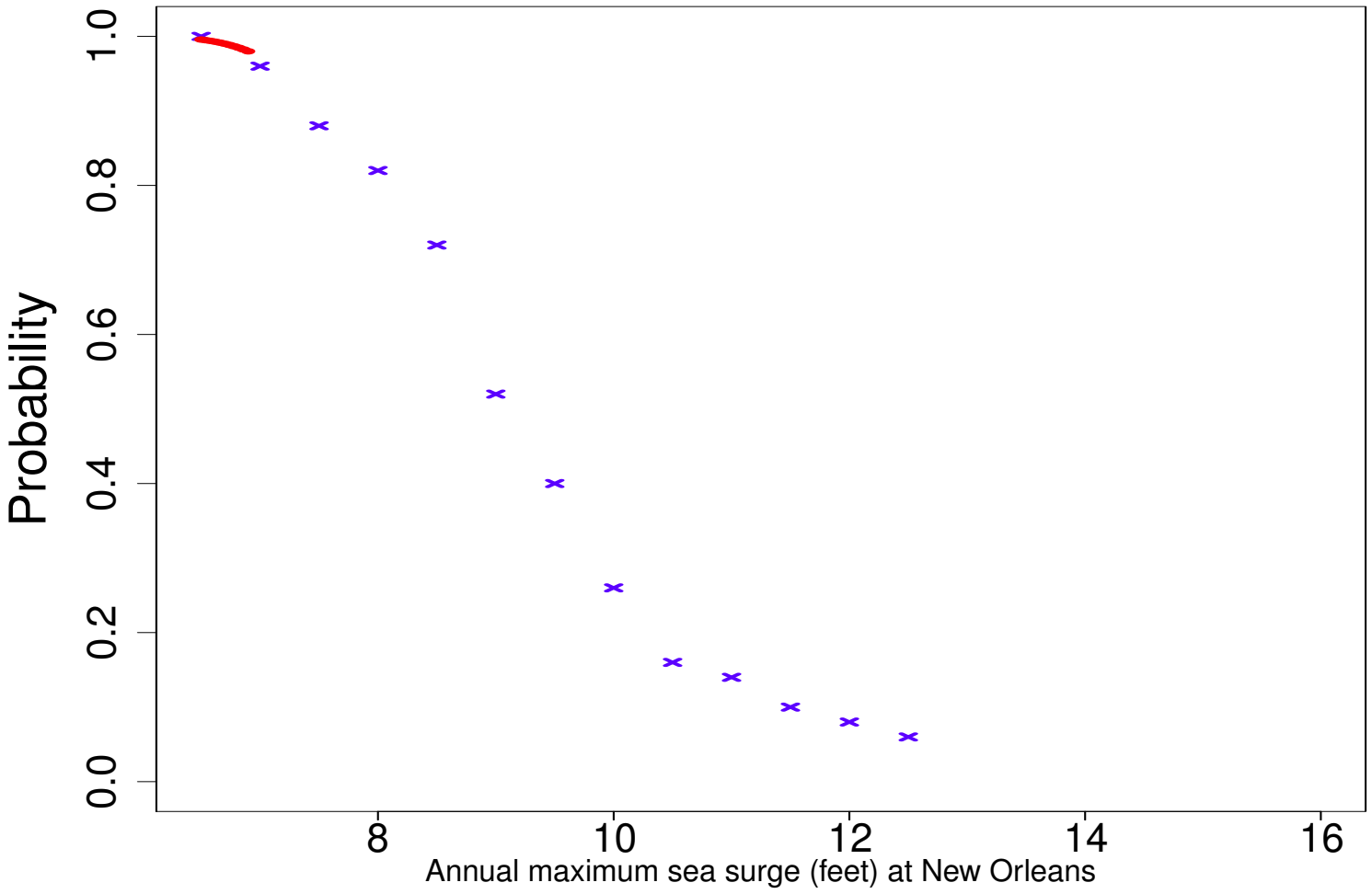

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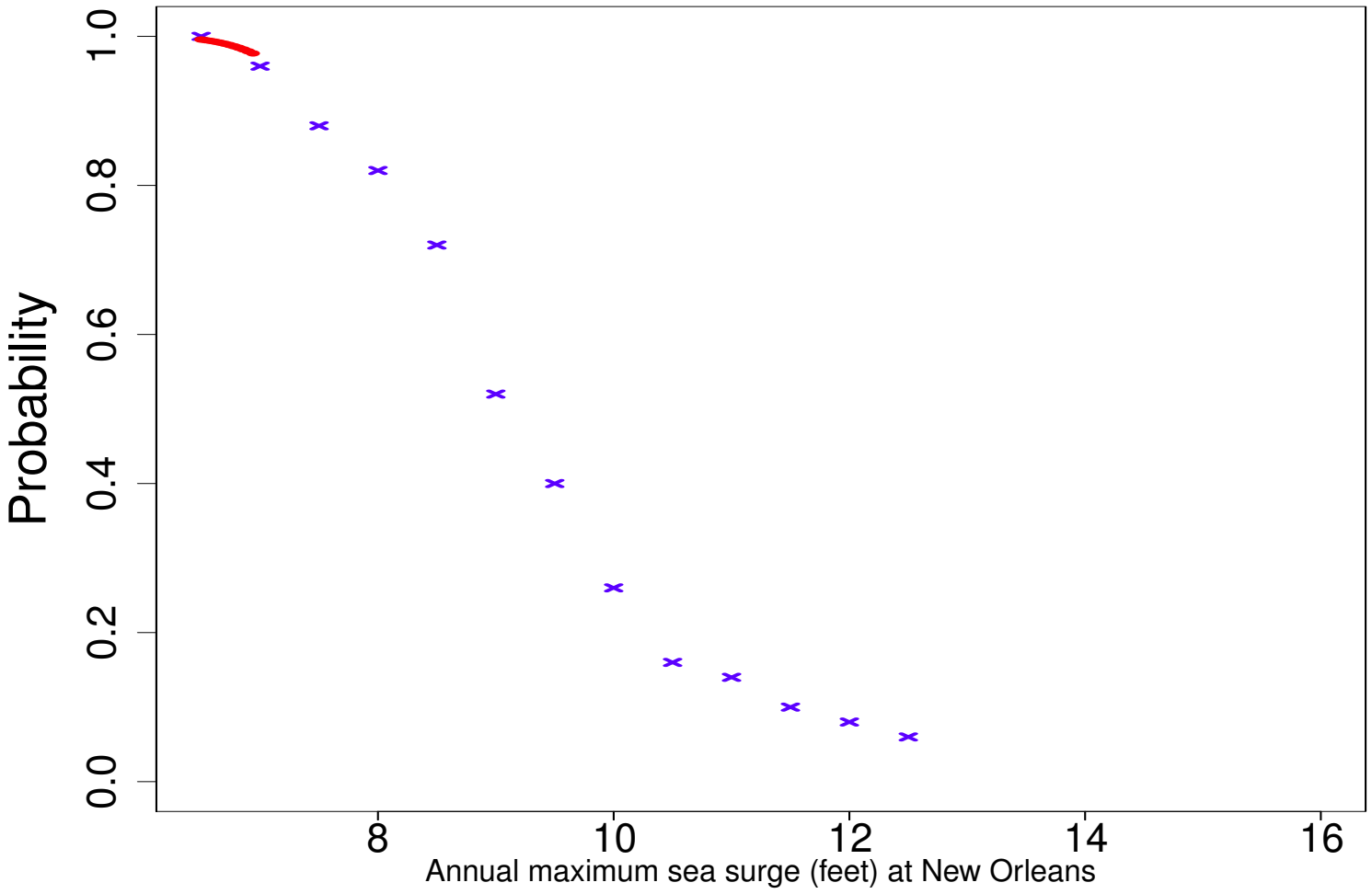

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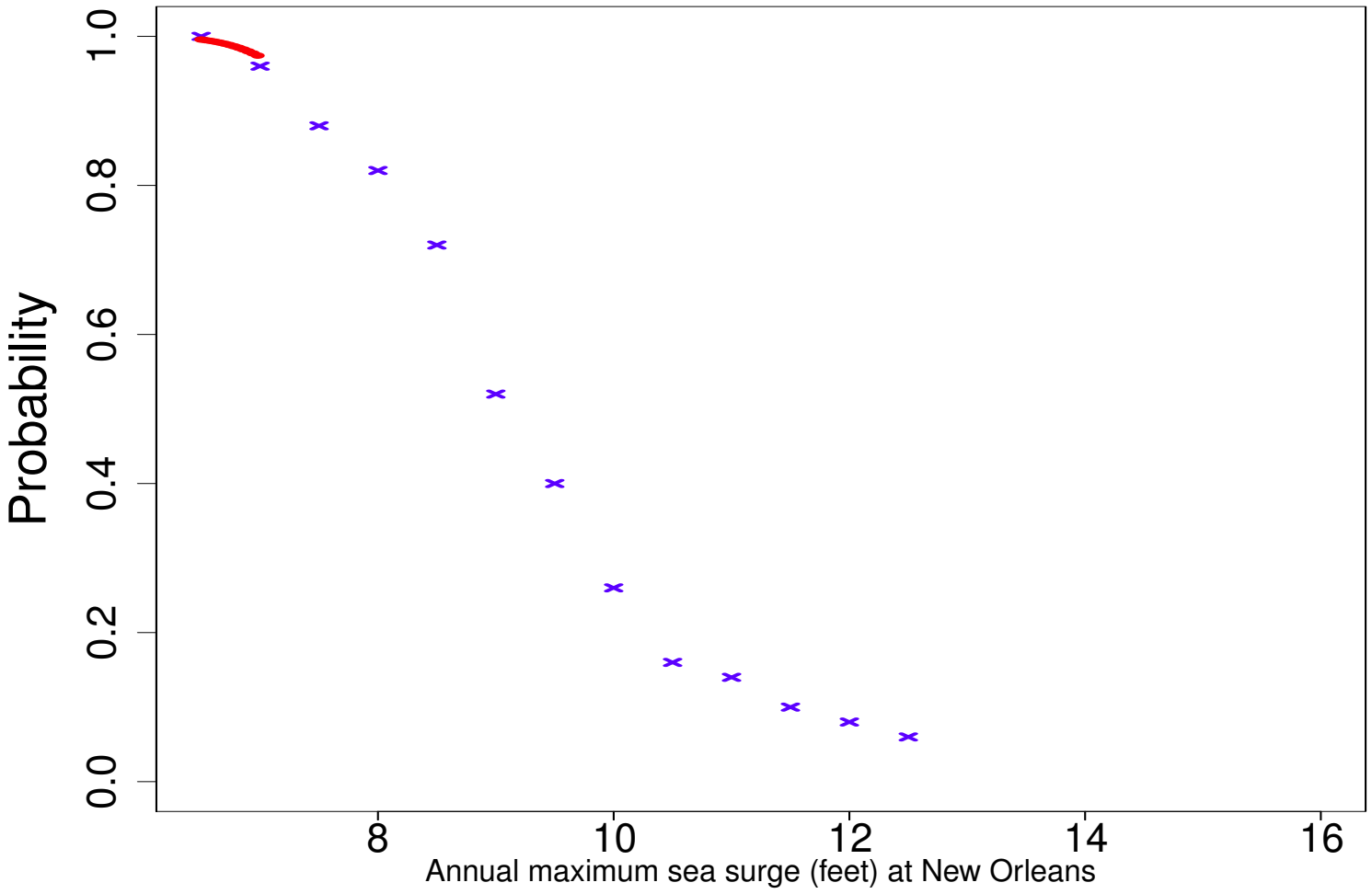

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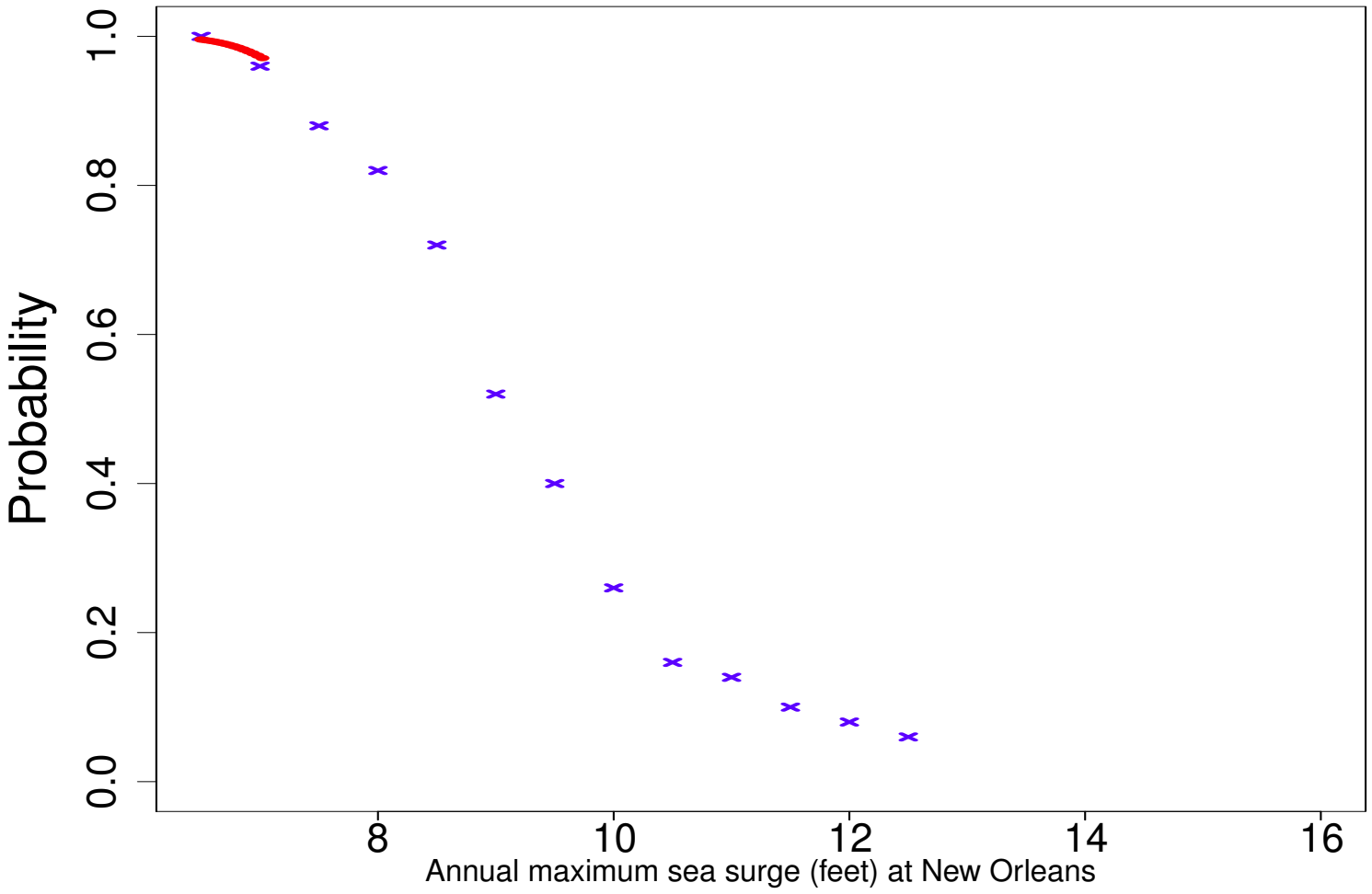

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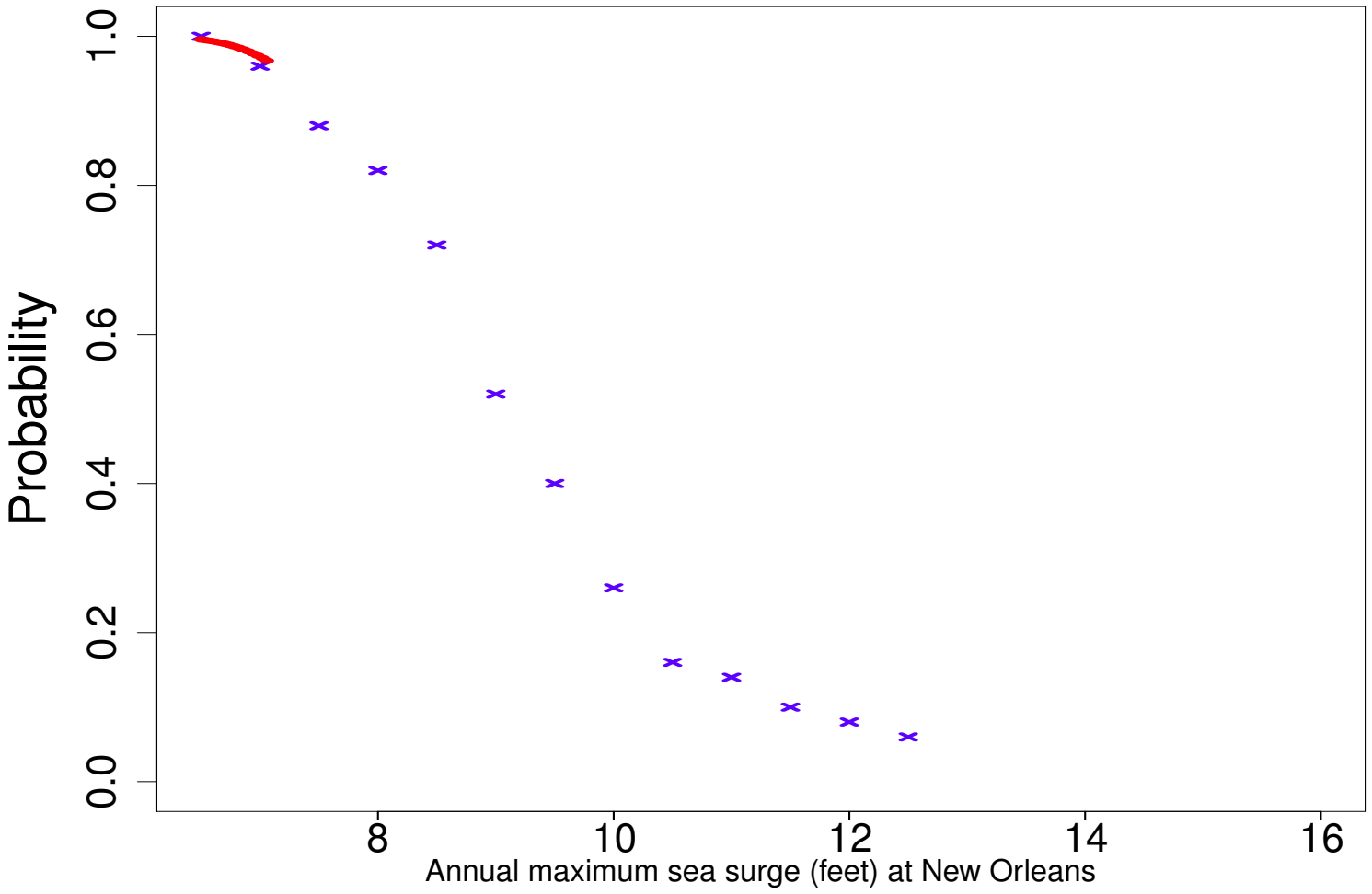

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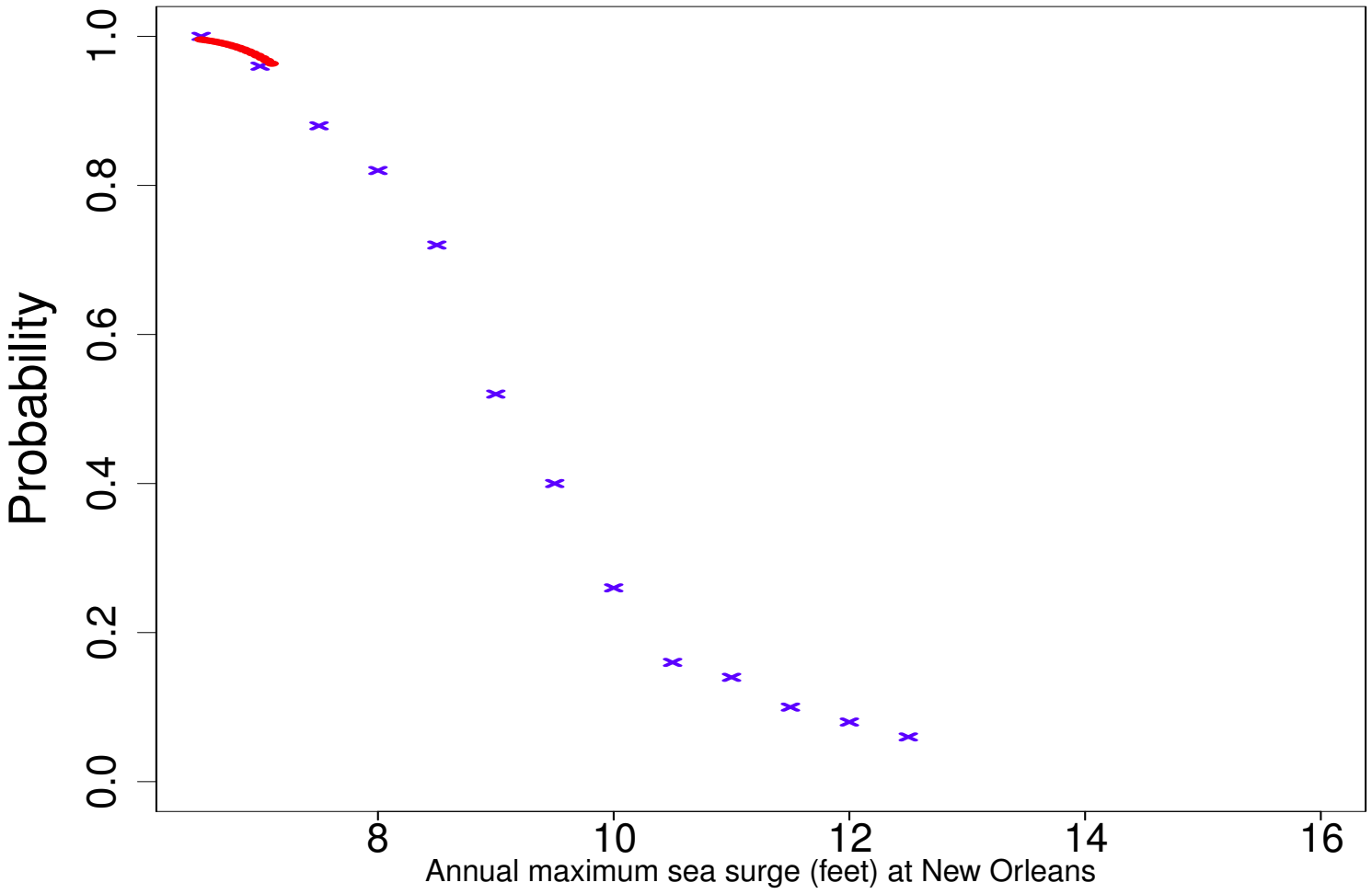

