# Introduction to Number Theory and Cryptography (MAS3214) 

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## Sending Secret Messages

## Transmitting over an Open Channel

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PLAINTEXT

## Sending Secret Messages

Transmitting over an Open Channel


PLAINTEXT
$\Downarrow$ Encode
ciphertext

## Sending Secret Messages

Transmitting over an Open Channel

$\Downarrow$ Encode
transmission
ciphertext $\rightarrow$ ciphertext

## Sending Secret Messages

Transmitting over an Open Channel


## Sending Secret Messages

## Transmitting over an Open Channel

Alice
$\Theta!$
$\leftarrow \Delta \rightarrow$
$\dagger\lfloor$
PLAINTEXT
$\Downarrow$ Encode
transmission
ciphertext $\rightarrow$ ciphertext

## Weaknesses

## Sending Secret Messages

## Transmitting over an Open Channel



PLAINTEXT
$\Downarrow$ Encode
ciphertext $\quad \rightarrow \quad$ transmission

## Weaknesses

- Bob needs to know how to decipher Alice's message.


## Sending Secret Messages

## Transmitting over an Open Channel



$$
\begin{array}{|l|}
\hline \text { PLAINTEXT } \\
\hline
\end{array}
$$

$\Downarrow$ Encode

$$
\text { ciphertext } \rightarrow \quad \text { ciphertext }
$$

## Weaknesses

- Bob needs to know how to decipher Alice's message.
- Someone else may work out how to decode the message.


## The Caesar Cipher

## Encryption by Shifting Letters

## The Caesar Cipher

## Encryption by Shifting Letters

| paanimexi |  | B | R | U | T | E | F | O | R | C |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |

encryption $\downarrow$ using $A \mapsto b$

| ciphertext |  | C | S | v | u | f | g | p | s | d | f |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |

## The Caesar Cipher

## Encryption by Shifting Letters

| plamarixt | B | R | U | T | E | F | O |  |  | C |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | encryption $\downarrow$ using $A \mapsto b$


| ciphertext |  | C | S | V | u | f | g | p | s | d | f |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |

## Weaknesses

## The Caesar Cipher

## Encryption by Shifting Letters

| platimext | B | R | U | T | E | F | O | R | C | E |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | encryption $\downarrow$ using $A \mapsto b$


| ciphertext |  | C | S | V | u | f | g | p | s | d | f |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |

## Weaknesses

- There are only 26 codes to try. [Rot13 is still used.]


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## Encryption by Shifting Letters

| platrizer |  | B | R | U | T | E | F | O | R | C |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | encryption $\downarrow$ using $A \mapsto b$


| ciphertext |  | C | S | V | u | f | g | P | S | d | f |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |

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## The Caesar Cipher

## Encryption by Shifting Letters

| paantrext | B | R | U | T | E | F | O | R | C | E |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | encryption $\downarrow$ using $A \mapsto b$


| ciphertext |  | C | S | V | u | f | g | P | S | d | f |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |

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- There are only 26 codes to try. [Rot13 is still used.]
- This is a rather easy process to do backwards. [zyxwvutsr...]
- Decode: mpm!


## The Caesar Cipher

## Encryption by Shifting Letters

| platrimex | B | R | U | T | E | F | O | R | C | E |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | encryption $\downarrow$ using $A \mapsto b$


| ciphertext | C | S | V | $\mathbf{u}$ | f | g | p | S | d | f |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |

## Weaknesses

- There are only 26 codes to try. [Rot13 is still used.]
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- LOL!


## The Caesar Cipher

## Encryption by Shifting Letters

| platimext | B | R | U | T | E | F | O | R | C | E |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | encryption using $A \mapsto b$


| ciphertext |  | C | S | V | $\mathbf{u}$ | f | g | p | s | d | f |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |

## Weaknesses

- There are only 26 codes to try. [Rot13 is still used.]
- This is a rather easy process to do backwards. [zyxwvutsr...]
- Decode: mpm!
- LOL!
- How can we make this code more difficult to break?


## The Permutation Cipher

## Encryption by Swapping Letters

## The Permutation Cipher

## Encryption by Swapping Letters

| I | R | E | A | D | T | H | E | N | O | T | E | S |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| encryption $\downarrow$ using a permutation |  |  |  |  |  |  |  |  |  |  |  |  |
| b | p | u | t | q | v | k | u | f | x | V | u | 0 |

## The Permutation Cipher

## Encryption by Swapping Letters

| I | R | E | A | D | T | H | E | N | O | T | E | S |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| encryption $\downarrow$ using a permutation |  |  |  |  |  |  |  |  |  |  |  |  |
| b | p | u | t | q | v | k | u | f | x | v | u | 0 |

There are $26!=26 \times 25 \times \ldots \times 2 \times 1 \approx 4 \times 10^{26}$ codes to try!

