Introduction to Number Theory and Cryptography (MAS3214)

Michael C. White

Newcastle University

2016

글 🕨 🖌 글

### Transmitting over an Open Channel

Michael C. White (Newcastle University)

イロト イポト イヨト イヨ

#### Transmitting over an Open Channel

Alice	Bob
$\Theta$ !	Θ?
$\leftarrow \Delta \rightarrow$	$\leftarrow \Box -$
Ţ	Π

 $\neg \rightarrow$ П

イロト イ理ト イヨト イヨト

#### Transmitting over an Open Channel

Alice	Bob
$\Theta$ !	Θ?
$\leftarrow \Delta \rightarrow$	$\leftarrow \Box \rightarrow$
μ	Π

 $\rightarrow$ 

イロト イ団ト イヨト イヨト

#### PLAINTEXT

### Transmitting over an Open Channel

Alice	Bob
$\Theta$ !	⊖ ?
$\leftarrow \Delta \rightarrow$	$\leftarrow \Box \rightarrow$
ΙL	П

PLAINTEXT

 $\Downarrow$  Encode

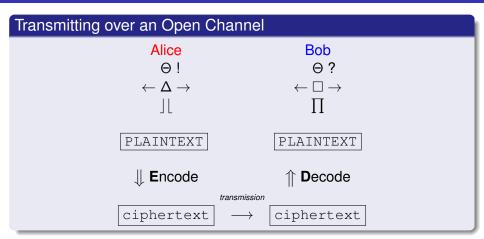
ciphertext

< ロ > < 同 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ >

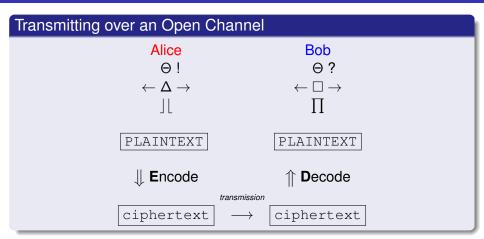
#### Transmitting over an Open Channel

Alice		Bob	
$\Theta$ !		$\Theta$ ?	
$\leftarrow \Delta \rightarrow$		$\leftarrow \Box \rightarrow$	
		Π	
PLAINTEXT			
$\Downarrow$ Encode			
	transmission		
ciphertext	$\longrightarrow$	ciphertext	

イロト イポト イヨト イヨ

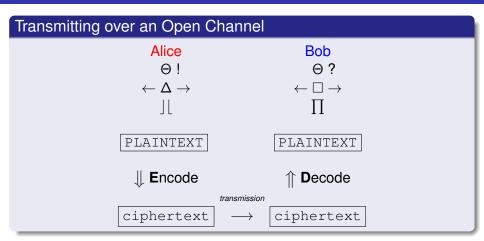


< ロ > < 同 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ >



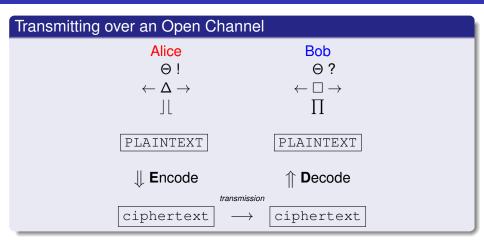
#### Weaknesses

Michael C. White (Newcastle University)



#### Weaknesses

Bob needs to know how to decipher Alice's message.



- Bob needs to know how to decipher Alice's message.
- Someone else may work out how to decode the message.

**Encryption by Shifting Letters** 

イロト イポト イヨト イヨ

### **Encryption by Shifting Letters**



크

イロト イ団ト イヨト イヨト

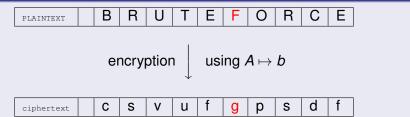
### **Encryption by Shifting Letters**



Weaknesses

イロト イ団ト イヨト イヨト

### **Encryption by Shifting Letters**



#### Weaknesses

• There are only 26 codes to try. [Rot13 is still used.]

イロト イ団ト イヨト イヨト

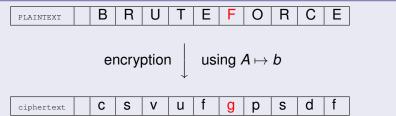
### **Encryption by Shifting Letters**



#### Weaknesses

- There are only 26 codes to try. [Rot13 is still used.]
- This is a rather easy process to do backwards. [zyxwvutsr...]