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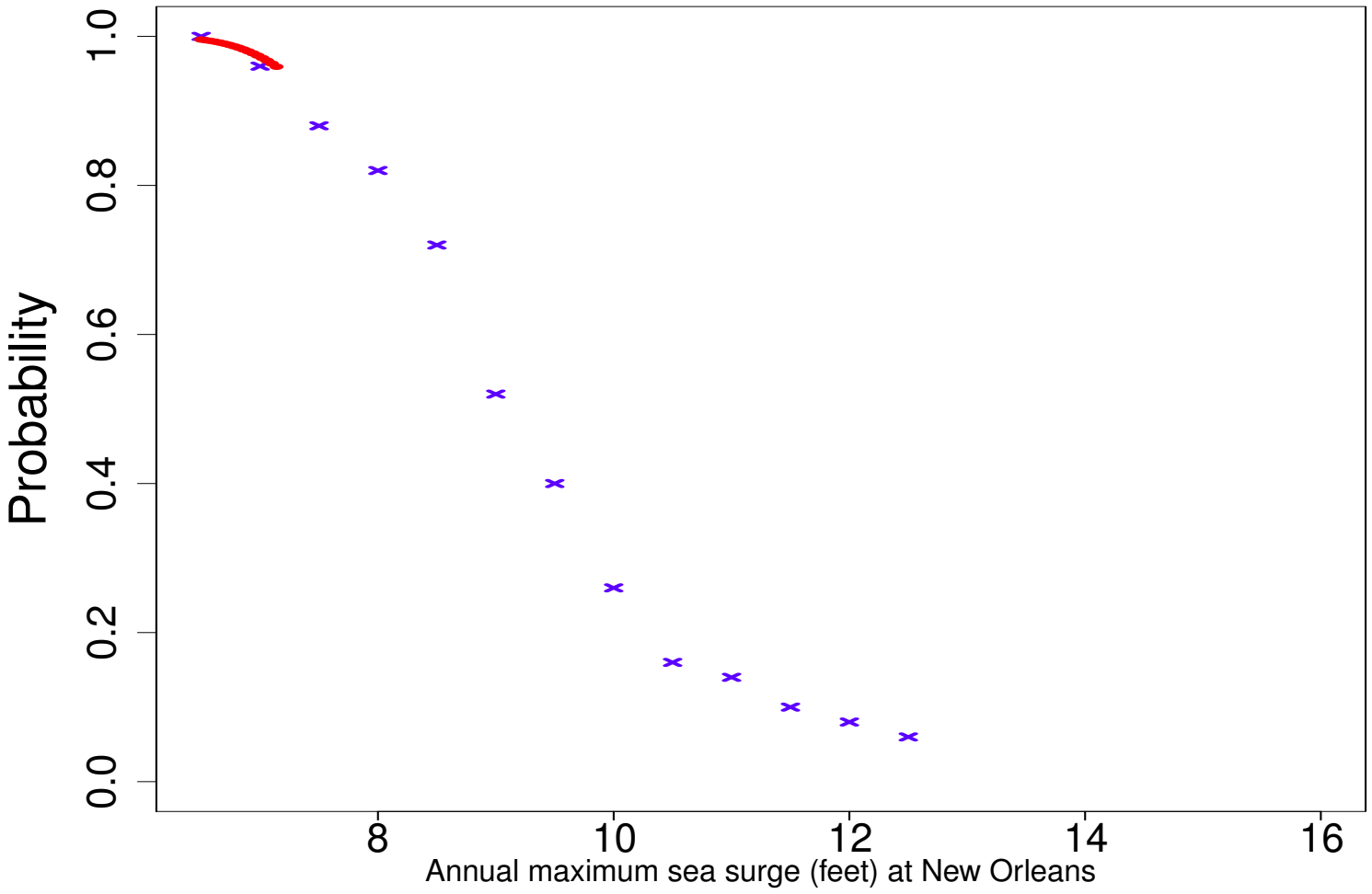

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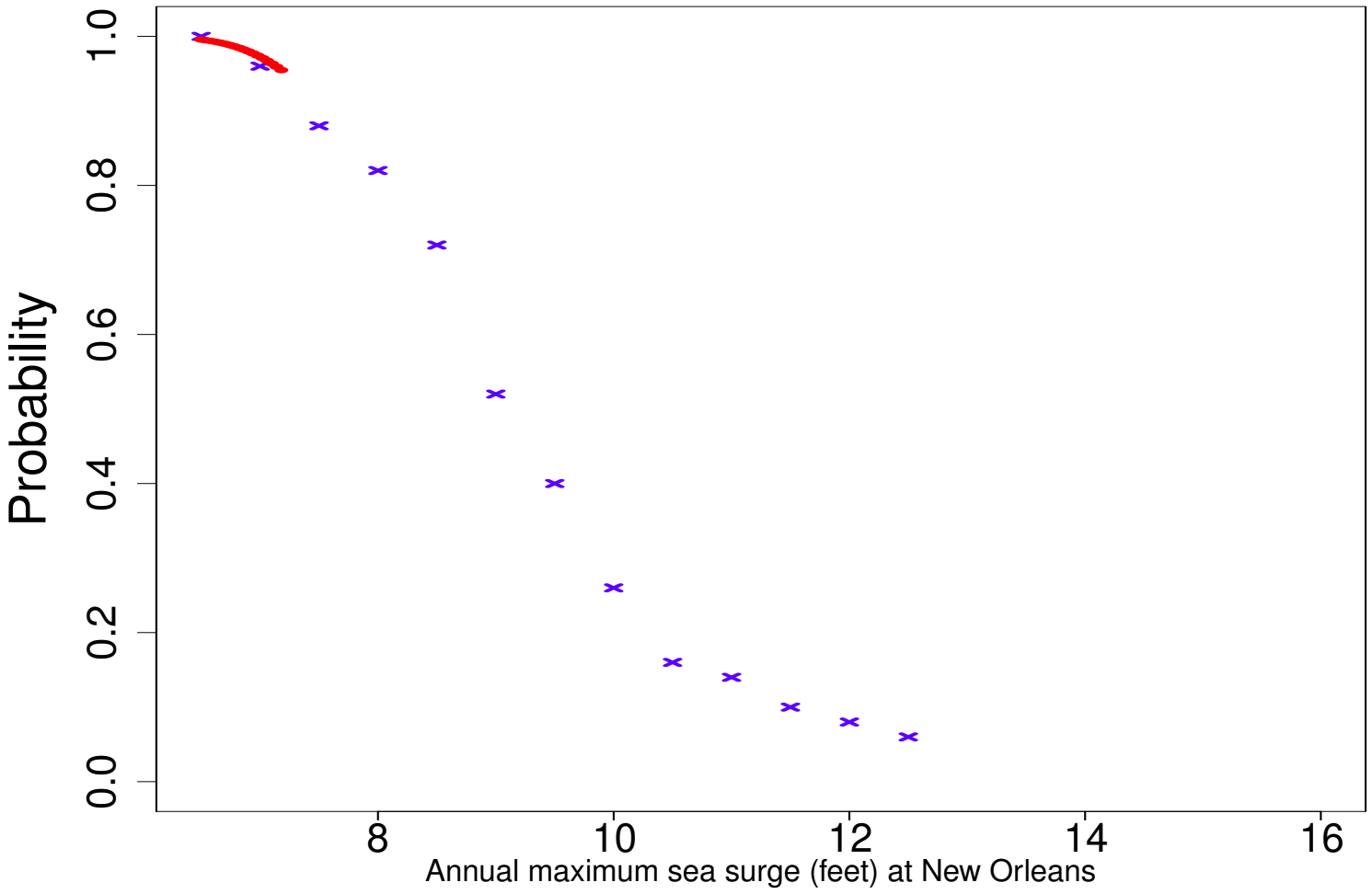

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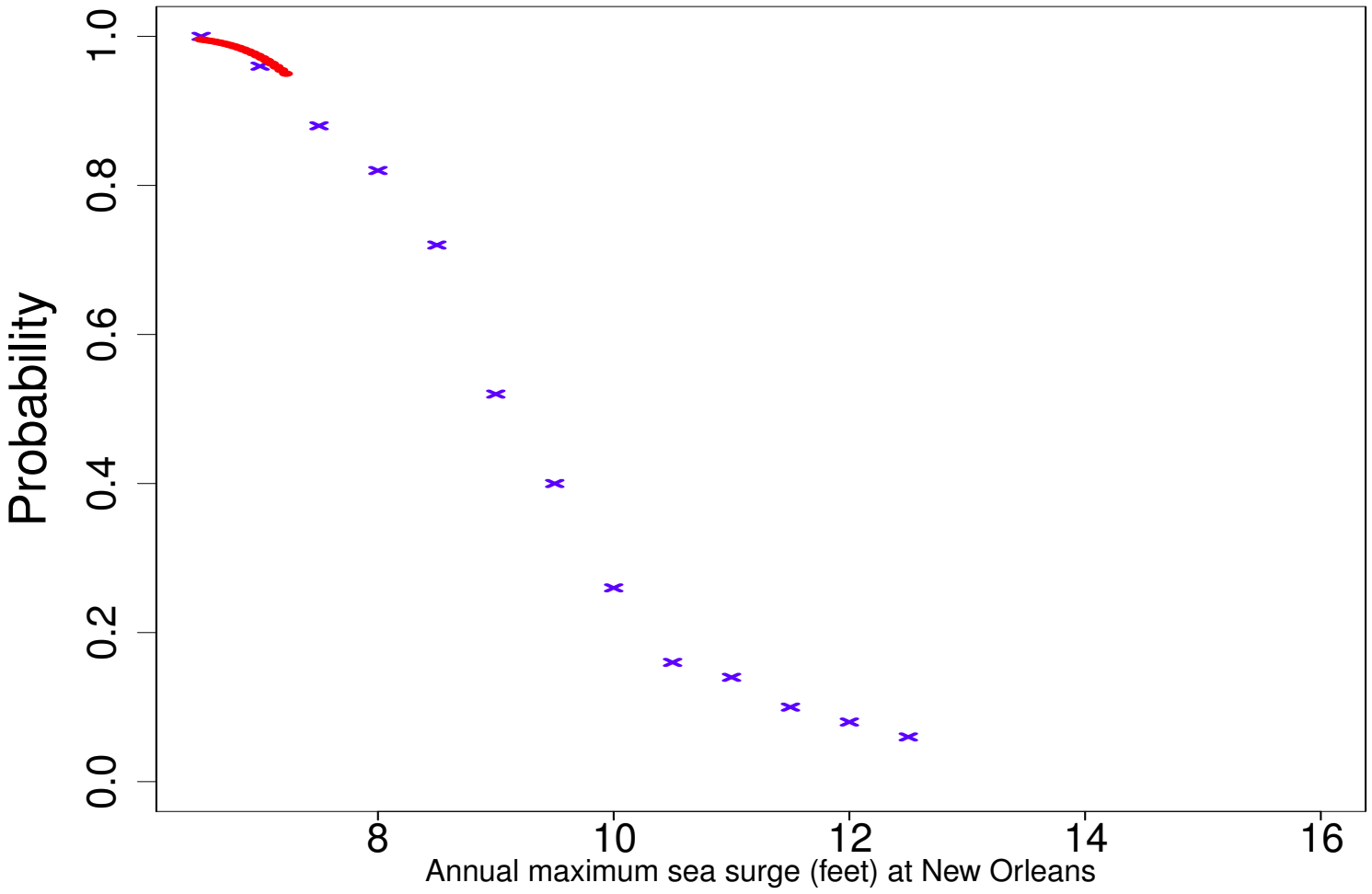

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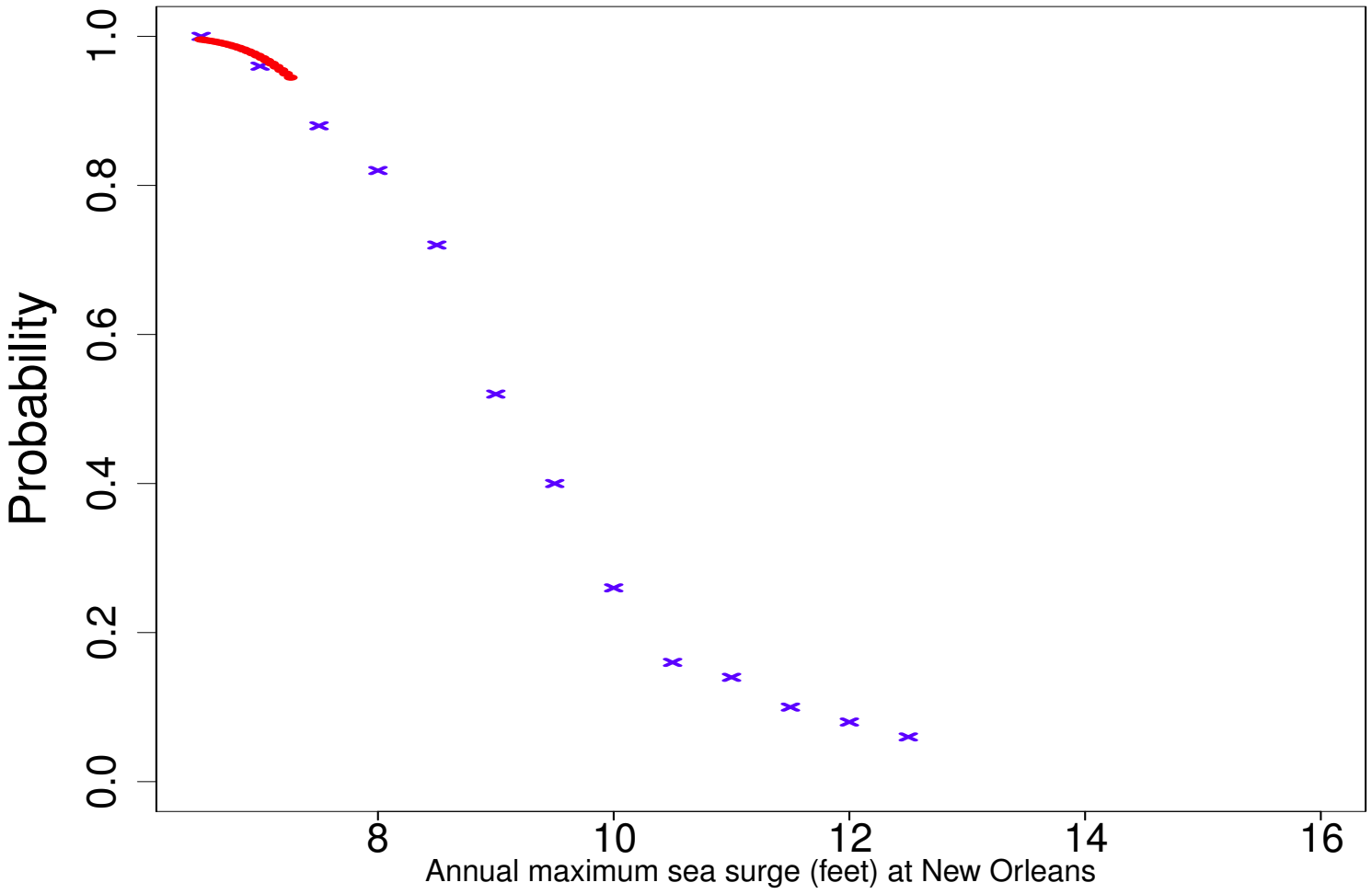

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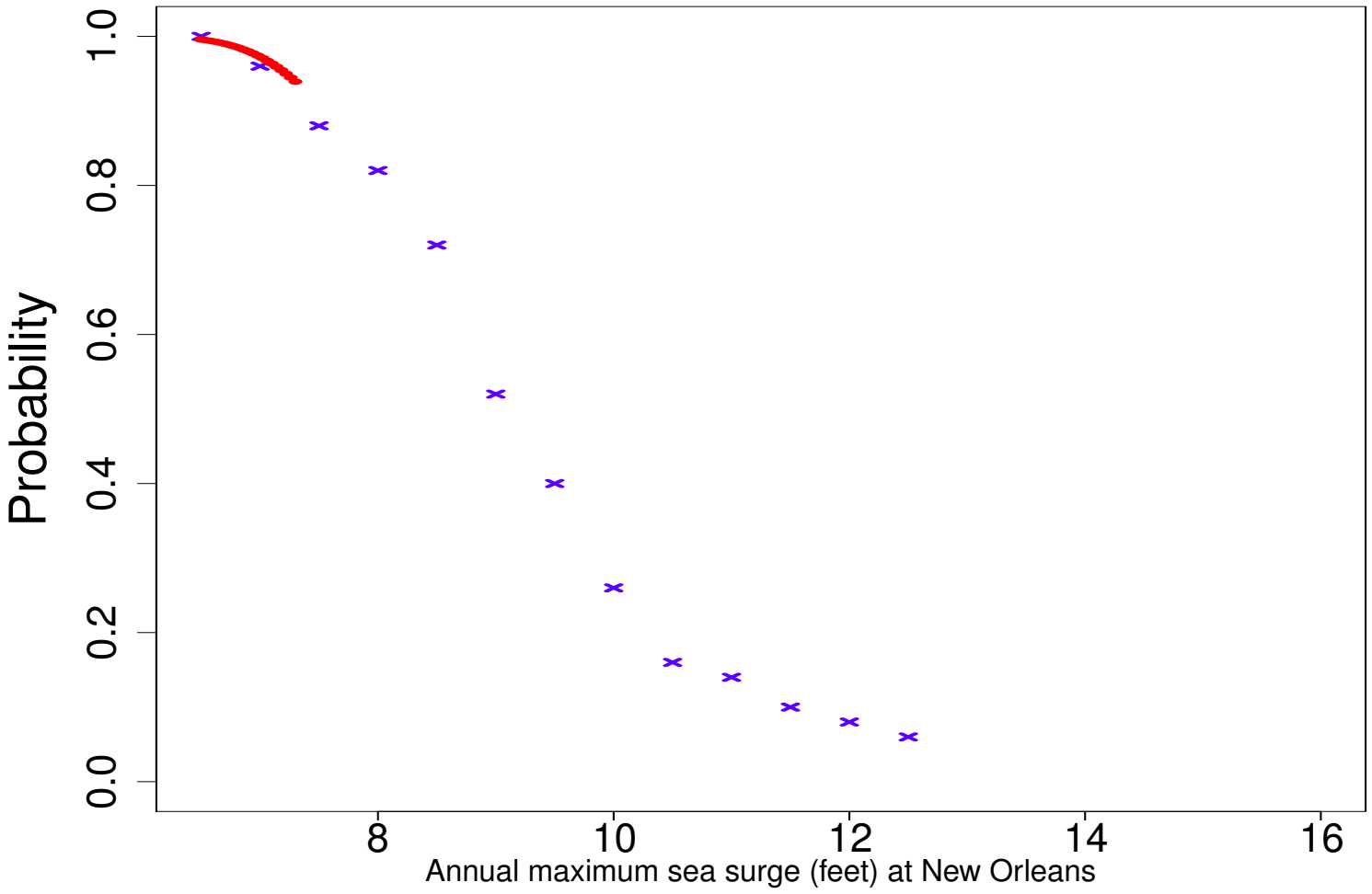

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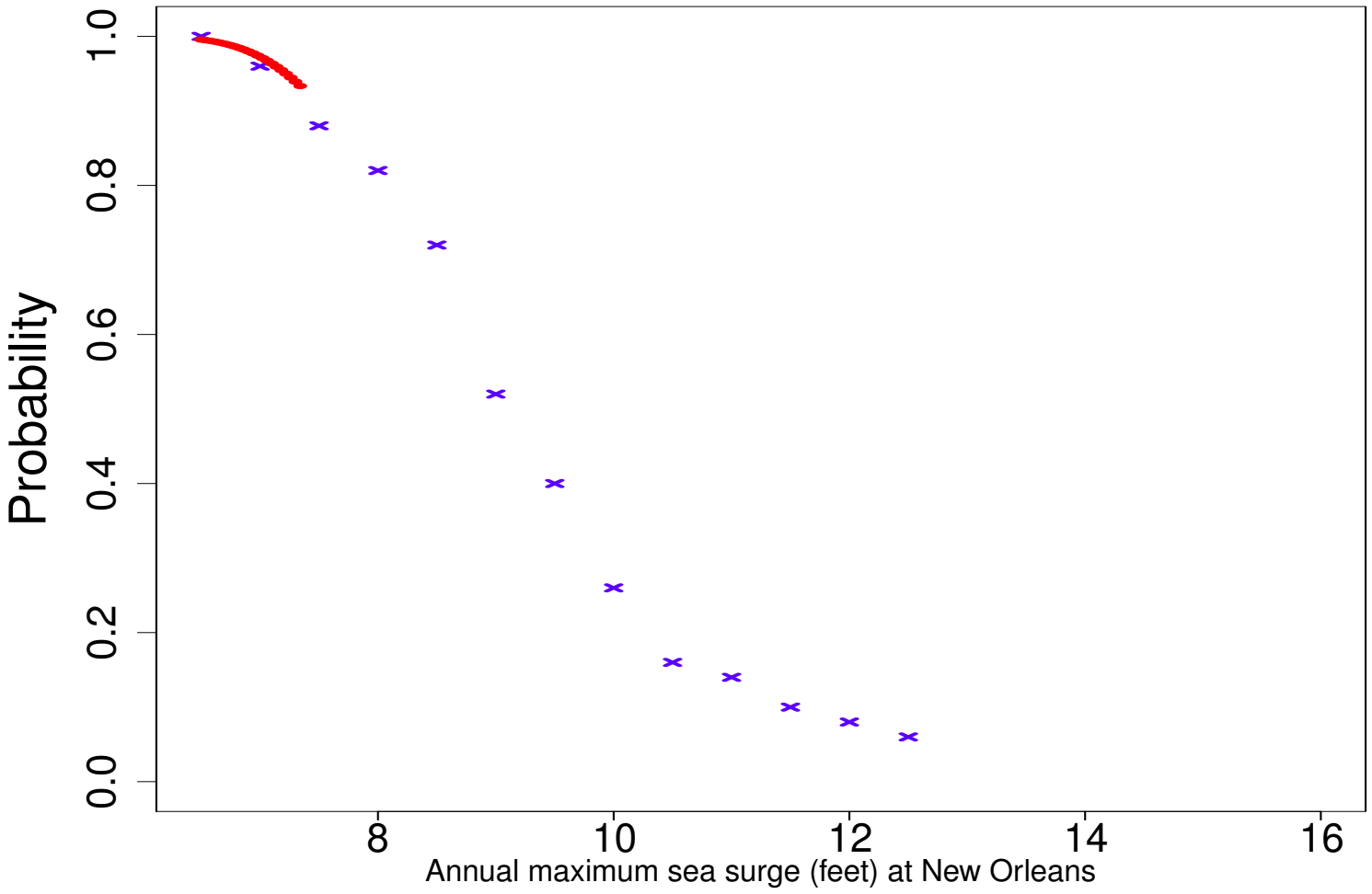

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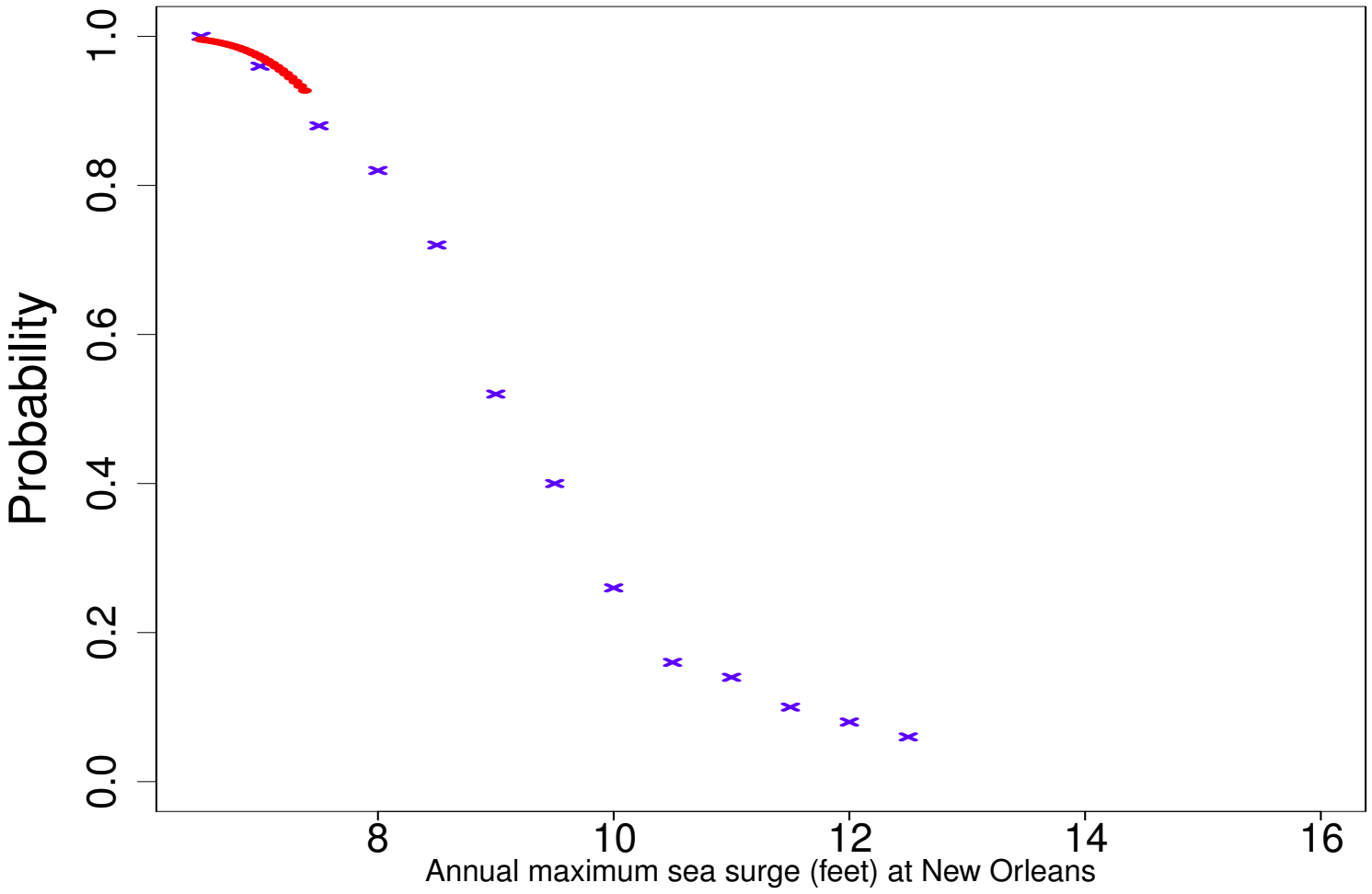