## The Permutation Cipher

## Encryption by Swapping Letters

| I | R | E | A | D | T | H | E | N | O | T | E | S |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| encryption $\downarrow$ using a permutation |  |  |  |  |  |  |  |  |  |  |  |  |
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## Weaknesses

- The single letter is probably "A" or "I";


## The Permutation Cipher

## Encryption by Swapping Letters

| I | R | E | A | D | T | H | E | N | O | T | E | S |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| encryption $\downarrow$ using a permutation |  |  |  |  |  |  |  |  |  |  |  |  |
| b | p | u | t | q | v | k | u | f | x | v | u | 0 |

There are $26!=26 \times 25 \times \ldots \times 2 \times 1 \approx 4 \times 10^{26}$ codes to try!

## Weaknesses

- The single letter is probably "A" or "I";
- The commonest letter is probably "E";


## The Permutation Cipher

## Encryption by Swapping Letters

| I | R | E | A | D | T | H | E | N | O | T | E | S |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| encryption $\downarrow$ using a permutation |  |  |  |  |  |  |  |  |  |  |  |  |
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There are $26!=26 \times 25 \times \ldots \times 2 \times 1 \approx 4 \times 10^{26}$ codes to try!

## Weaknesses

- The single letter is probably "A" or "I";
- The commonest letter is probably "E";
- Words are likely to end with "S", if not "E";


## The Permutation Cipher

## Encryption by Swapping Letters

| I | R | E | A | D | T | H | E | N | O | T | E | S |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| encryption $\downarrow$ using a permutation |  |  |  |  |  |  |  |  |  |  |  |  |
| b | p | u | t | q | v | k | u | f | x | v | u | 0 |

There are $26!=26 \times 25 \times \ldots \times 2 \times 1 \approx 4 \times 10^{26}$ codes to try!

## Weaknesses

- The single letter is probably "A" or "I";
- The commonest letter is probably "E";
- Words are likely to end with "S", if not "E";
- The commonest (English) letters are: E, T, A, O, I, N, S, H, R, D.


## The Permutation Cipher

## Encryption by Swapping Letters

| I | R | E | A | D | T | H | E | N | O | T | E | S |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| encrryption $\downarrow$ using a permutation |  |  |  |  |  |  |  |  |  |  |  |  |
| b | p | u | t | q | V | k | u | $f$ | x | v | u | 0 |

There are $26!=26 \times 25 \times \ldots \times 2 \times 1 \approx 4 \times 10^{26}$ codes to try!

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- We might guess:


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- The commonest (English) letters are: E, T, A, O, I, N, S, H, R, D.
- We might guess:

| I |  | $?$ | E | $?$ | $?$ |  | T | H | E |  | $?$ | $?$ | T | E | S |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |

## Breaking the Permutation Cipher

The Frequency Attack for long messages - Al Kindi

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| I | T |  | I | S |  | A |  | T | R | U | T | H |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\cdots$ |  |  |  |  |  |  |  |  |  |  |  |  |
| b | v |  | b | o |  | t |  | v | p | I | v | k |

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| I | T |  | I | S |  | A |  | T | R | U | T | H |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\cdots$ |  |  |  |  |  |  |  |  |  |  |  |  |
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| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\cdots$ |  |  |  |  |  |  |  |  |  |  |  |  |
| b | v |  | b | o |  | t |  | v | p | I | v | k |

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- The commonest letters in English (decreasing order) are:
- E, T, A, O, I, N, S, H, R, D.


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| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\cdots$ |  |  |  |  |  |  |  |  |  |  |  |  |
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## Weaknesses

- The commonest letters in English (decreasing order) are:
- E, T, A, O, I, N, S, H, R, D.
- The commonest encrypted letters (in our full message) are:
- u, v, t, x, b, f, o, k, p, q.