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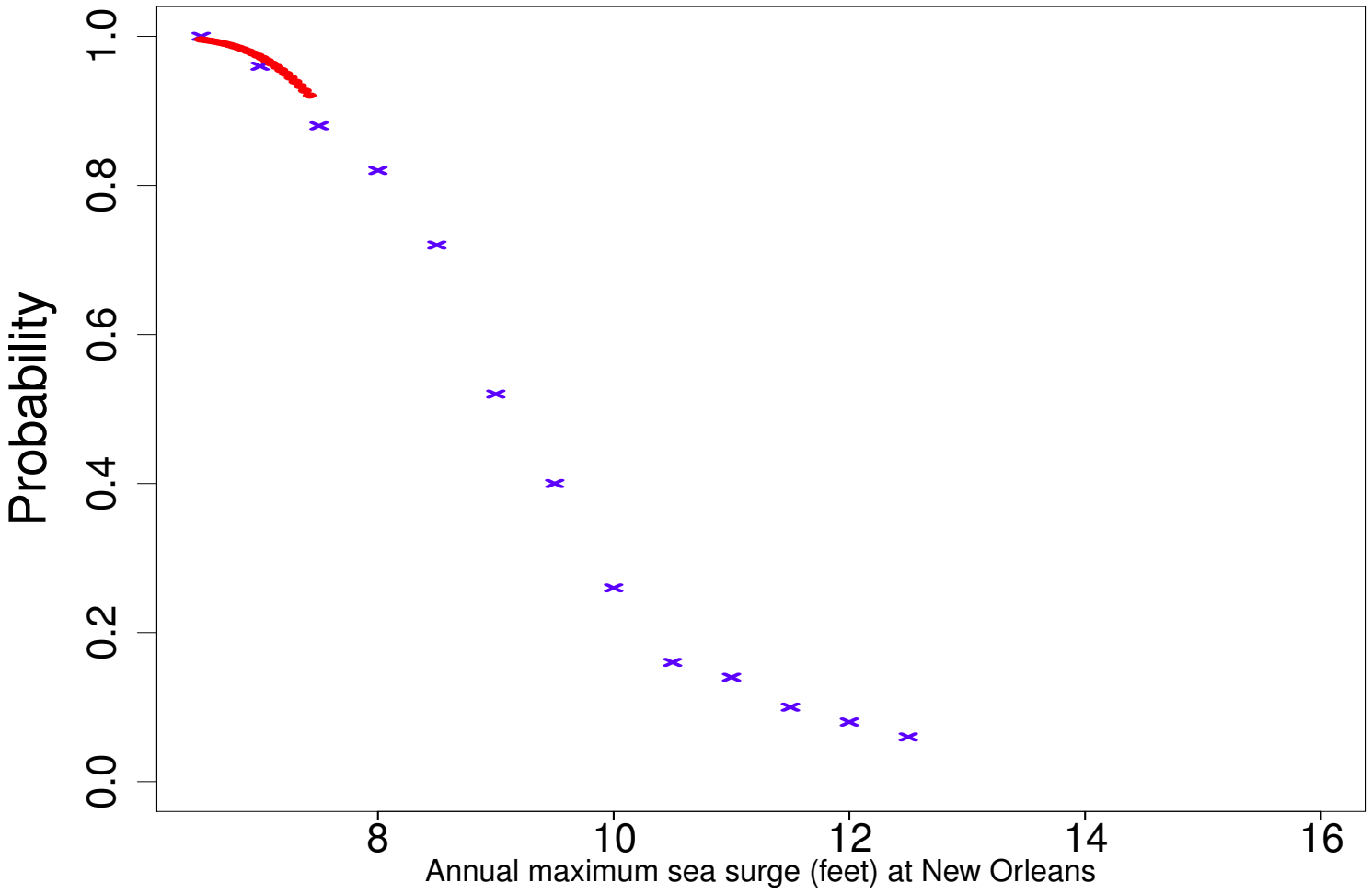

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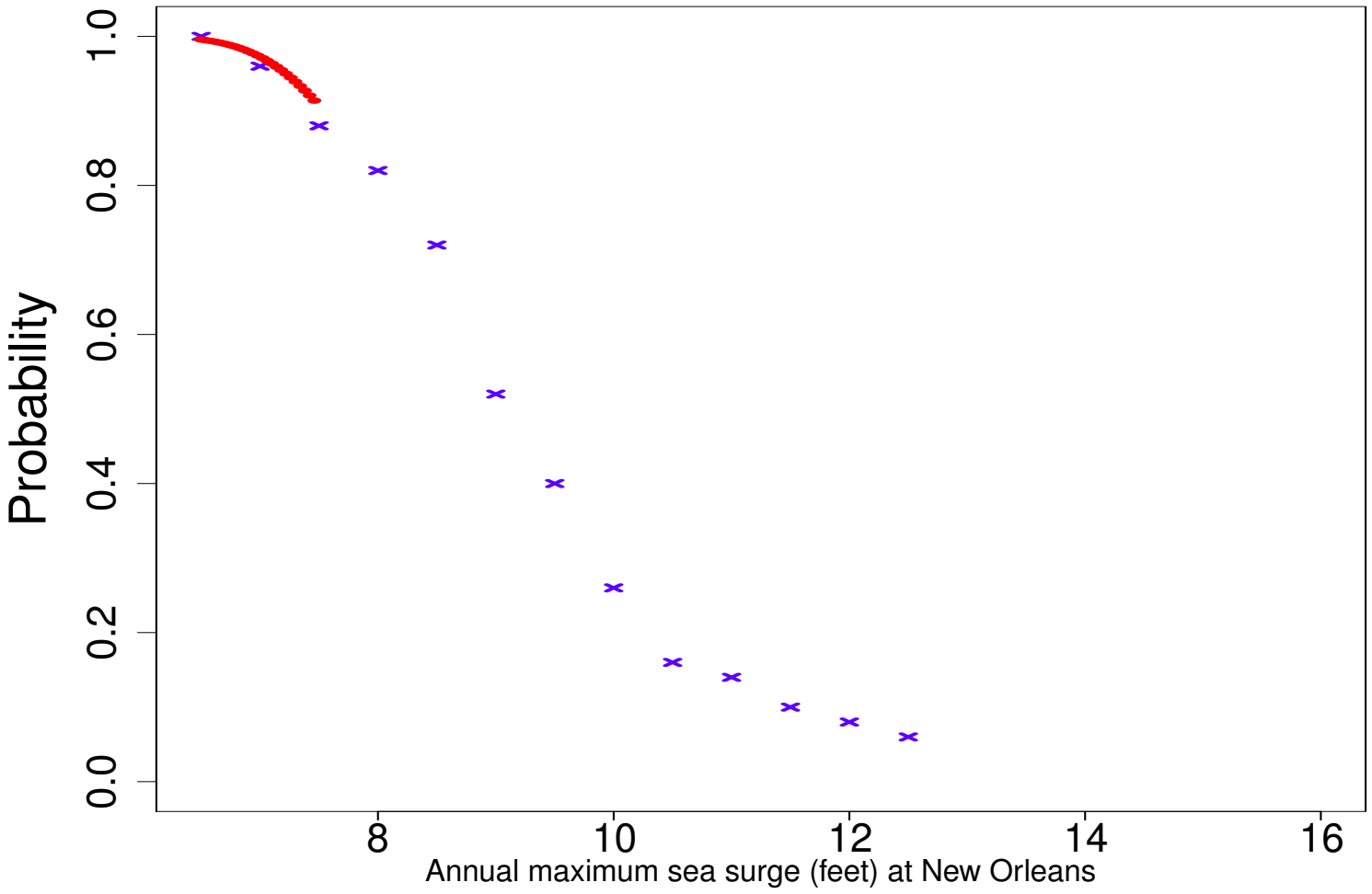

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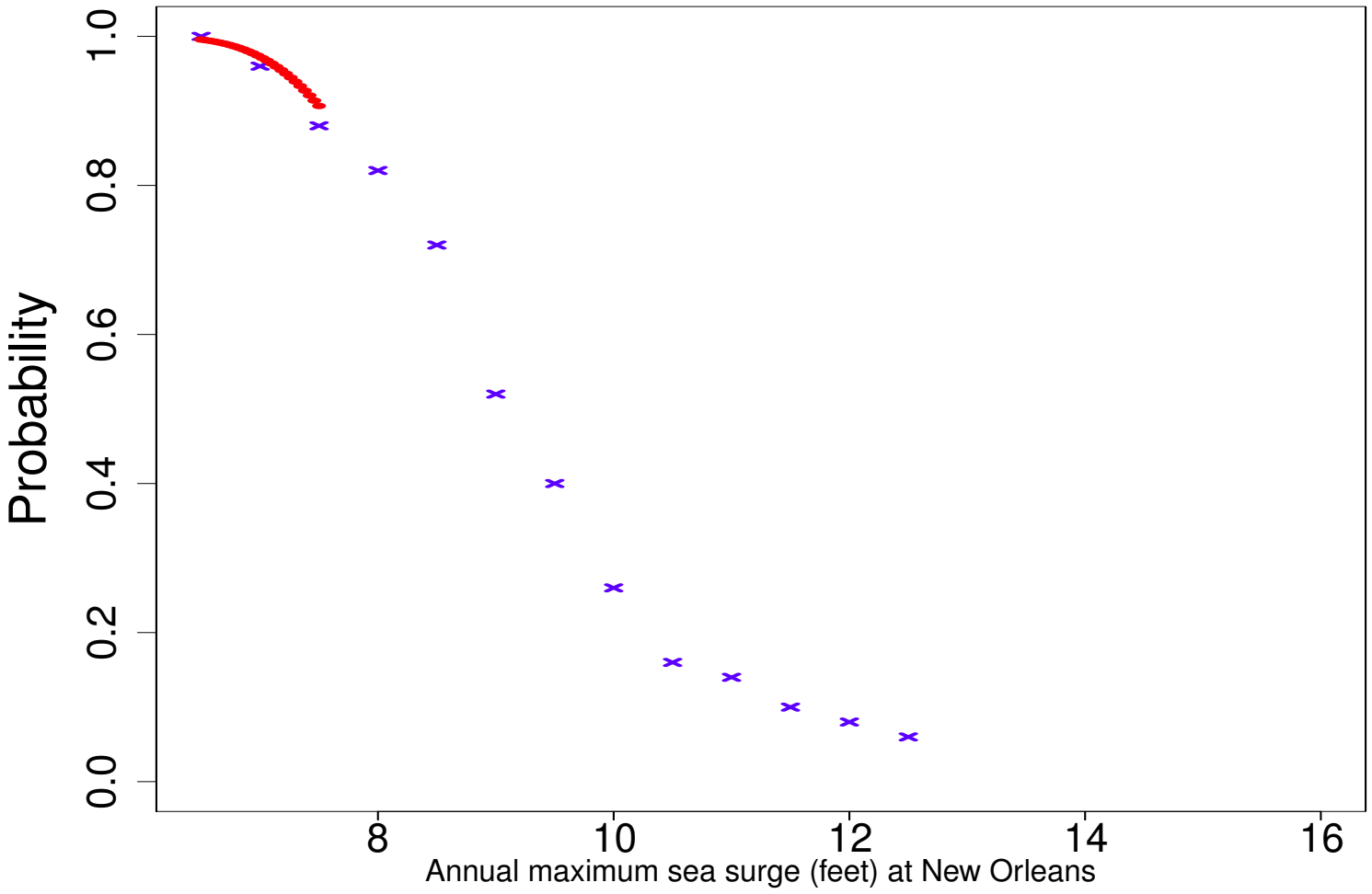

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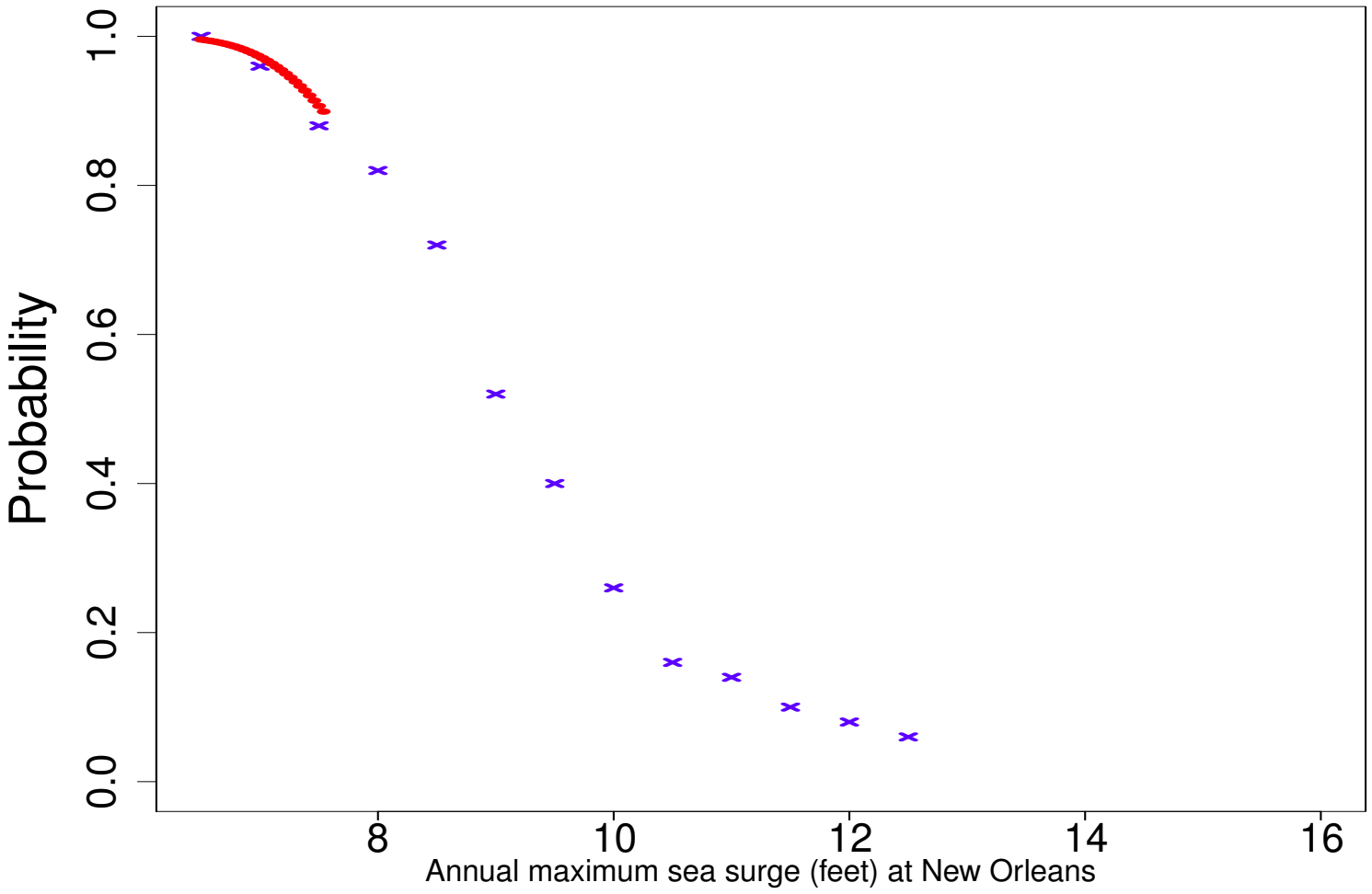

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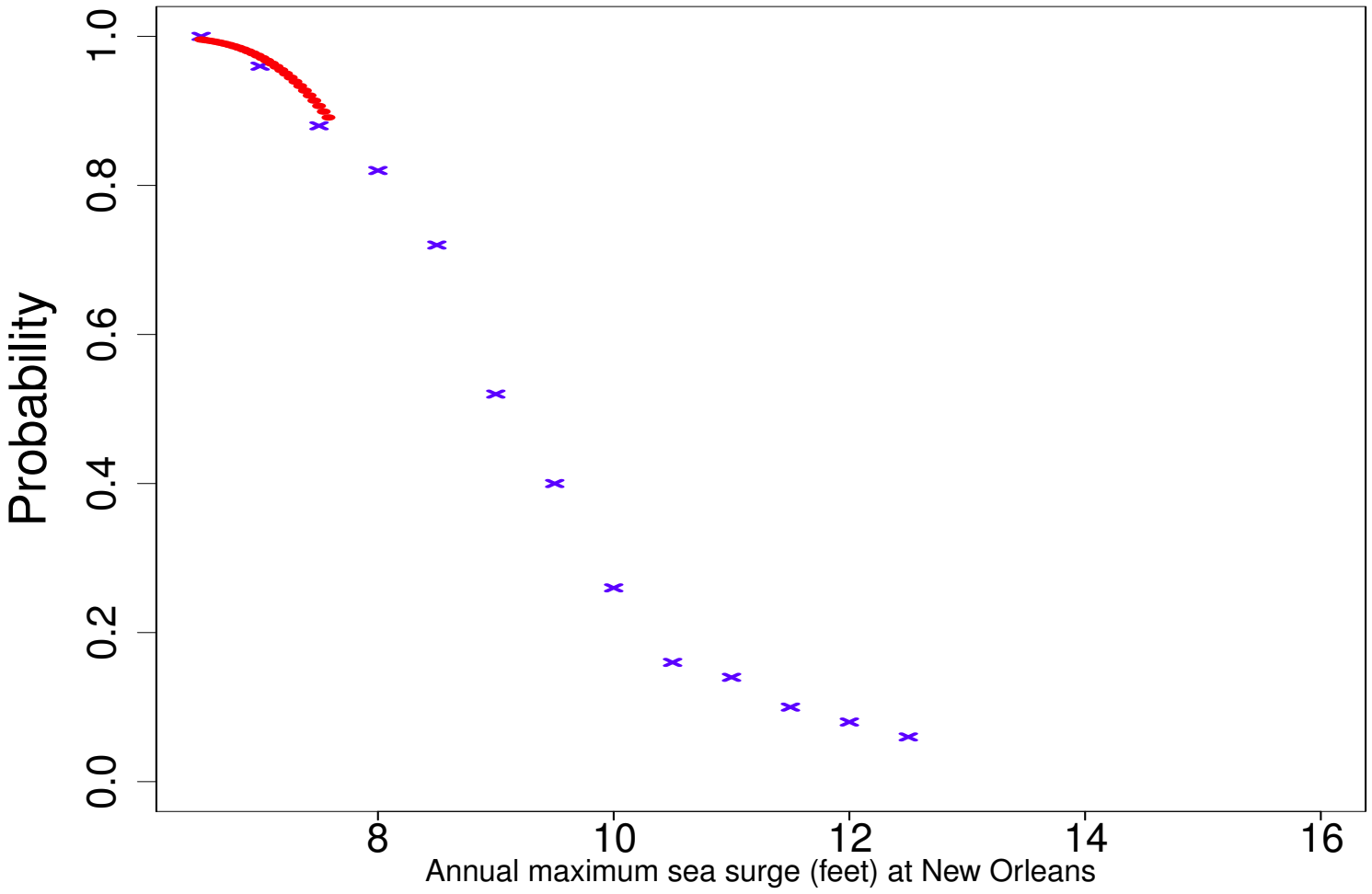

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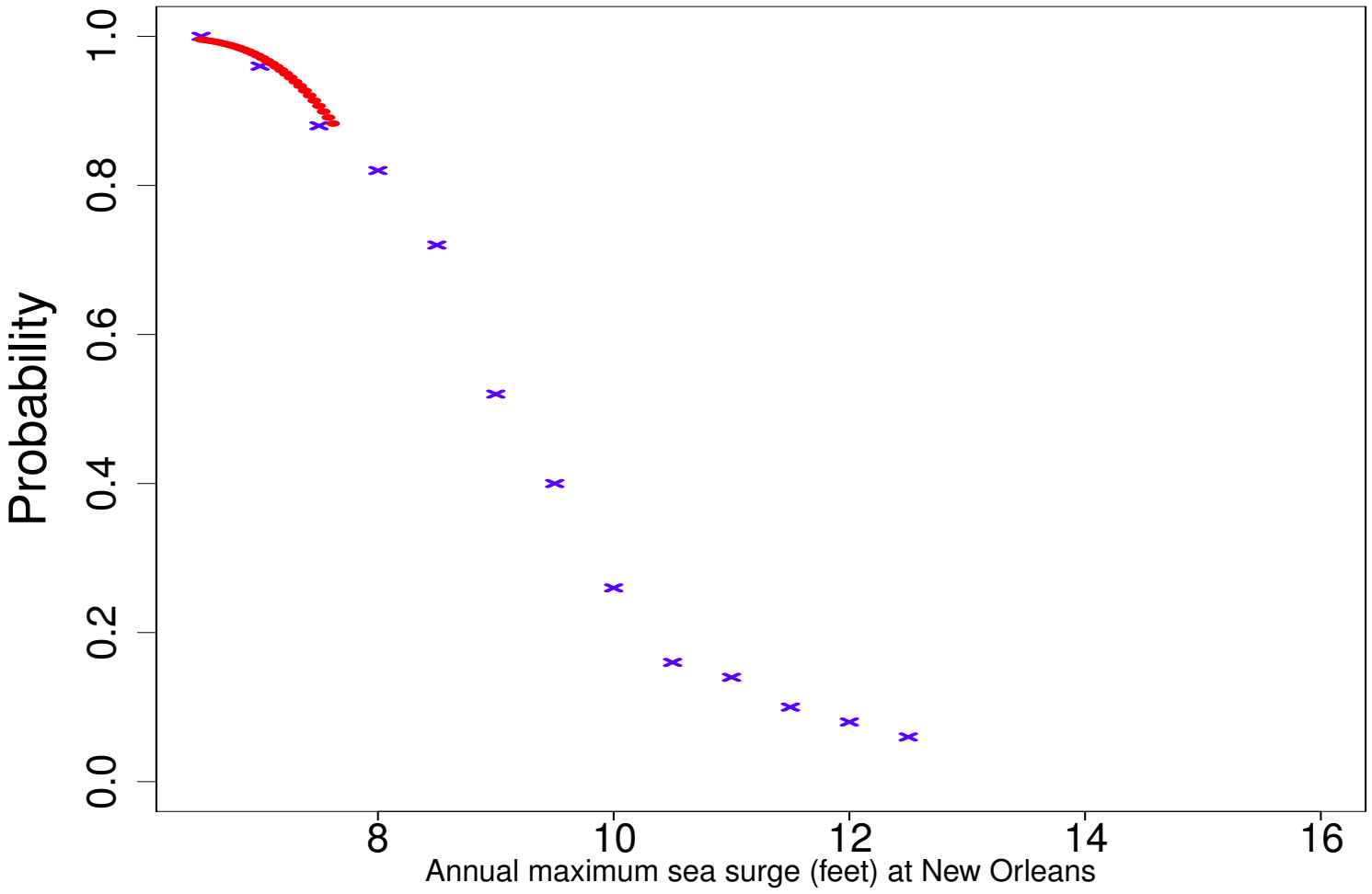

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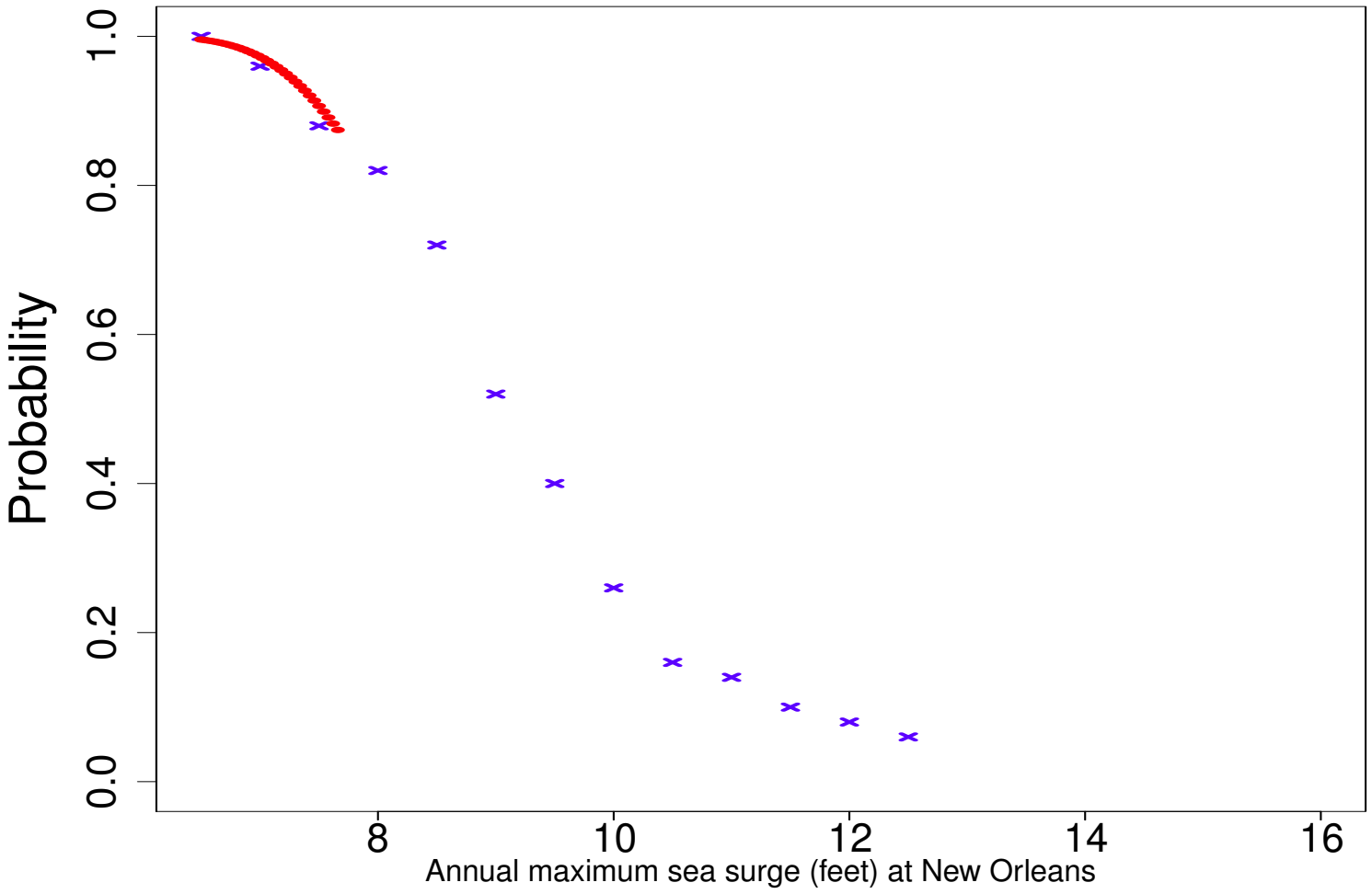

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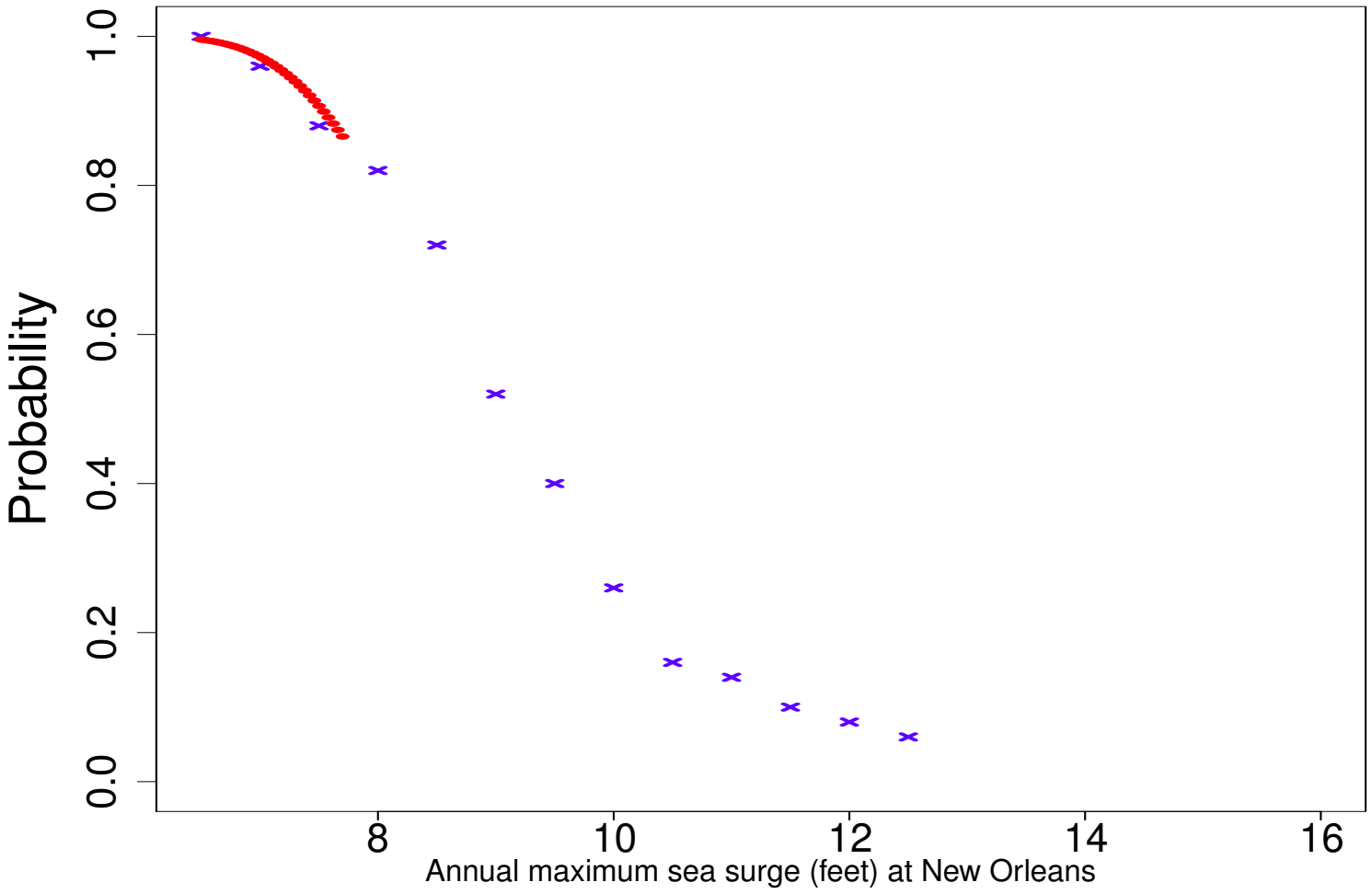

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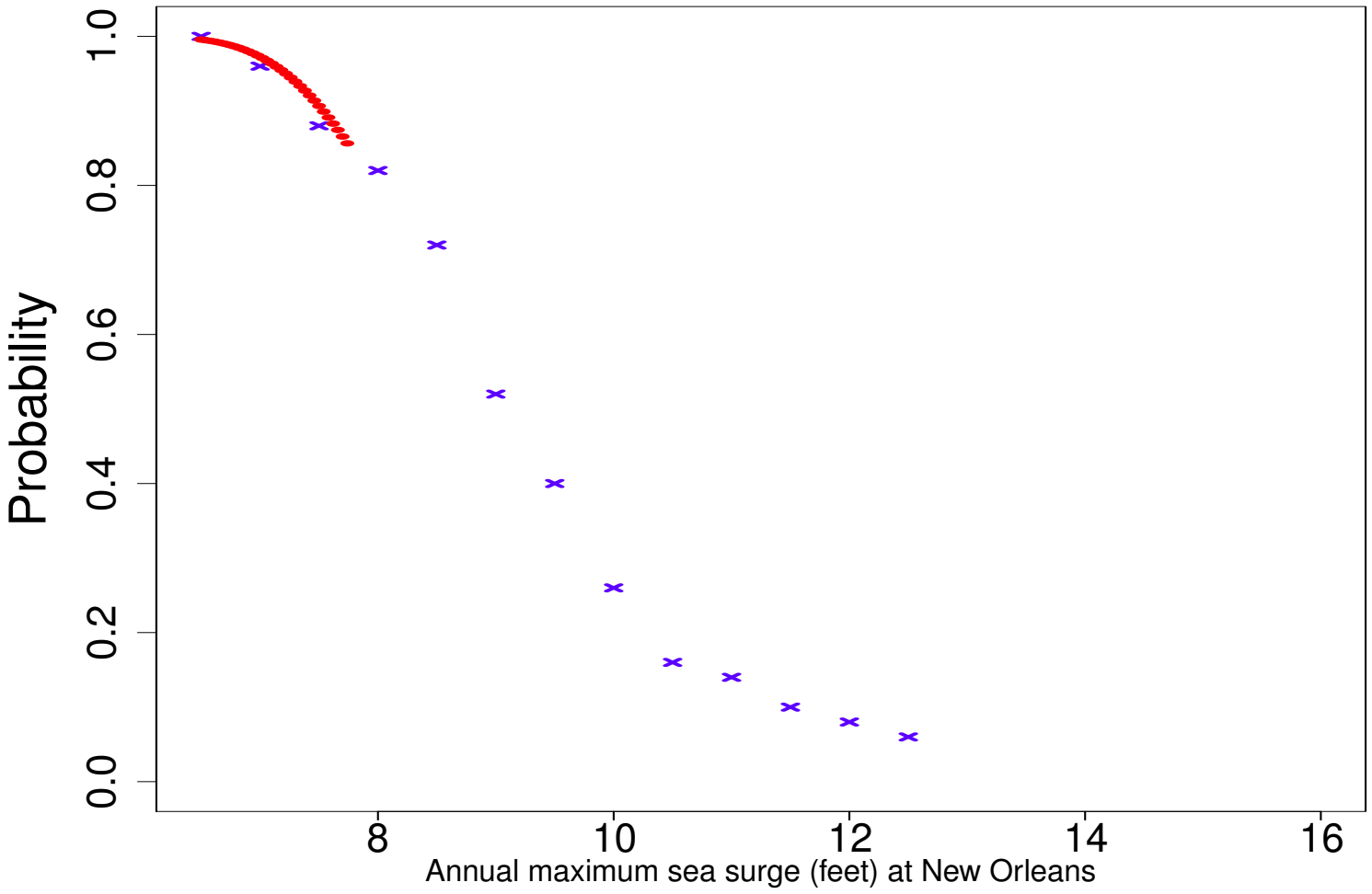

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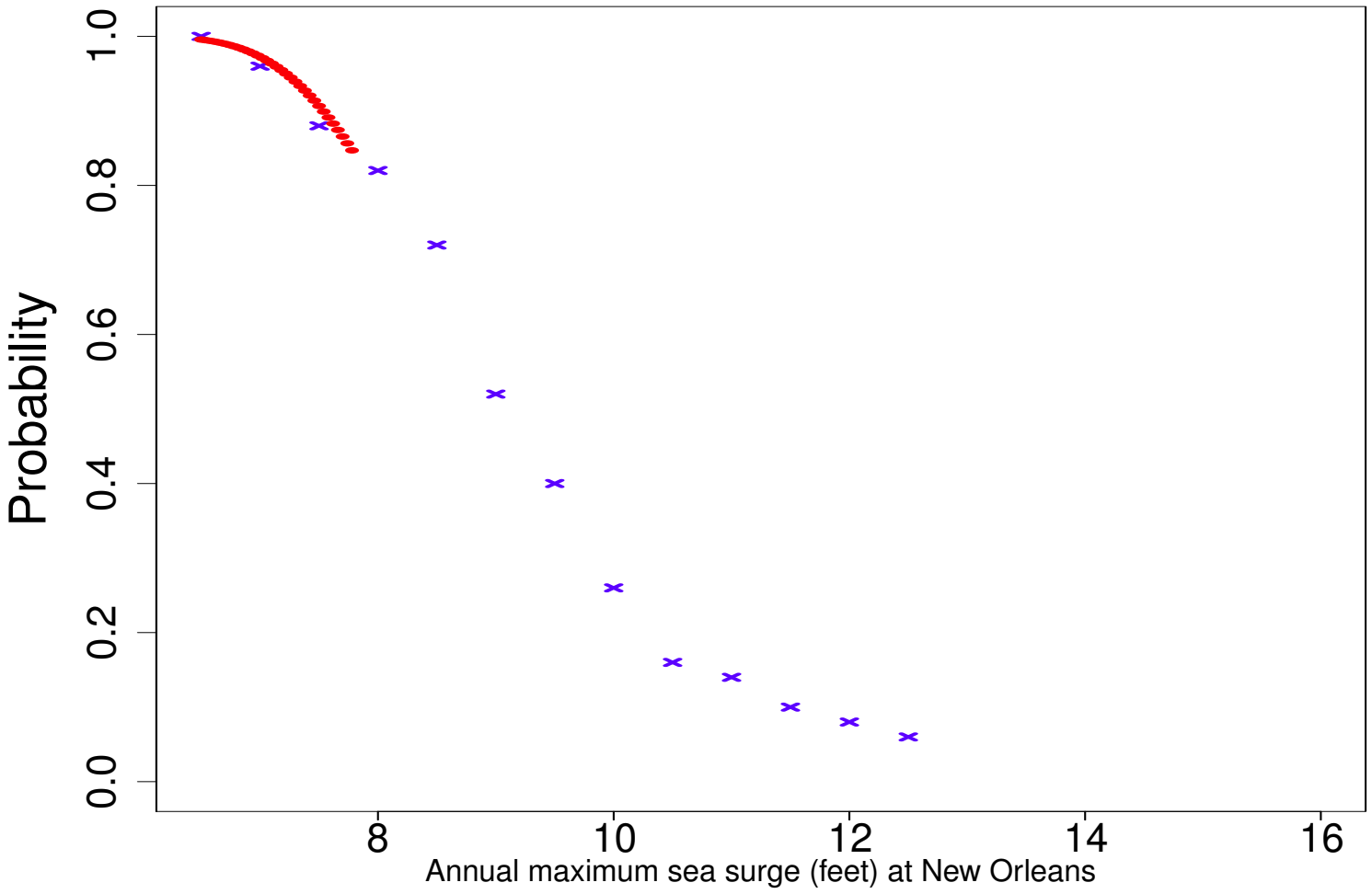

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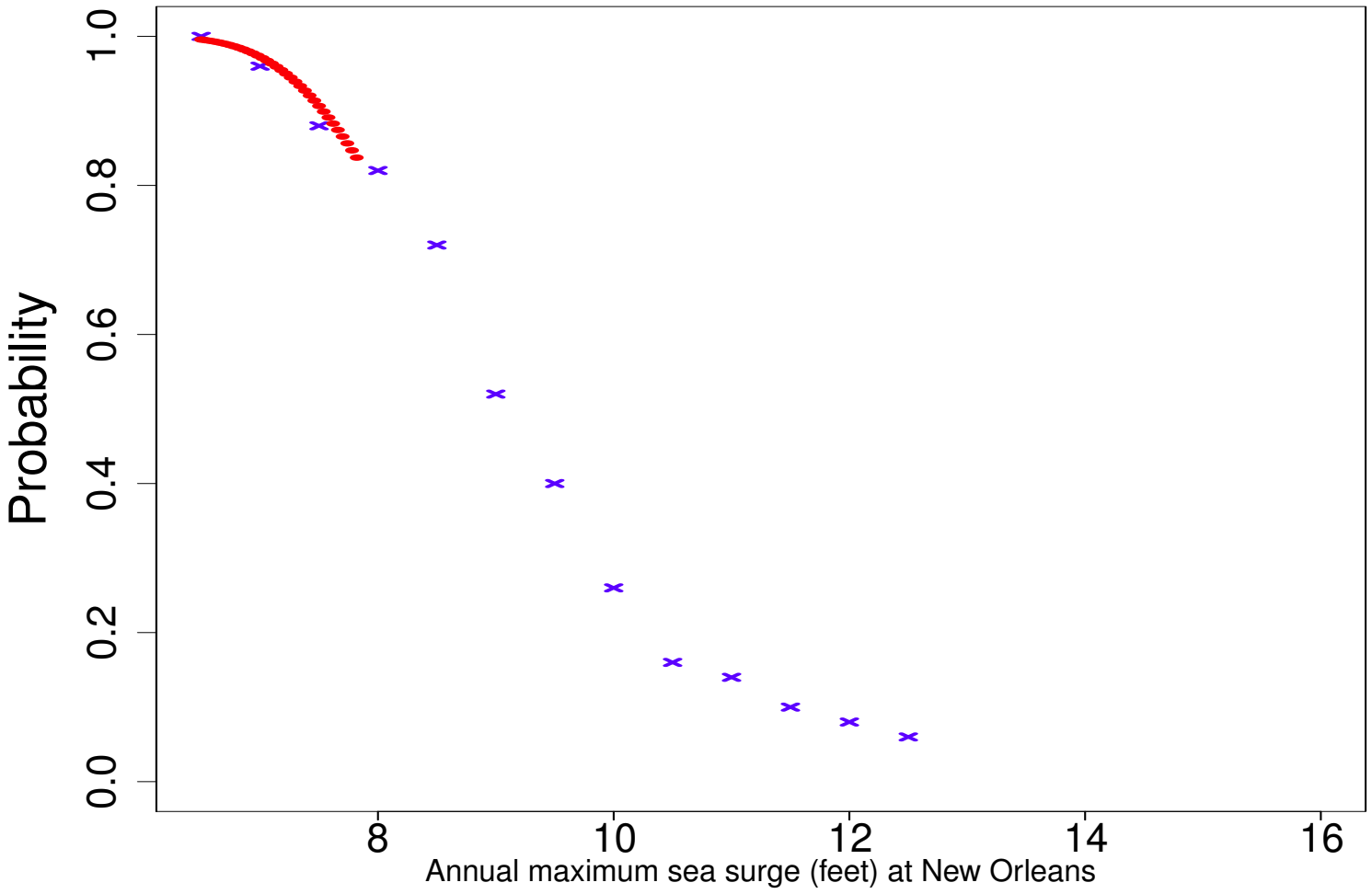

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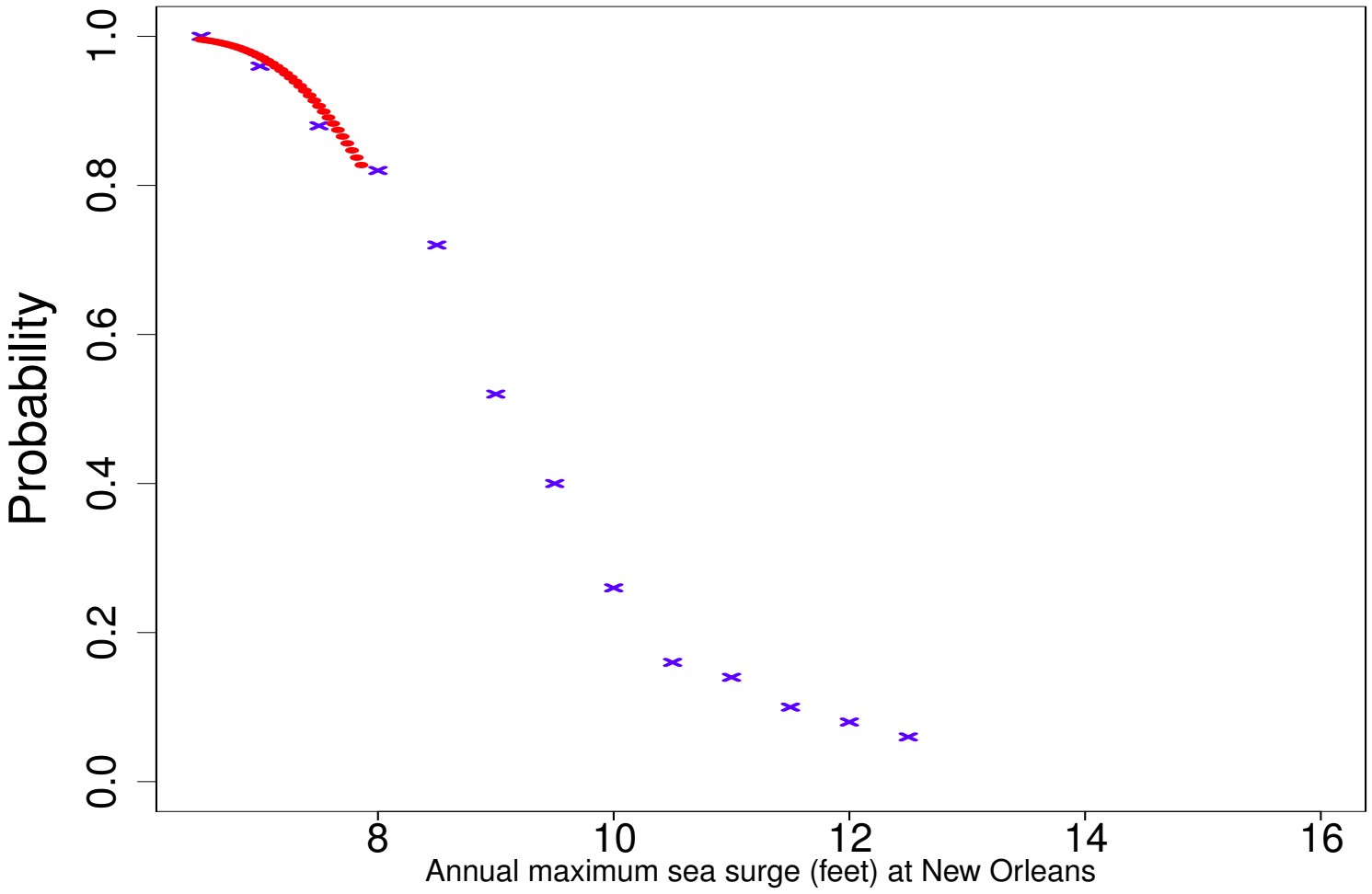