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| I | T |  | I | S |  | A |  | T | R | U | T | H |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\cdots$ |  |  |  |  |  |  |  |  |  |  |  |  |
| b | v |  | b | o |  | t |  | v | P | I | v | k |

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| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\cdots$ |  |  |  |  |  |  |  |  |  |  |  |  |
| b | v |  | b | o |  | t |  | v | P | I | v | k |

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- NOTE


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| I | T |  | I | S |  | A |  | T | R | U | T | H |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\cdots$ |  |  |  |  |  |  |  |  |  |  |  |  |
| b | v |  | b | o |  | t |  | v | p | I | v | k |

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- NOTE: This leads us to guess the following decryption:


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| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| H | $\cdots$ |  |  |  |  |  |  |  |  |  |  |
| b | v |  | b | o |  | t |  | v | p | I | v |
| k | k | $\cdots$ |  |  |  |  |  |  |  |  |  |

## Weaknesses

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- Decode: fxvu
- NOTE: This leads us to guess the following decryption:
- IT IS A TR?TH ?NI?ERSA??? A??NO??ED?ED THAT A SIN??E ?AN IN ?OSSESSION O? A ?OOD ?ORT?NE ??ST ?E IN ?ANT O? A ?I?E.


## Vigenère Cipher

## Defending against the frequency attack

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- Keep changing your code!


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- "ACE" tells us to move letters on by $1,3,5,1,3,5, \ldots$


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## Vigenère Example

## Vigenère Cipher

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## Vigenère Example

| 1 | T | I | S | A | T | R | U | T | H |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| A | C | E | A | C | E | A | C | E | A |
| $\downarrow$ |  |  |  |  |  |  |  |  |  |
| j | w | n | t | d | $y$ | S | x | $y$ | i |

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## Defending against the frequency attack

- Keep changing your code!
- If you change the code with every letter it is hard to break.
- In the Vigenère Cipher your have a 'Codeword’, say "ACE".
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| I | T | I | S | A | T | R | U | T | H |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| A | C | E | A | C | E | A | C | E | A |
|  |  |  |  |  |  |  |  |  |  |
| j | w | n | t | d | y | S | x | y | i |

This is harder to decode. However, in this example we just need 3 frequency tables. One for each letter of the Codeword.

## One Time Pad

## Use Random Shifts for each letter

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- If your Codeword is long, then the frequency attack is hard.
- It is tempting to use a long text, like "The Bible", as your Codeword.


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## Vigenère Example

| I | T | 1 | S | A | T | R | U | T | H |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| I | N | T | H | E | B | E | G | I | N |
| $\downarrow$ |  |  |  |  |  |  |  |  |  |
| r | h | C | a | f | v | w | b | C | V |

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## Vigenère Example

| I | T | I | S | A | T | R | U | T | H |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| I | N | T | H | E | B | E | G | I | N |
| $\downarrow$ |  |  |  |  |  |  |  |  |  |
| r | h | C | a | f | v | w | b | c | v |

- BUT, someone might guess your CodeText.


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## Vigenère Example

| I | T | I | S | A | T | R | U | T | H |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| I | N | T | H | E | B | E | G | 1 | N |
| $\downarrow$ |  |  |  |  |  |  |  |  |  |
| r | h | C | a | f | v | w | b | C | v |

- BUT, someone might guess your CodeText.
- Better to use random shifts, from a One Time Pad.


## One Time Pad

## Use Random Shifts for each letter

- If your Codeword is long, then the frequency attack is hard.
- It is tempting to use a long text, like "The Bible", as your Codeword.


## Vigenère Example

| I | T | 1 | S | A | T | R | U | T | H |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| I | N | T | H | E | B | E | G | I | N |
| $\downarrow$ |  |  |  |  |  |  |  |  |  |
| r | h | C | a | f | v | w | b | c | v |

- BUT, someone might guess your CodeText.
- Better to use random shifts, from a One Time Pad.
- How can I securely send something random to you? (like a number in a One Time Pad)


## Public Key Cryptography

## Sending Secret Numbers

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- How can I tell you how to encrypt a message, without telling you how to decrypt a message?


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## The $(3,100)$ Cipher

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## Sending Secret Numbers

- How can I tell you how to encrypt a message, without telling you how to decrypt a message?

The $(3,100)$ Cipher

- To send 17 send the last two digits of $17^{3}=17 \times 17 \times 17=49113$;


## Public Key Cryptography

## Sending Secret Numbers

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The $(3,100)$ Cipher

- To send 17 send the last two digits of $17^{3}=17 \times 17 \times 17=49113$;
- Encode: 17 as the number 13 .


## Public Key Cryptography

## Sending Secret Numbers

- How can I tell you how to encrypt a message, without telling you how to decrypt a message?


## The $(3,100)$ Cipher

- To send 17 send the last two digits of $17^{3}=17 \times 17 \times 17=49113$;
- Encode: 17 as the number 13.
- The Public Key is $(3,100)$.


## Public Key Cryptography

## Sending Secret Numbers

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## The $(3,100)$ Cipher

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- In practice we use big numbers, which are hard to factor.


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