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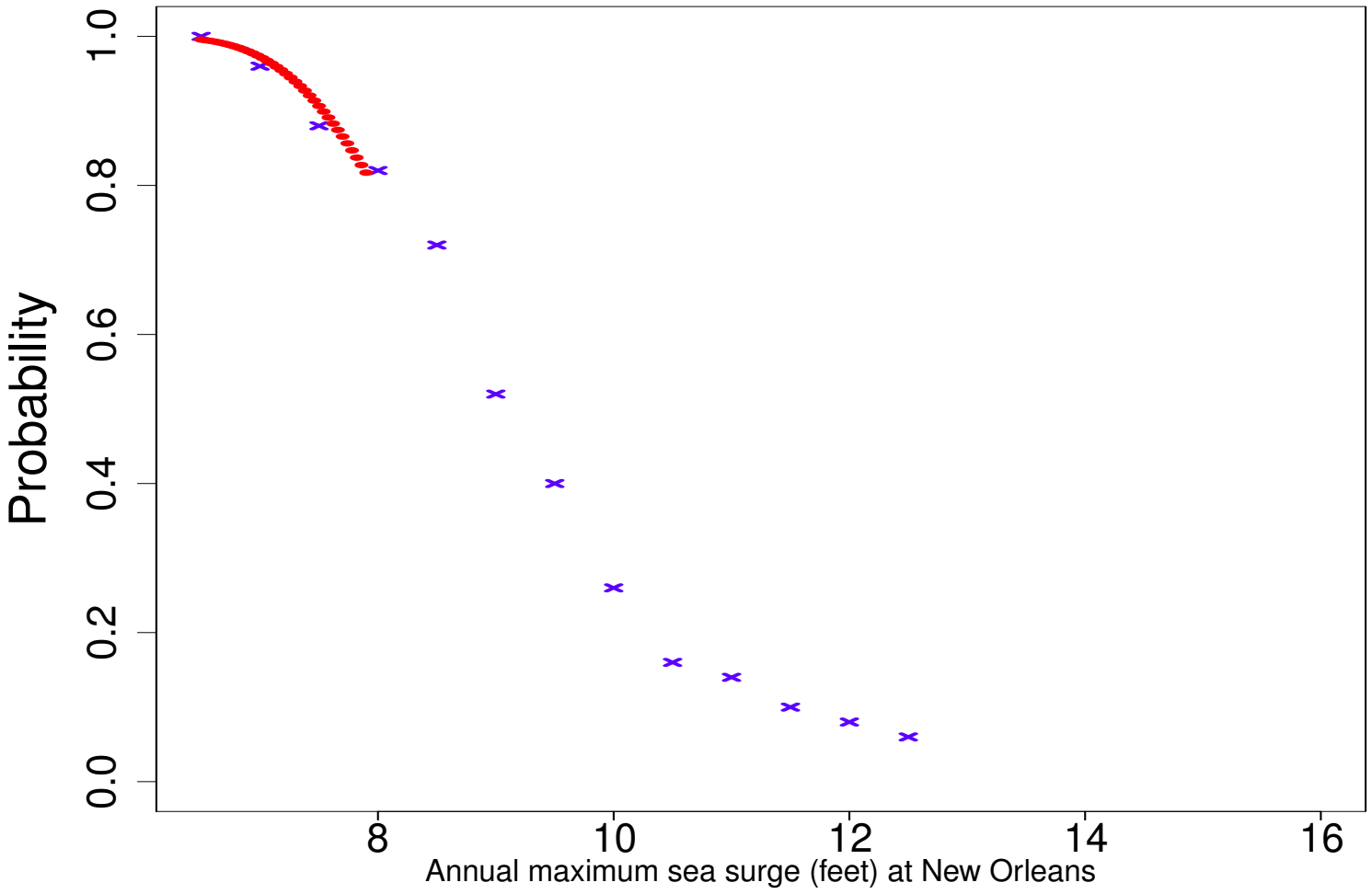

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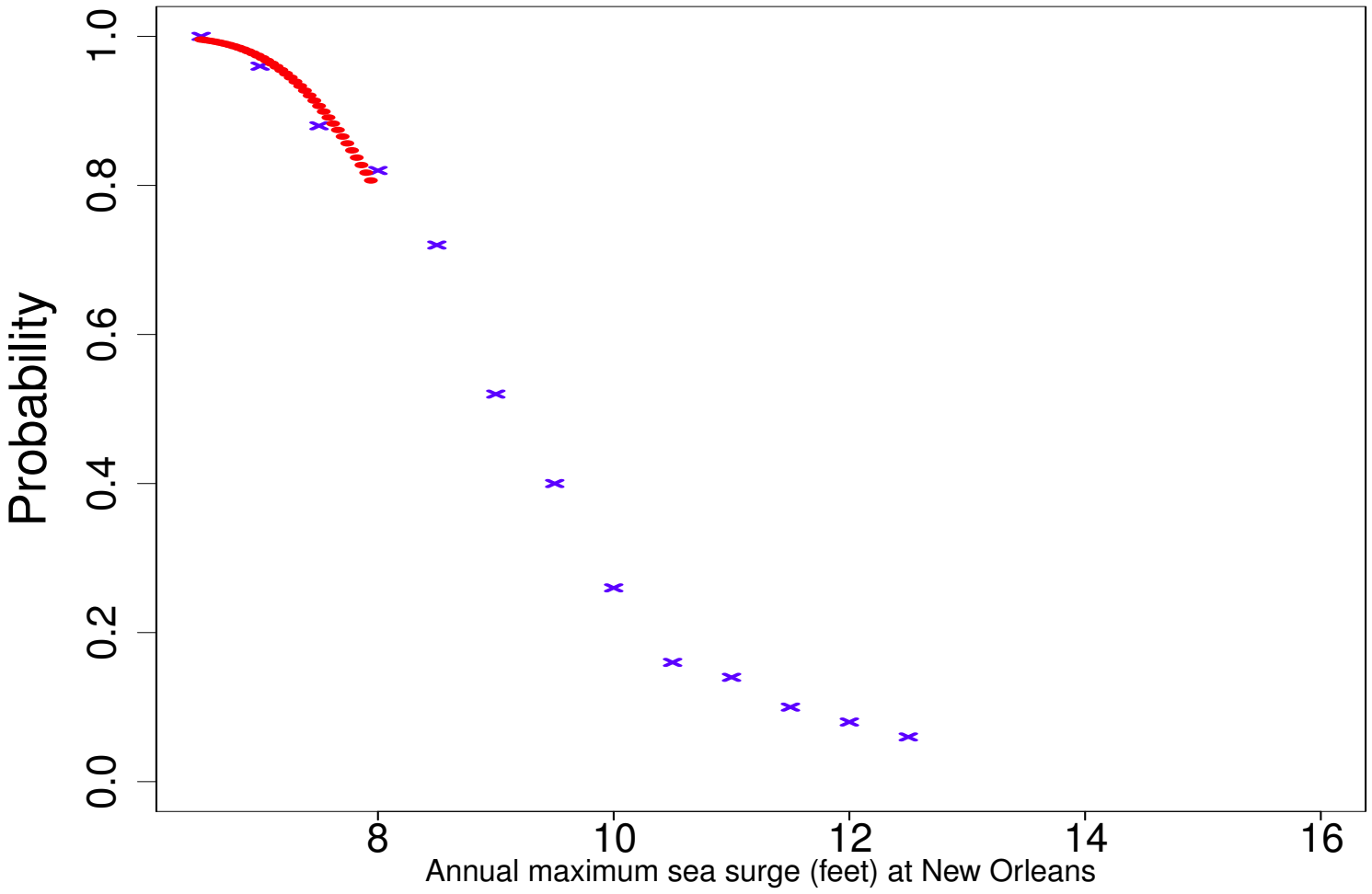

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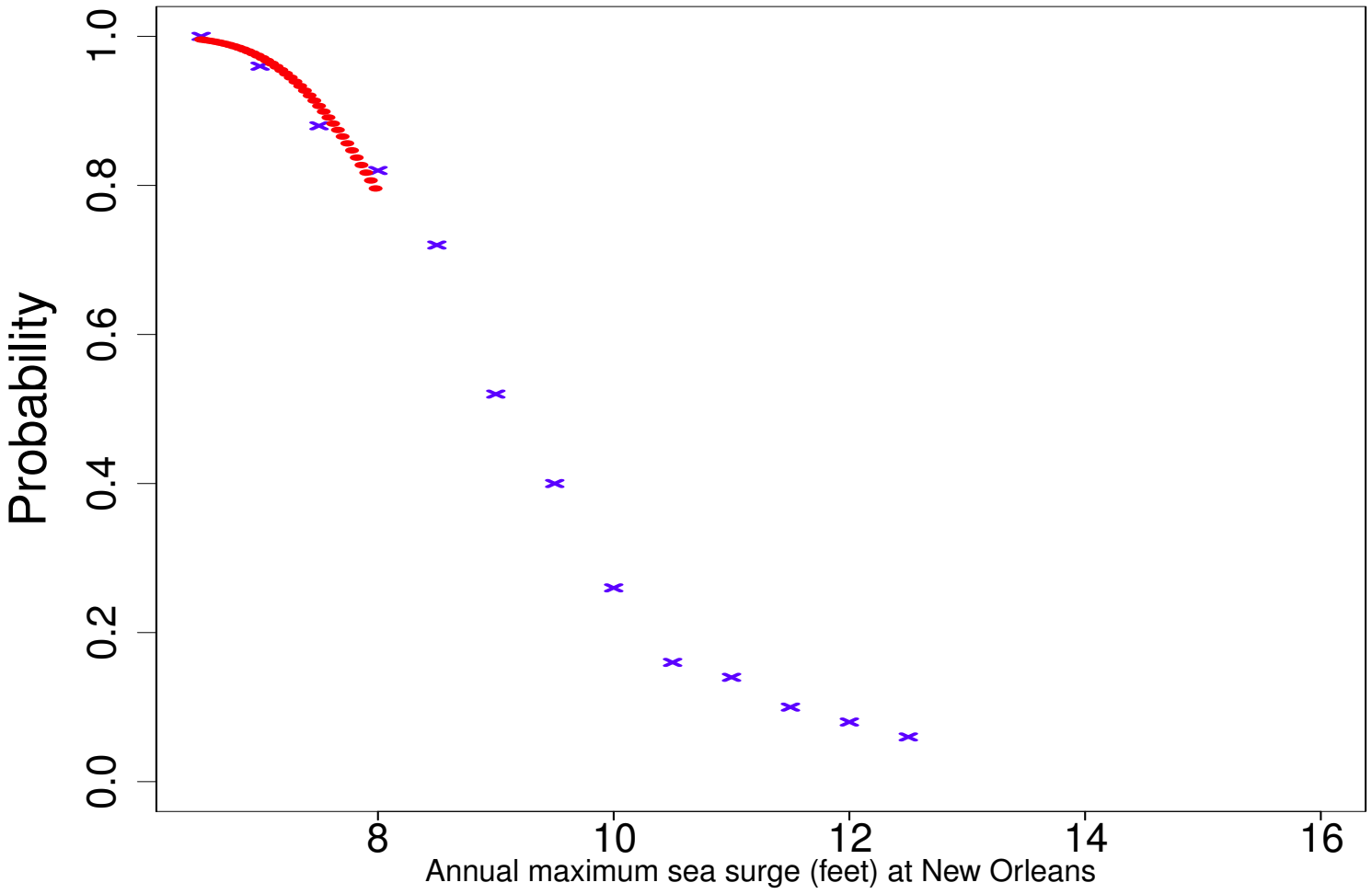

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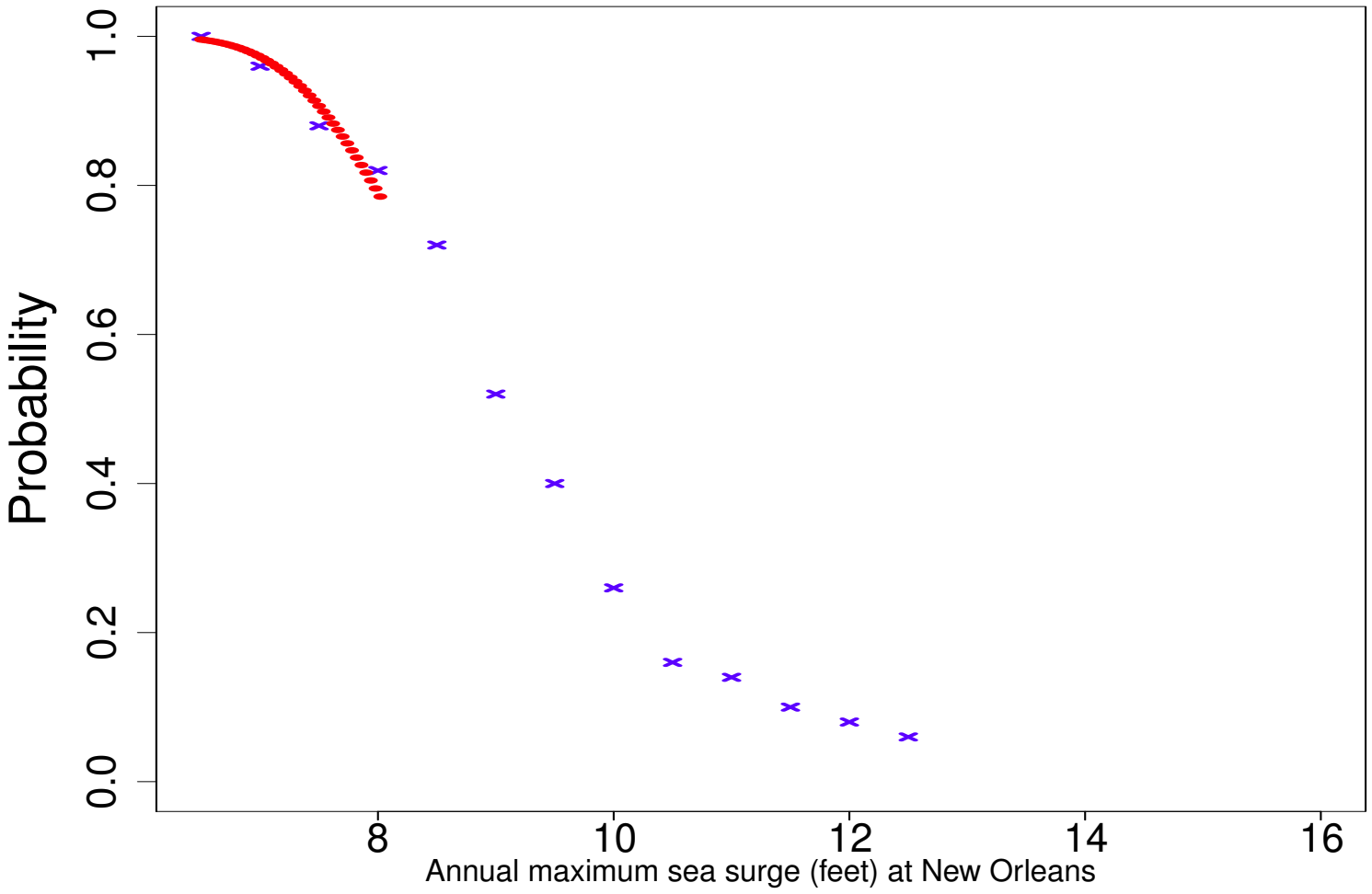

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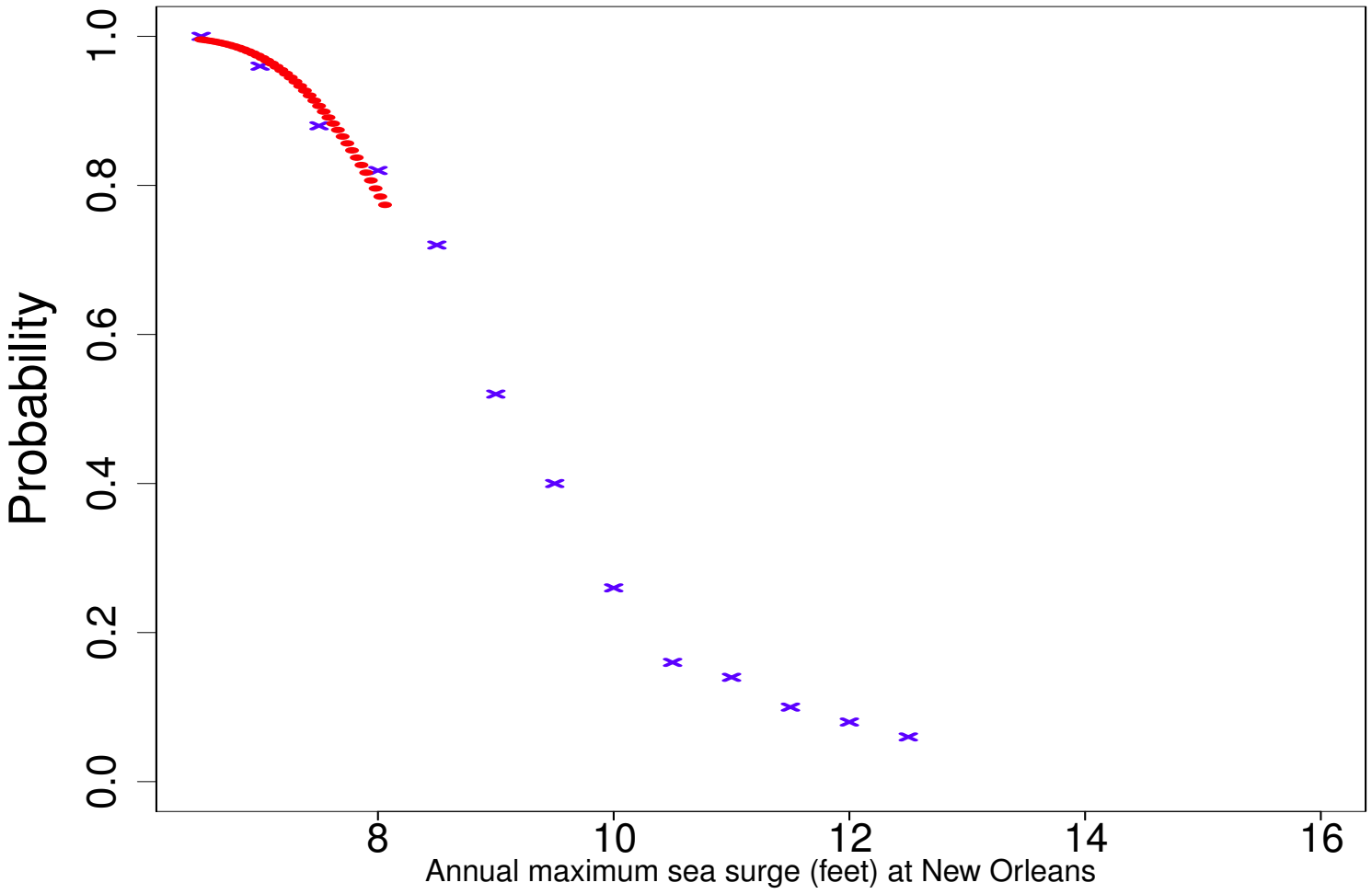

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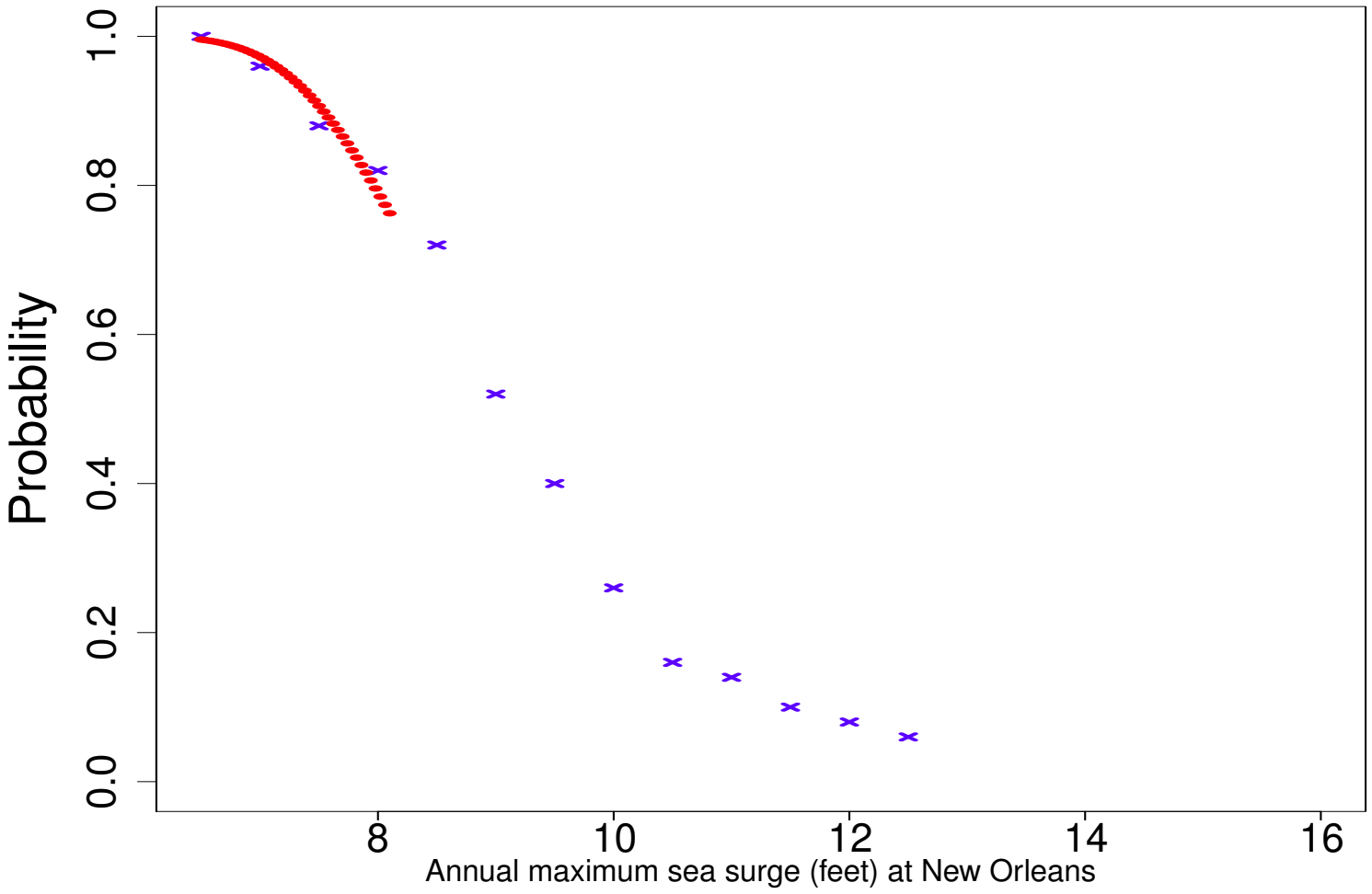

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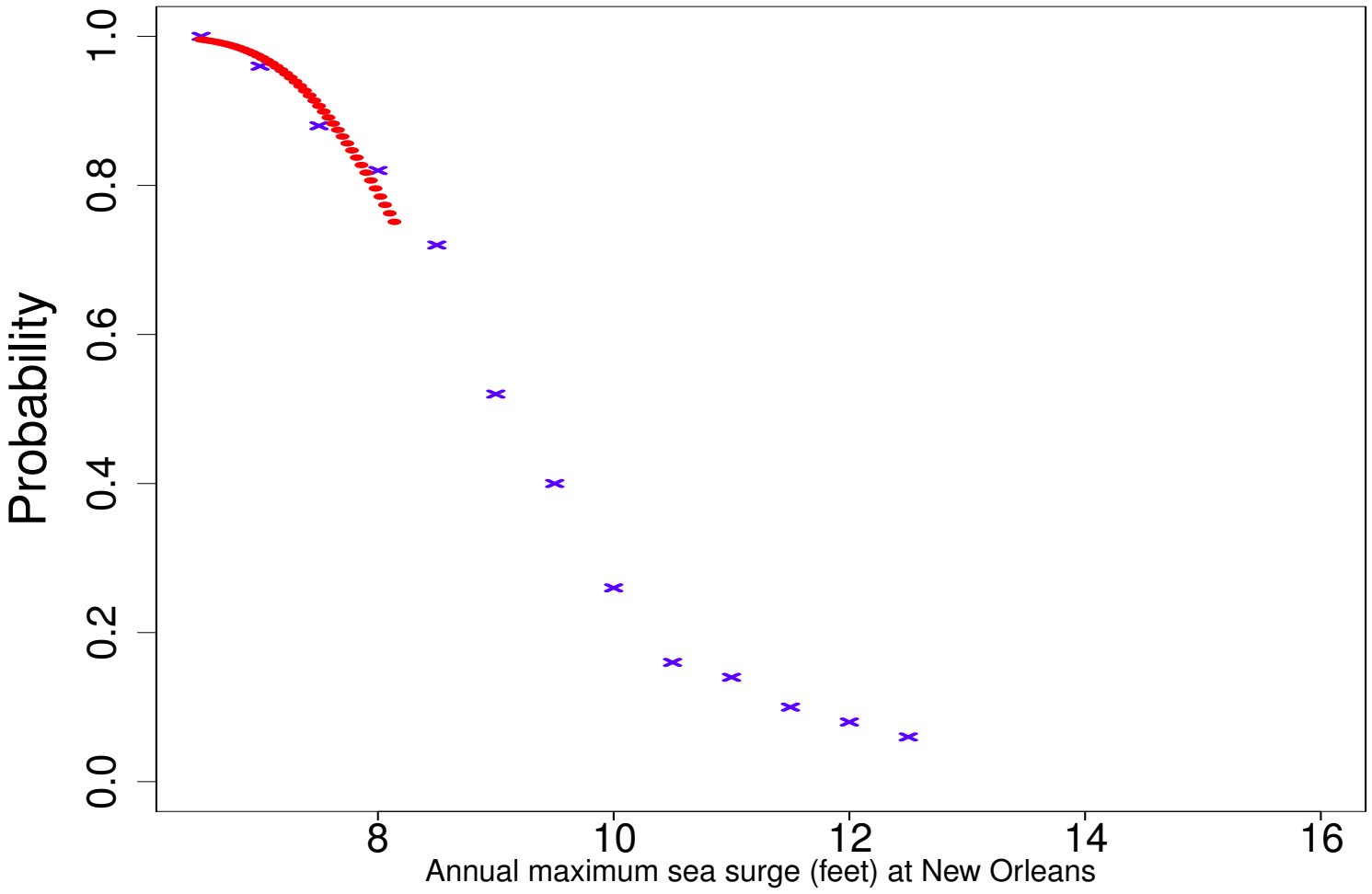

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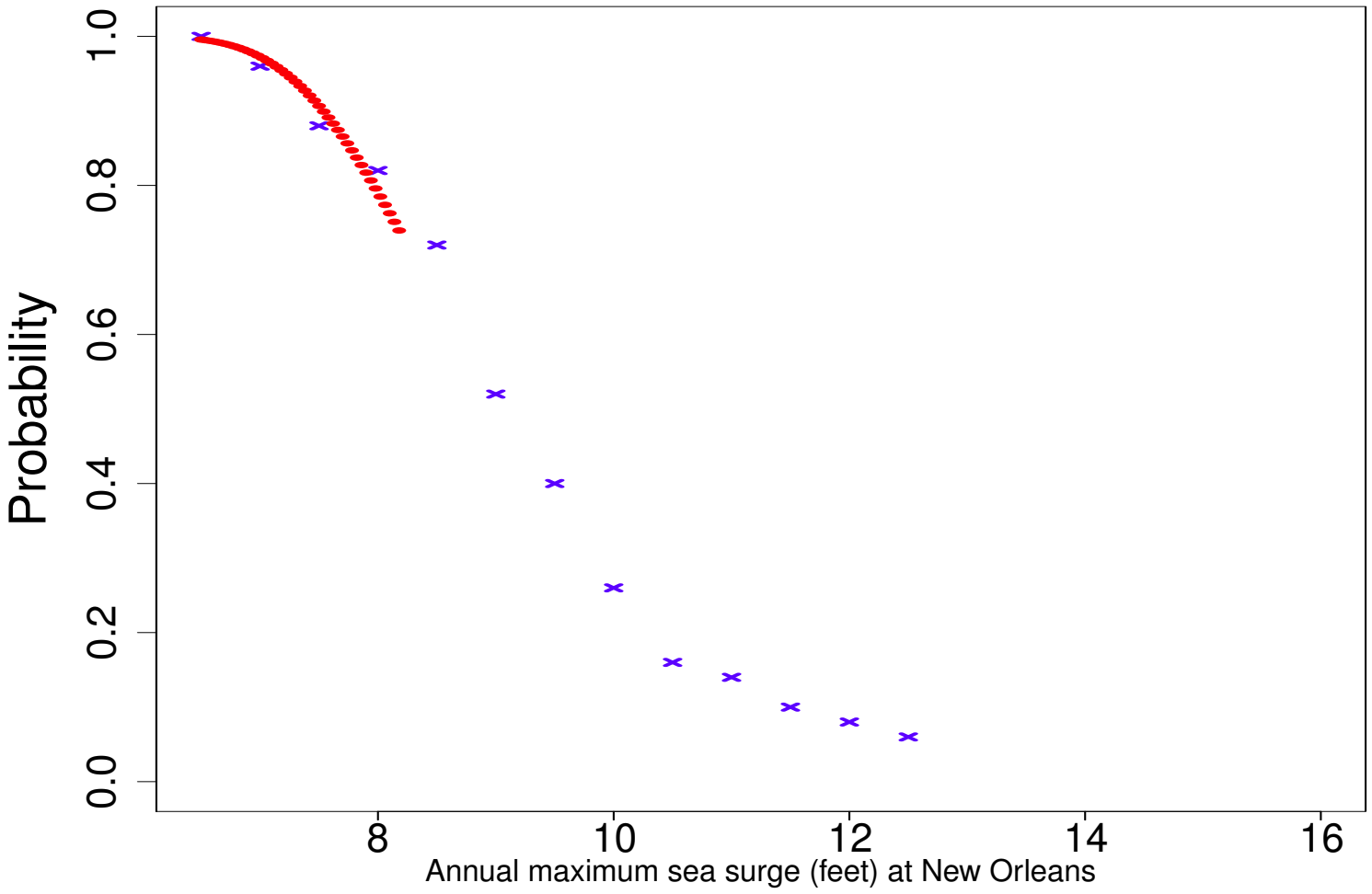

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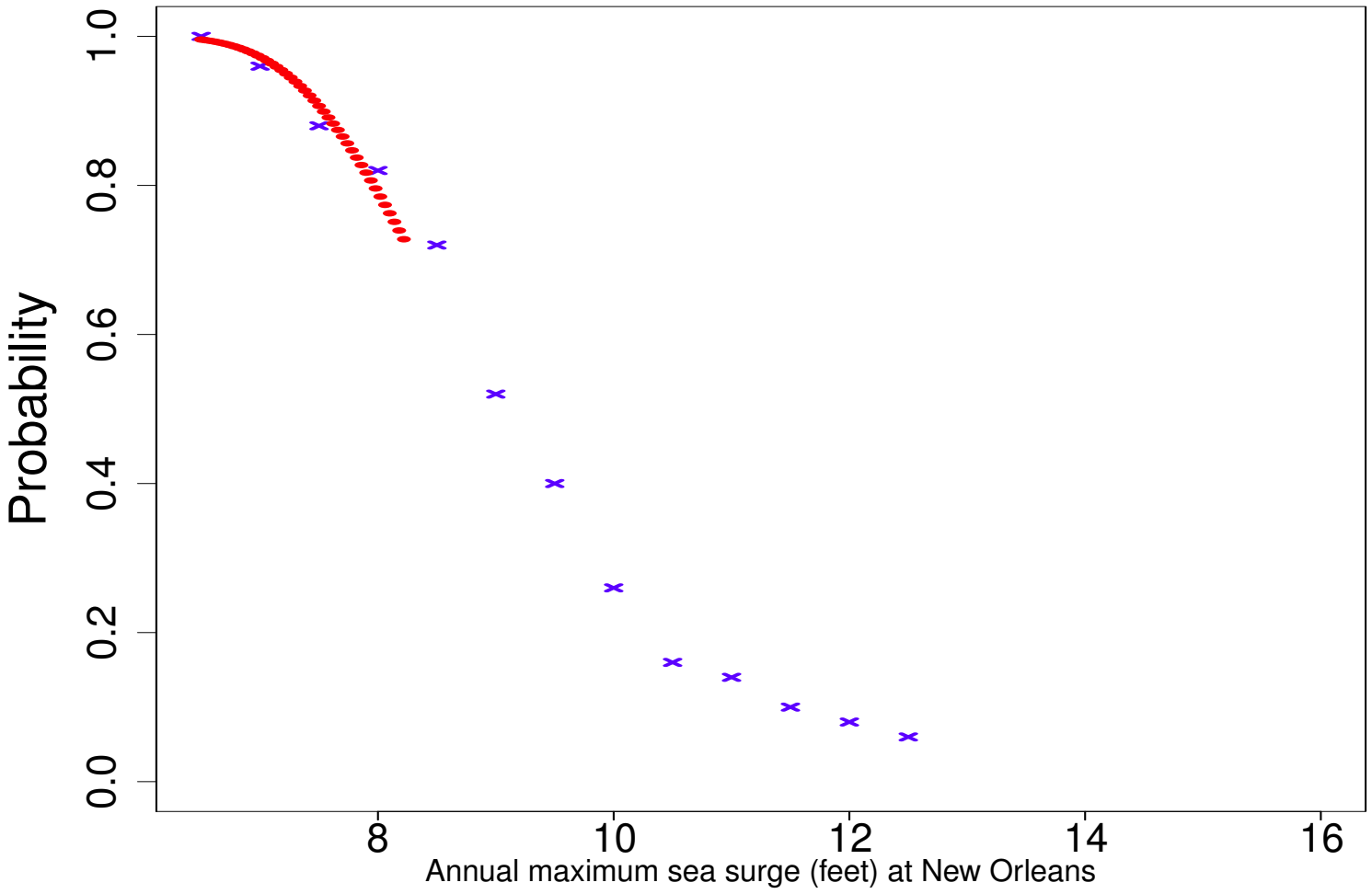

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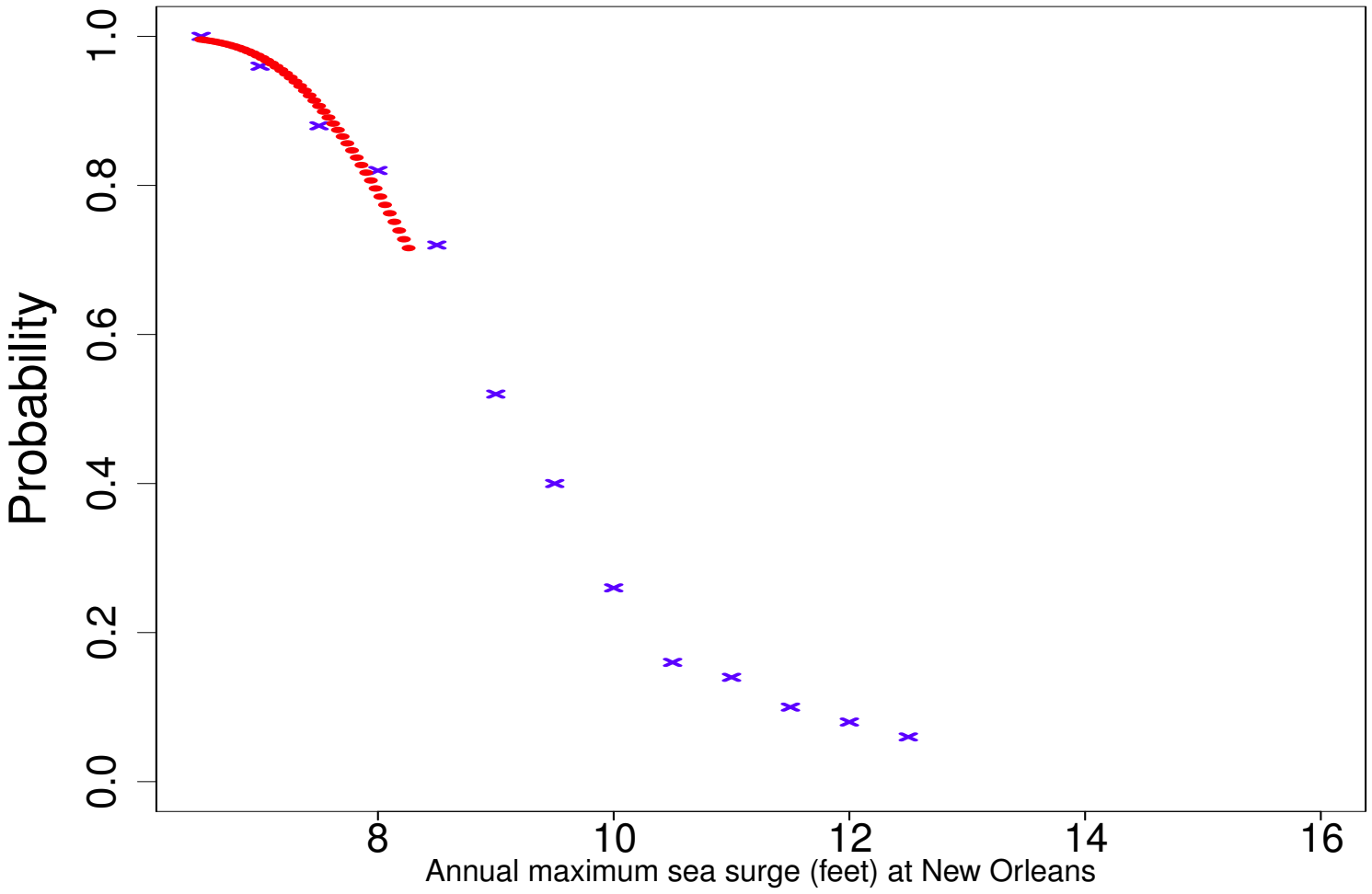

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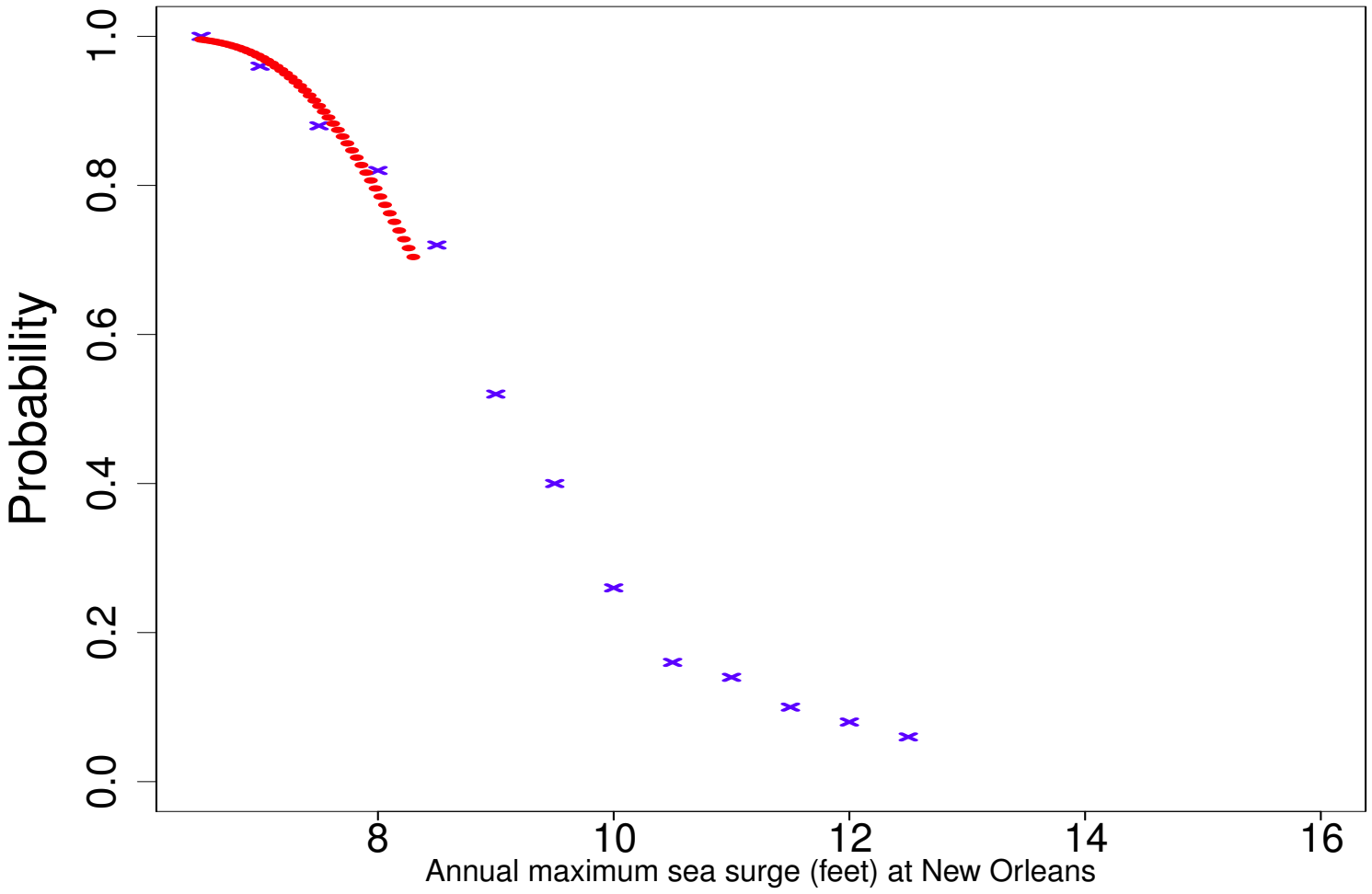

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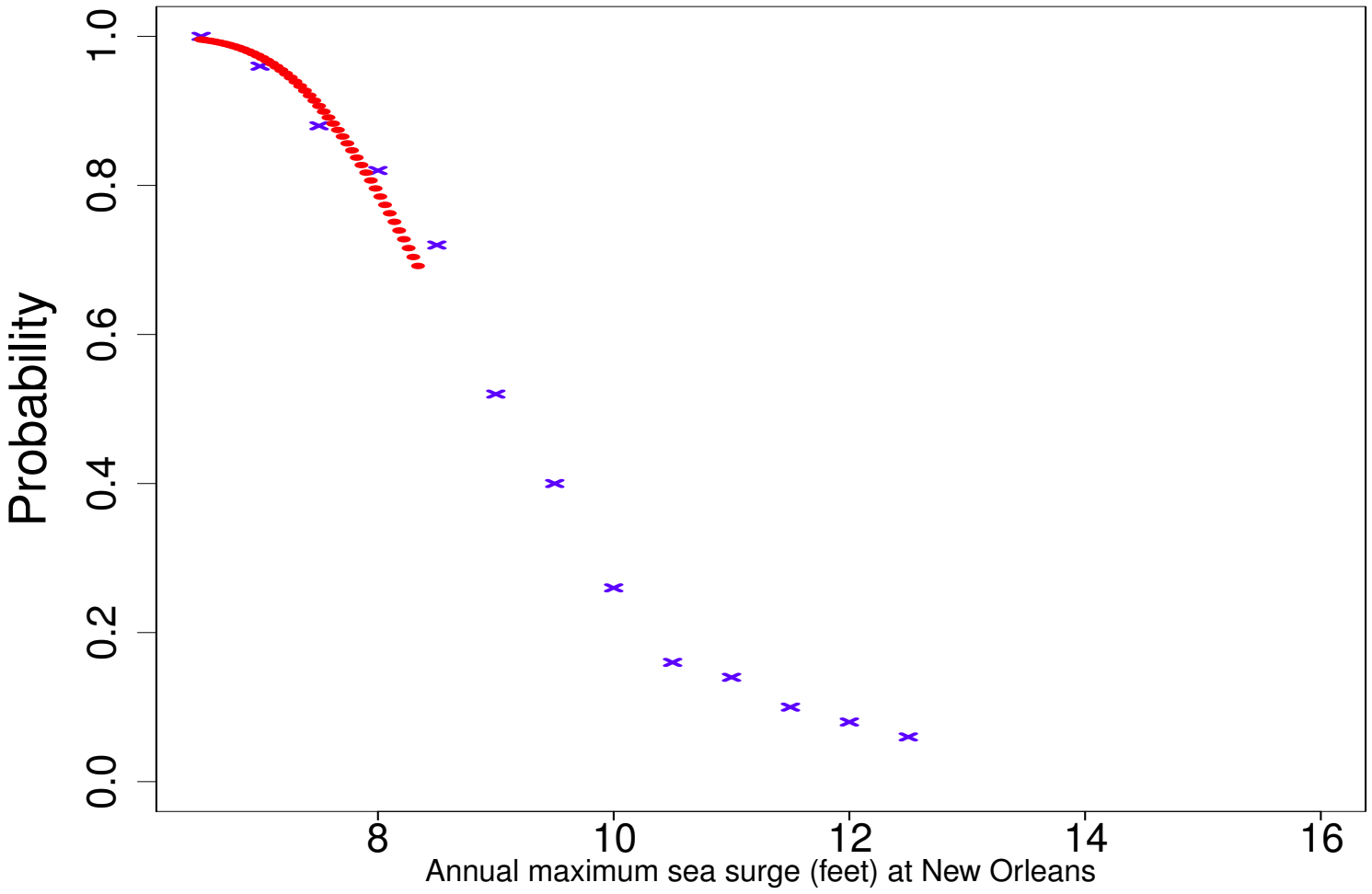

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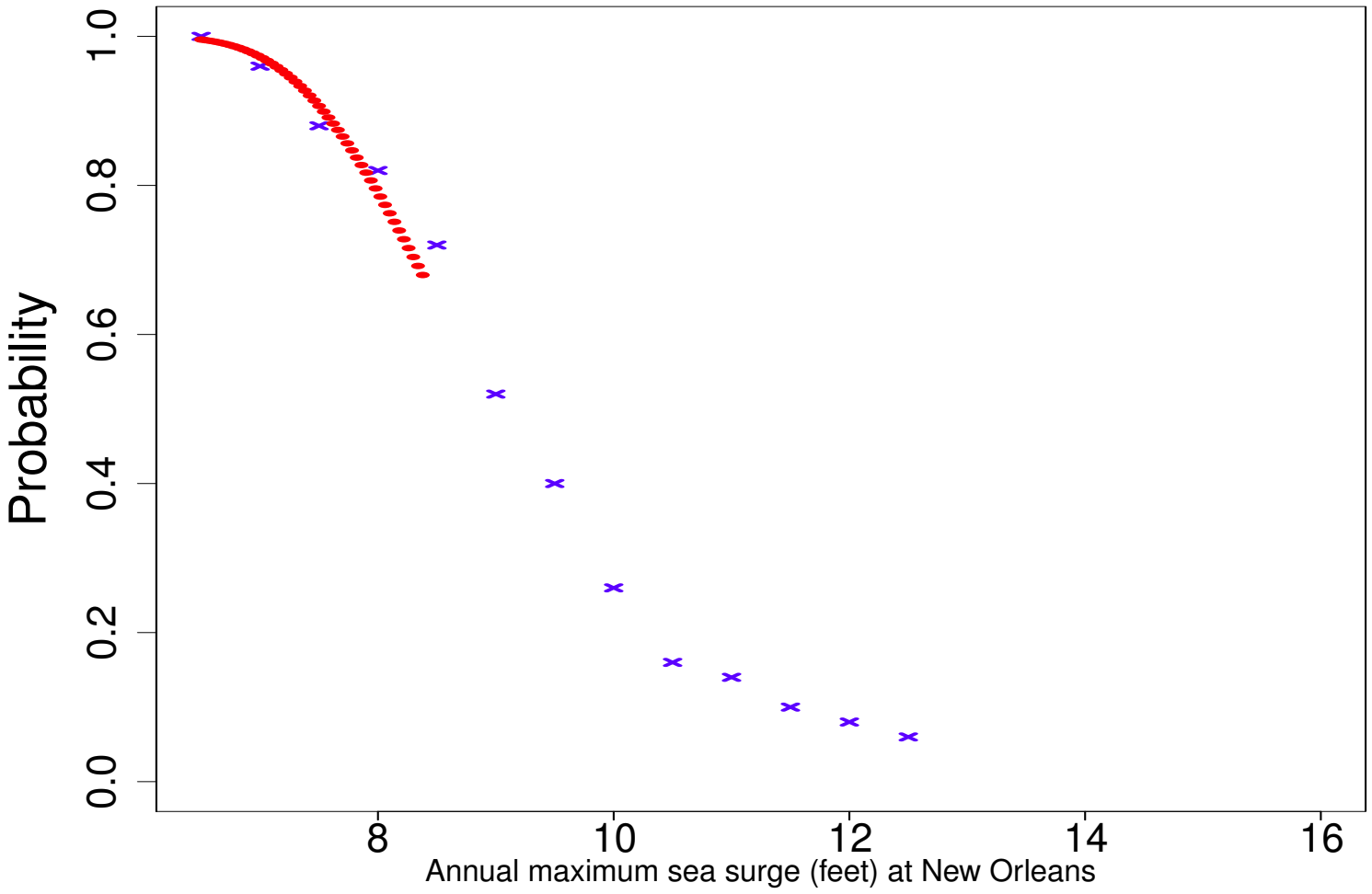

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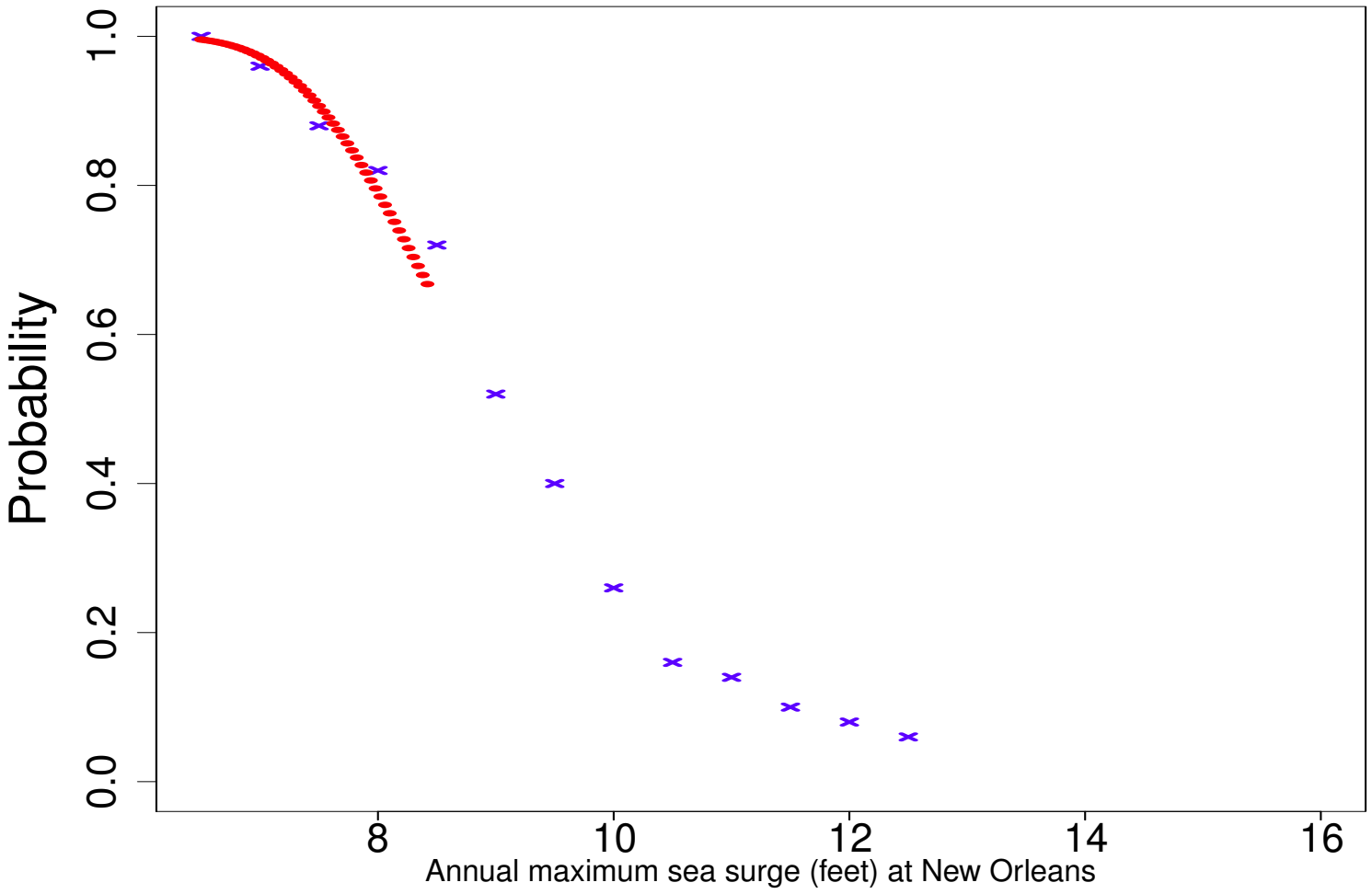

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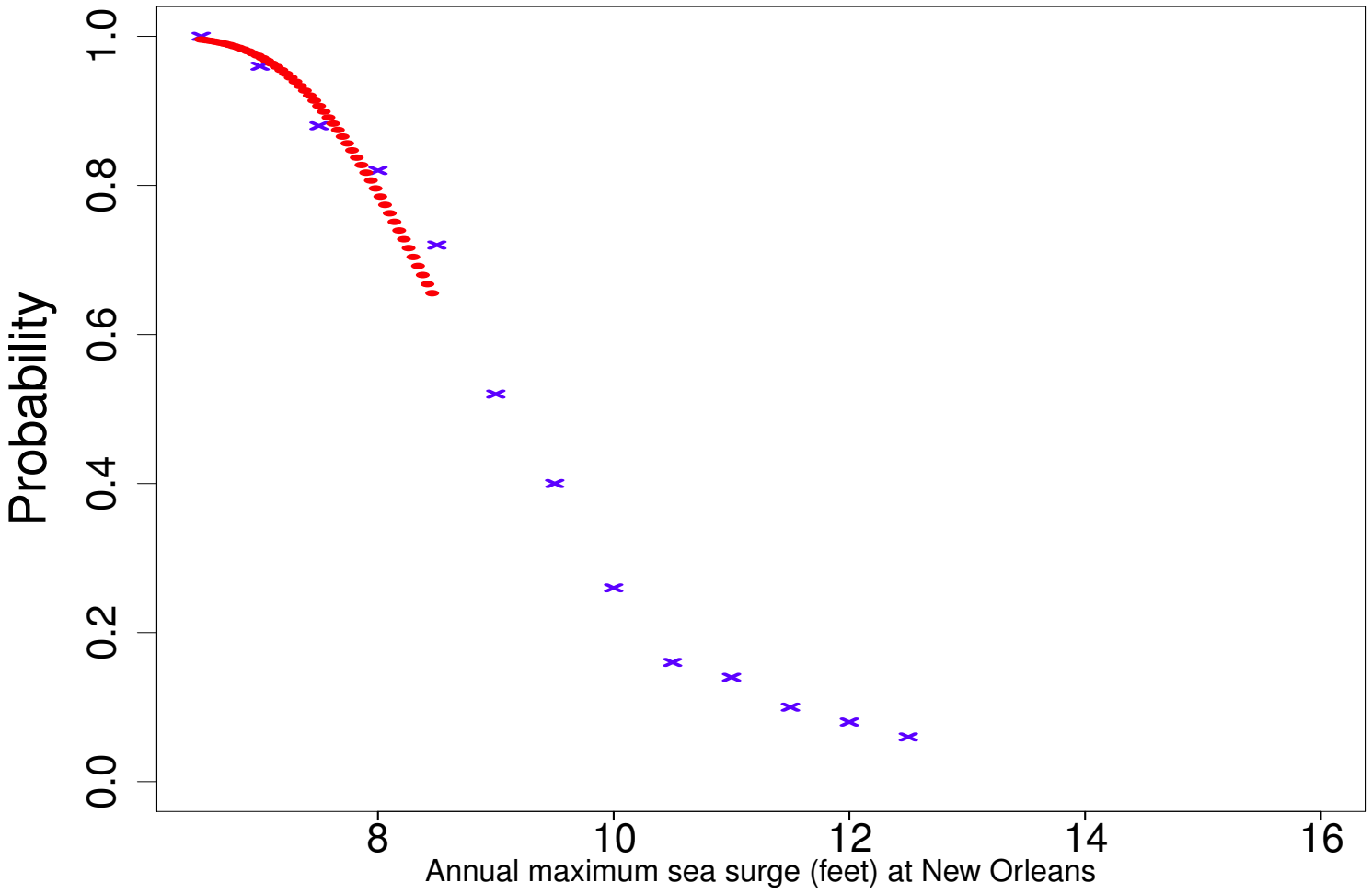

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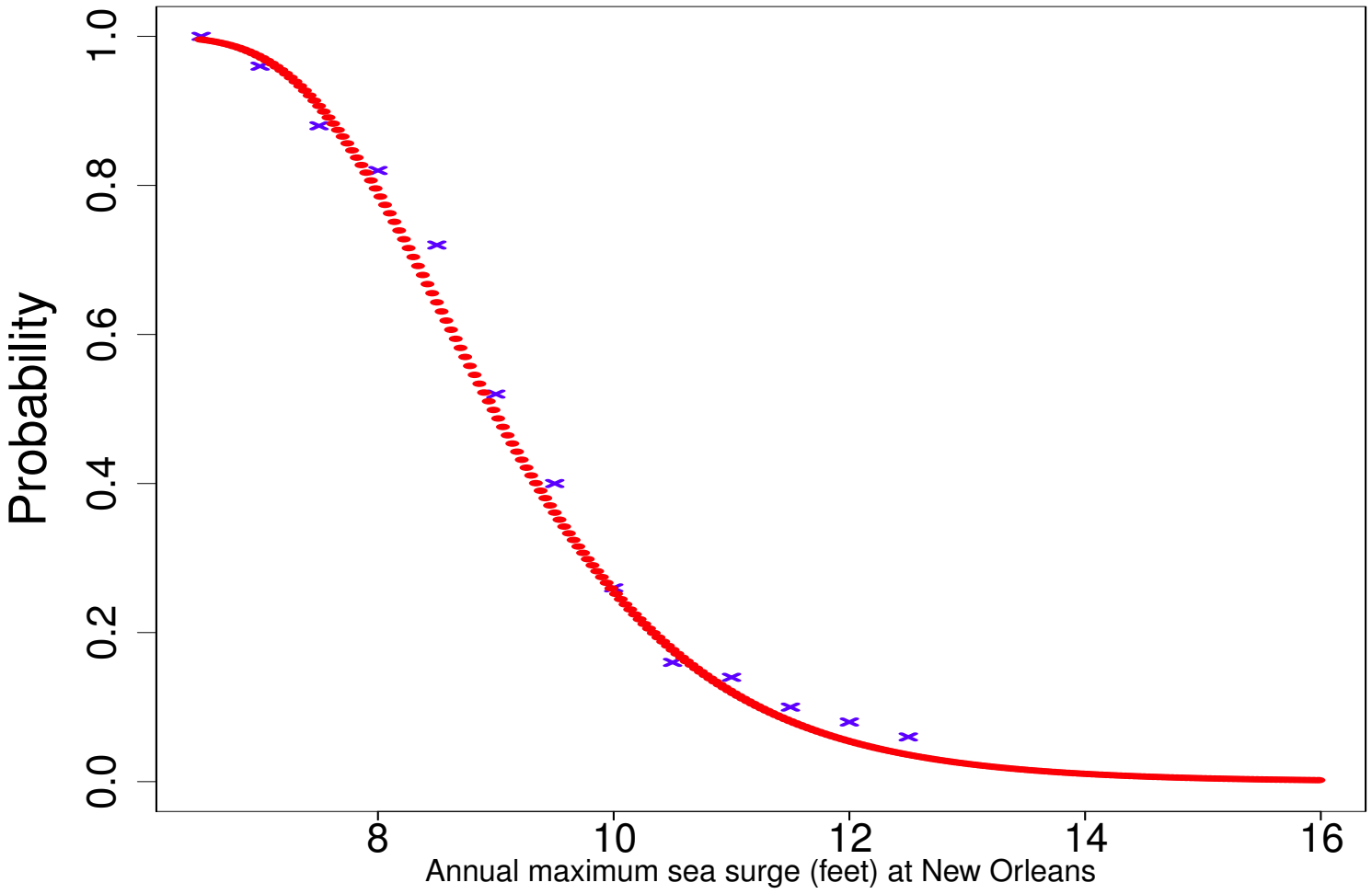

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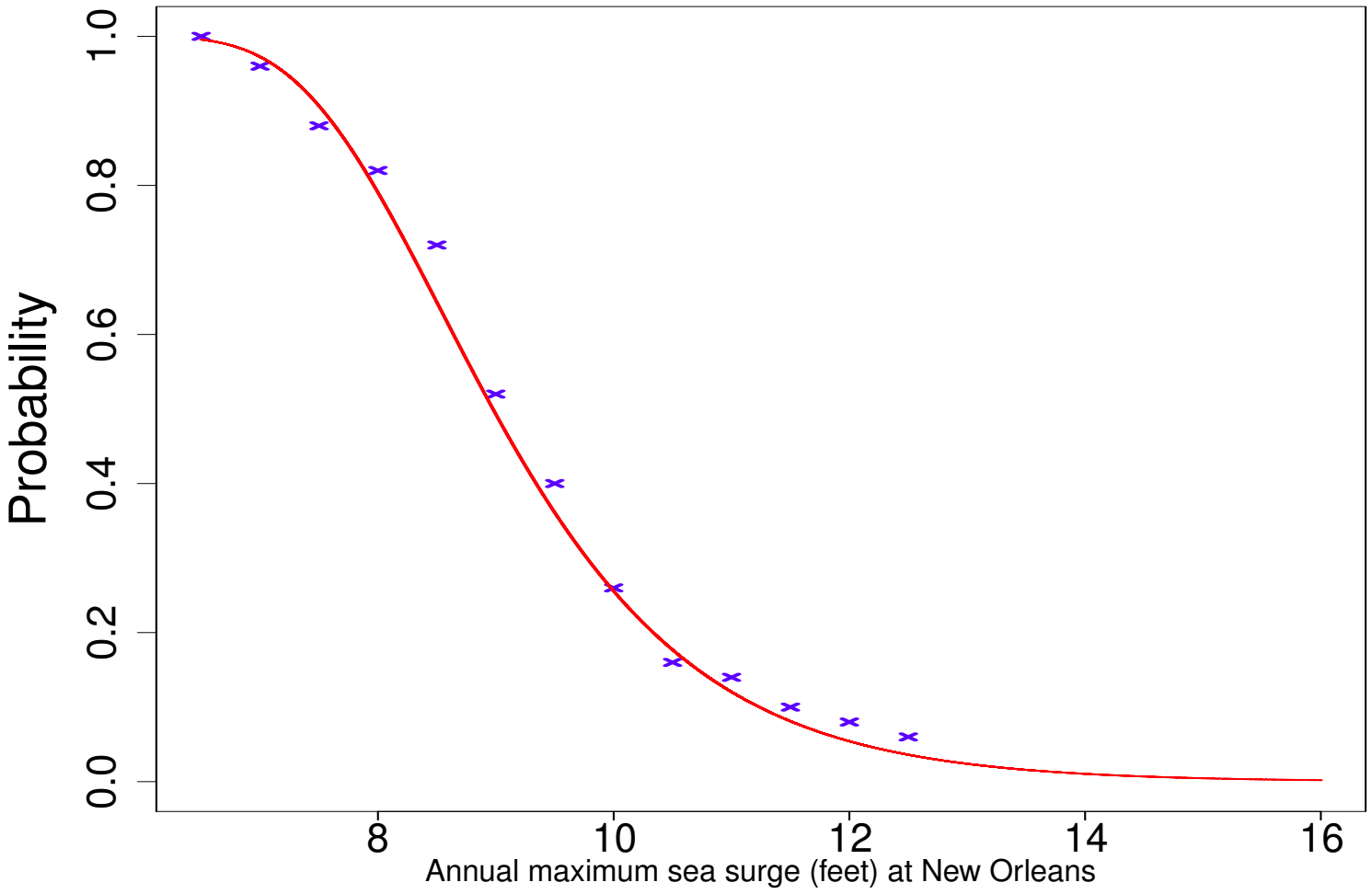

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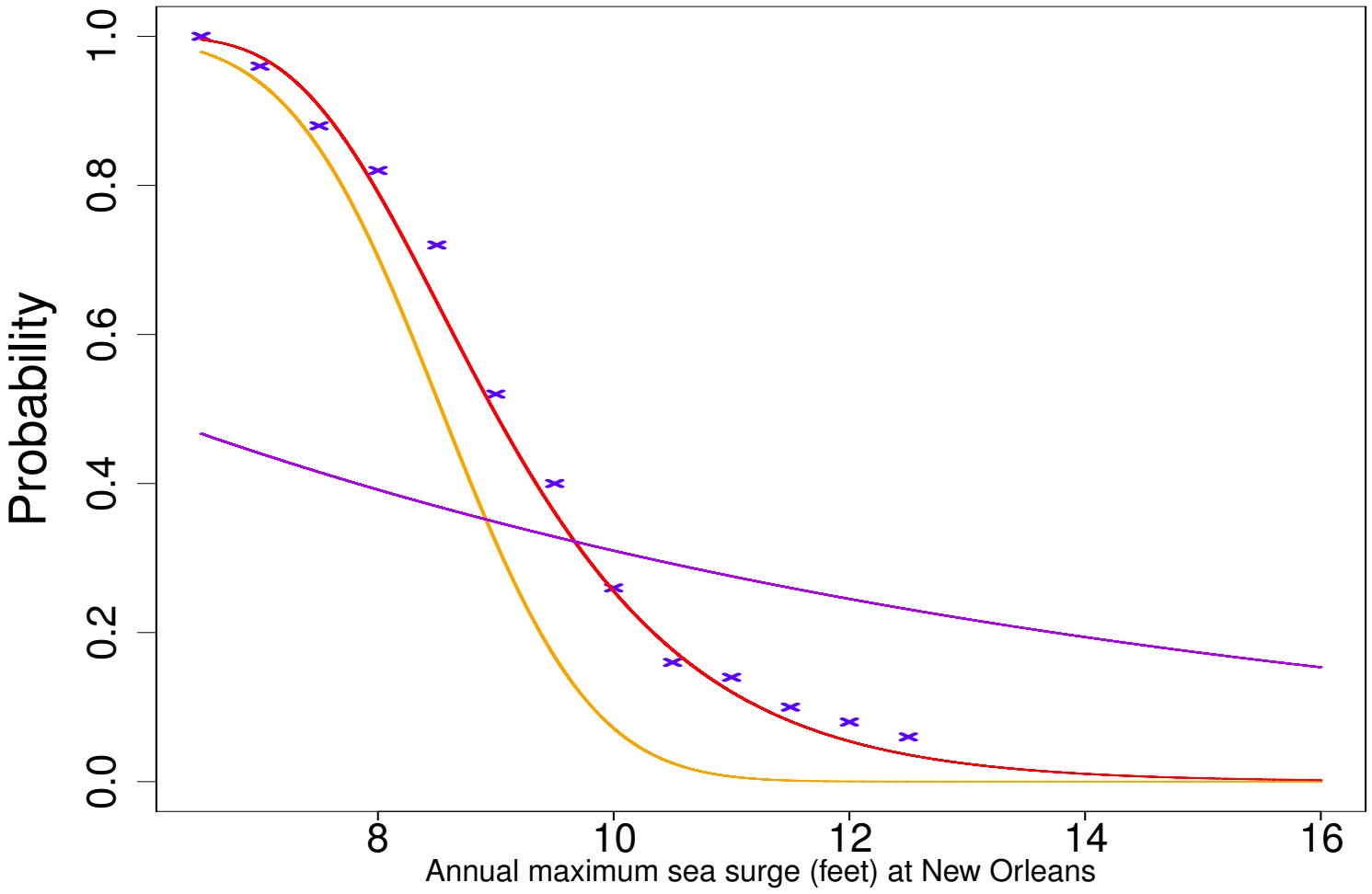

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What sort of curve would the **Normal distribution** give?

The mean of the data is $\bar{x} = 9.346$ and the standard deviation is $s = 1.5162$. Therefore:

$$\begin{aligned} P(X > 7) &= P\left(Z > \frac{7 - 9.346}{1.5162}\right) \\ &= P(Z > -1.55) \\ &= 1 - P(Z < -1.55) \\ &= 1 - 0.0606 \\ &= \mathbf{0.9394}. \end{aligned}$$

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Similarly,

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Complete the following table and then plot the Normal curve for exceedance probabilities on your graph:

| Sea–surge | Normal distribution exceedance probability |
|---|---|
| 7 | 0.939 |
| 8 | 0.813 |
| 9 | |
| 10 | |
| 11 | |
| 12 | |
| 13 | |
| 14 | |

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Complete the following table and then plot the Normal curve for exceedance probabilities on your graph:

| Sea–surge | Normal distribution exceedance probability |
|---|---|
| 7 | 0.939 |
| 8 | 0.813 |
| 9 | 0.590 |
| 10 | 0.333 |
| 11 | 0.138 |
| 12 | 0.040 |
| 13 | 0.008 |
| 14 | 0.001 |

# Part B: A probability model for extremes

The Gumbel model does really well at predicting the sea–surge exceedance probabilities at New Orleans!

We can also estimate probabilities of events more extreme than those we have observed via **extrapolation**.

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We can use our graph, or the values calculated using Gumbel's formula, to estimate the probability that, this year, the sea–surge at New Orleans will:

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3. exceed 14 feet.

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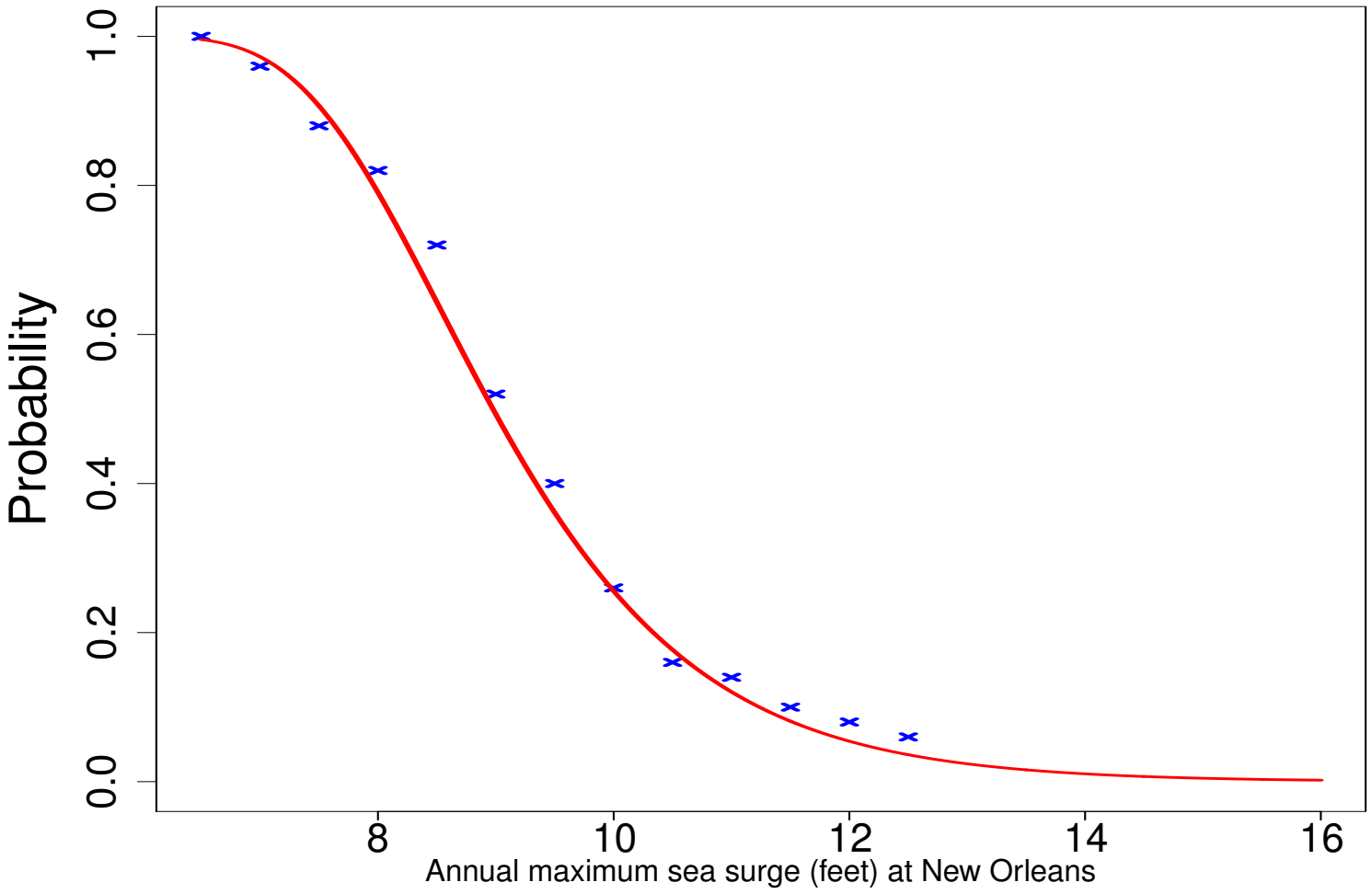
3. exceed 14 feet.

# Part B: A probability model for extremes

We can use our graph, or the values calculated using Gumbel's formula, to estimate the probability that, this year, the sea–surge at New Orleans will:

1. exceed 8.75 feet;
2. exceed 11.25 feet;
3. exceed 14 feet.

# Part B: A probability model for extremes



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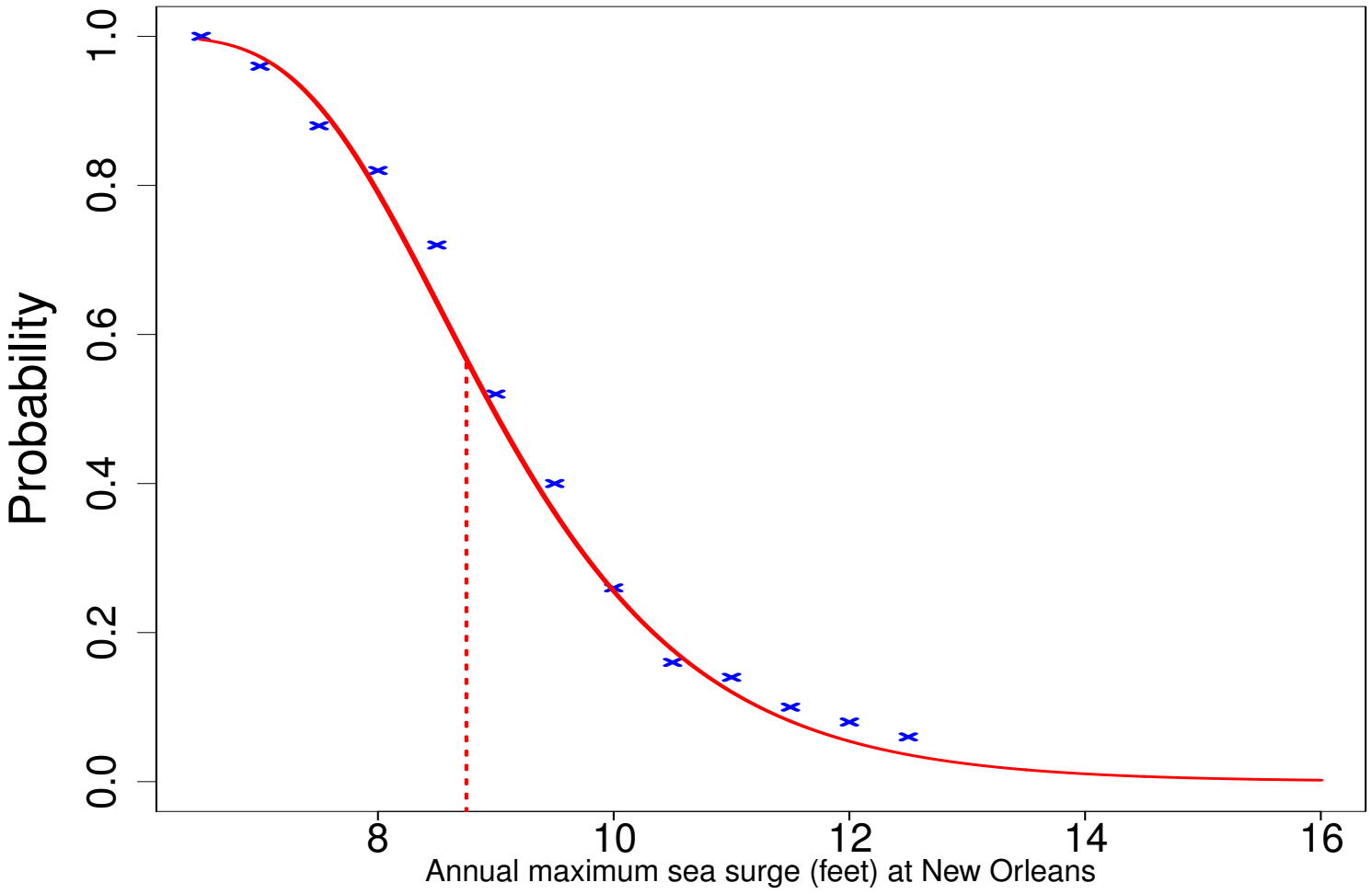

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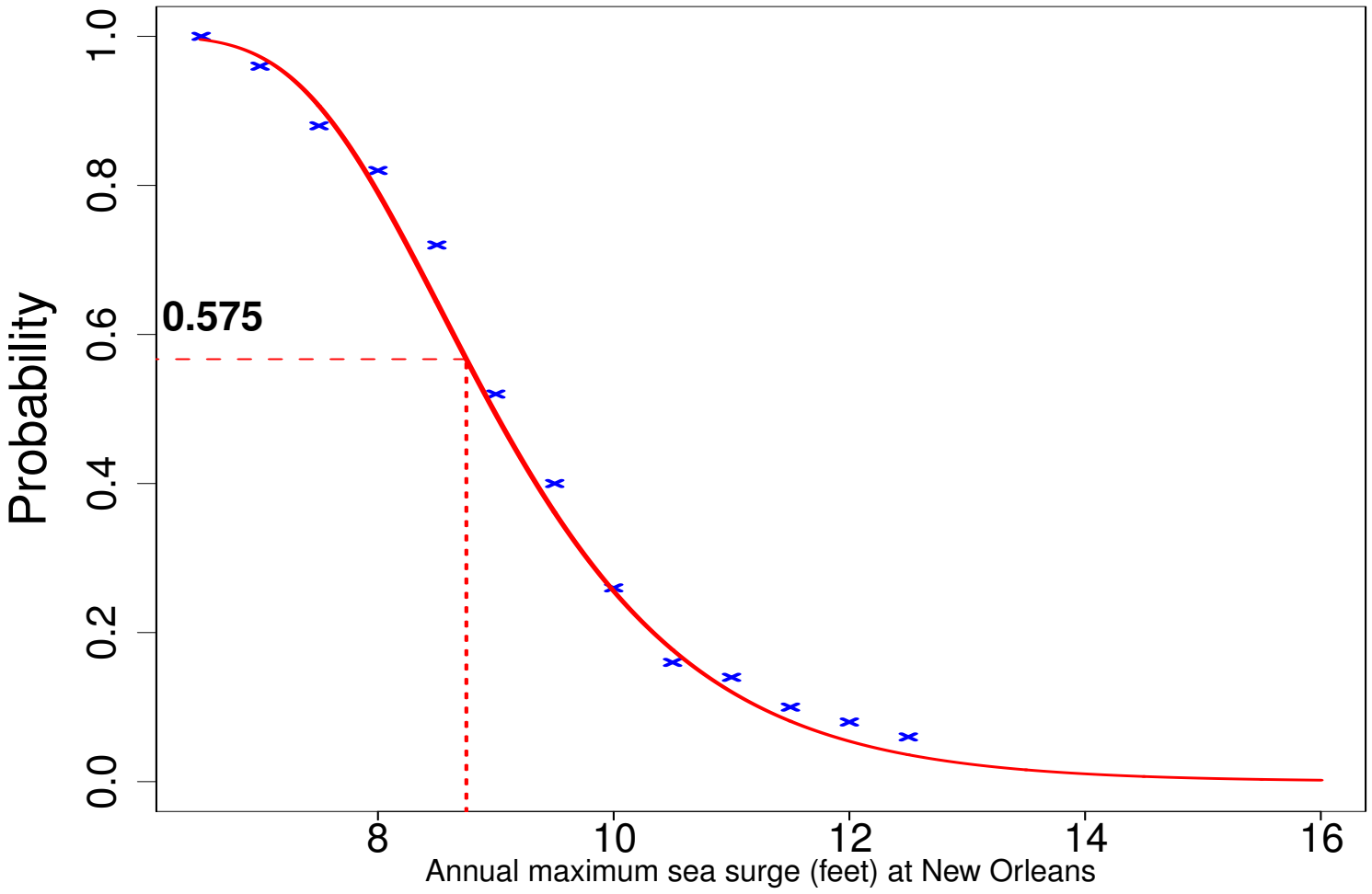

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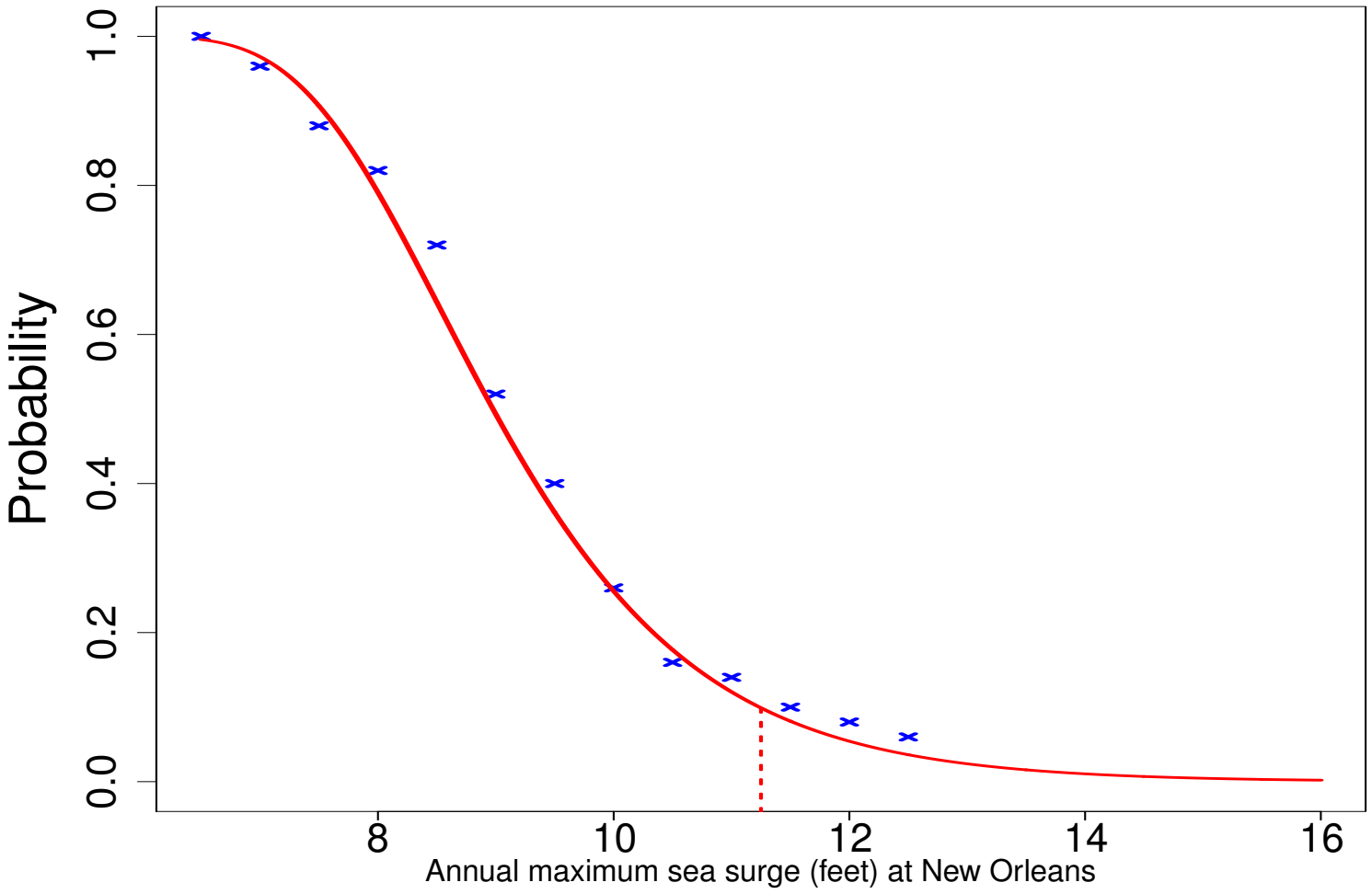

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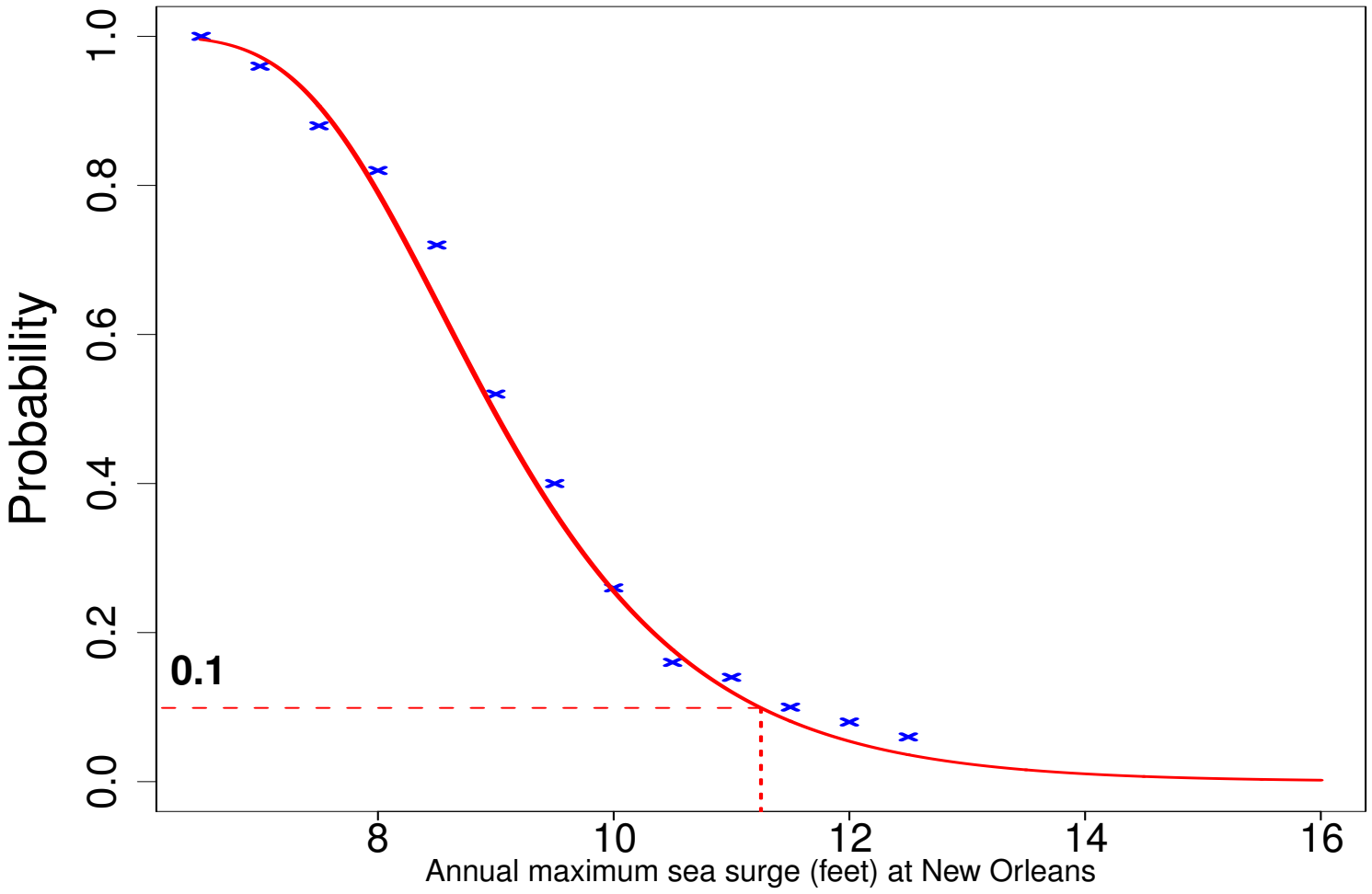

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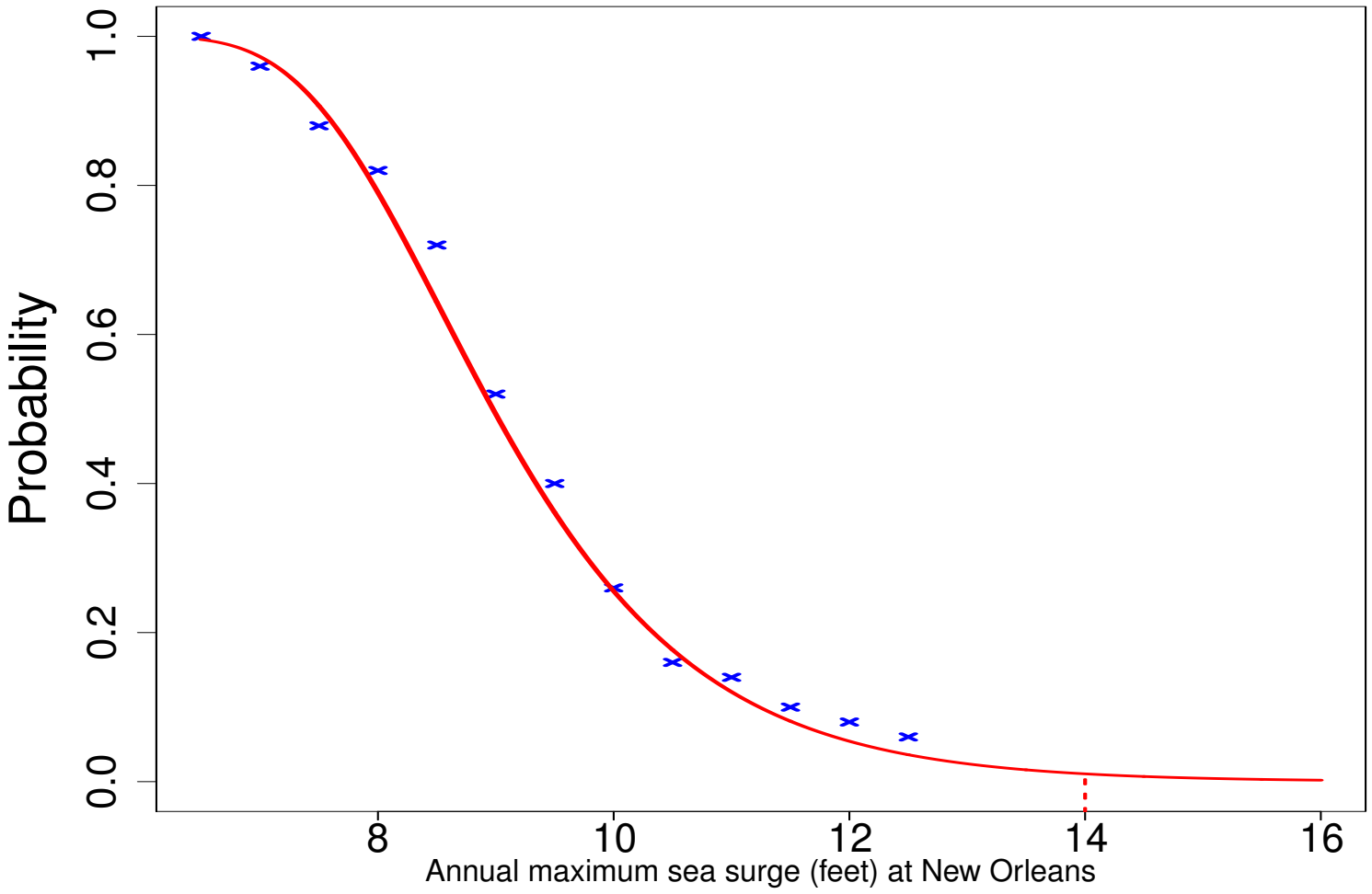

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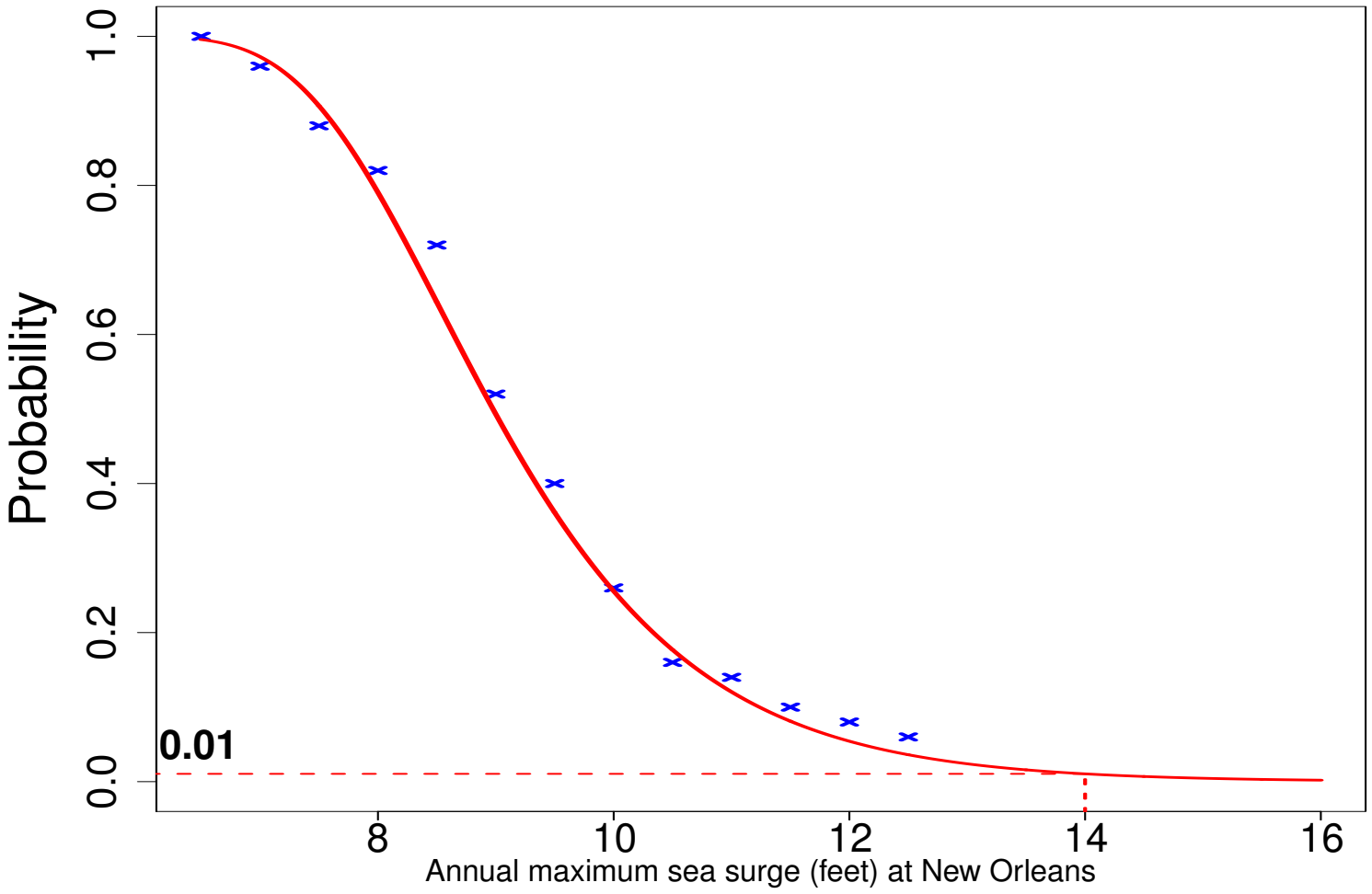

# Part B: A probability model for extremes

So we get:

| Probabilities | Exceeds | | |
|---|---|---|---|
| | 8.75 feet | 11.25 feet | 14 feet |
| Data alone | 0.66 | 0.12 | 0 |
| Gumbel model | 0.575 | 0.1 | 0.01 |
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Can you see why Katrina was billed as the "**storm of the century**"?

# Part C: Application to structural design

As you might remember from the first part of this session, during Hurricane Katrina sea–surges exceeded 14 feet and parts of the sea wall system protecting the city were breached.

A new flood defence system is to be built; as the mathematician, you are asked how tall the sea wall should be to protect against the storm we might expect to see, on average, **once every 500 years**.

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# Part C: Application to structural design

Use the Gumbel model to help estimate the height of the new sea wall.

We want $x$ such that

$$\boldsymbol{P}(\text{Sea–surge} > x) = \frac{1}{500}.$$

Using Gumbel's formula, this gives

$$1 - \exp\left[-\exp\left\{-\left(\frac{x - 8.536}{1.2}\right)\right\}\right] = \frac{1}{500}.$$

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$$= 15.99233 \approx 16 \text{ feet.}$$

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# Part C: Application to structural design

**What would the estimated height be assuming Normality?**

# Further study in Maths

Why take Maths any further than GCSE?

- A level Maths gives you further "grounding" in the basics, and is seen as a 'prestigious' A level to have!
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# Other talks and references

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