## **Encryption by Shifting Letters**



#### Weaknesses

- There are only 26 codes to try. [Rot13 is still used.]
- This is a rather easy process to do backwards. [zyxwvutsr...]
- Decode: mpm!

## **Encryption by Shifting Letters**



#### Weaknesses

- There are only 26 codes to try. [Rot13 is still used.]
- This is a rather easy process to do backwards. [zyxwvutsr...]
- Decode: mpm!

#### • LOL!

## **Encryption by Shifting Letters**



#### Weaknesses

- There are only 26 codes to try. [Rot13 is still used.]
- This is a rather easy process to do backwards. [zyxwvutsr...]
- Decode: mpm!

### • LOL!

• How can we make this code more difficult to break?

Michael C. White (Newcastle University)

## The Permutation Cipher

Encryption by Swapping Letters

• • • • • • • • • • • • •

## The Permutation Cipher

## Encryption by Swapping Letters



(I) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1))

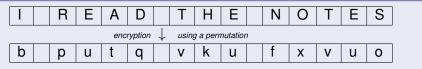


There are  $26! = 26 \times 25 \times \ldots \times 2 \times 1 \approx 4 \times 10^{26}$  codes to try!

(I) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1))



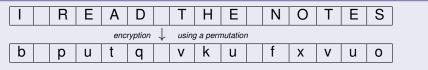
There are  $26! = 26 \times 25 \times \ldots \times 2 \times 1 \approx 4 \times 10^{26}$  codes to try!



There are  $26! = 26 \times 25 \times \ldots \times 2 \times 1 \approx 4 \times 10^{26}$  codes to try!

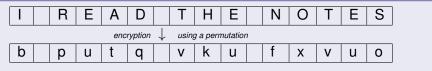
#### Weaknesses

• The single letter is probably "A" or "I";



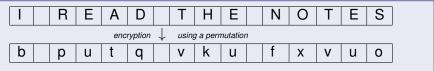
There are  $26! = 26 \times 25 \times \ldots \times 2 \times 1 \approx 4 \times 10^{26}$  codes to try!

- The single letter is probably "A" or "I";
- The commonest letter is probably "E";



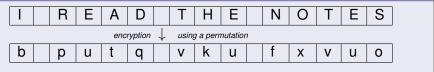
There are  $26! = 26 \times 25 \times \ldots \times 2 \times 1 \approx 4 \times 10^{26}$  codes to try!

- The single letter is probably "A" or "I";
- The commonest letter is probably "E";
- Words are likely to end with "S", if not "E";



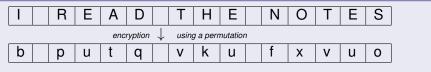
There are  $26! = 26 \times 25 \times \ldots \times 2 \times 1 \approx 4 \times 10^{26}$  codes to try!

- The single letter is probably "A" or "I";
- The commonest letter is probably "E";
- Words are likely to end with "S", if not "E";
- The commonest (English) letters are: E, T, A, O, I, N, S, H, R, D.



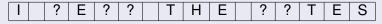
There are  $26! = 26 \times 25 \times \ldots \times 2 \times 1 \approx 4 \times 10^{26}$  codes to try!

- The single letter is probably "A" or "I";
- The commonest letter is probably "E";
- Words are likely to end with "S", if not "E";
- The commonest (English) letters are: E, T, A, O, I, N, S, H, R, D.
- We might guess:



There are  $26! = 26 \times 25 \times \ldots \times 2 \times 1 \approx 4 \times 10^{26}$  codes to try!

- The single letter is probably "A" or "I";
- The commonest letter is probably "E";
- Words are likely to end with "S", if not "E";
- The commonest (English) letters are: E, T, A, O, I, N, S, H, R, D.
- We might guess:



### The Frequency Attack for long messages – Al Kindi

э

• • • • • • • • • • • • •

### The Frequency Attack for long messages – Al Kindi

Ι	Т	Ι	S	А	Т	R	U	Т	Η	
b	V	b	0	t	V	р		V	k	

э

• • • • • • • • • • • • •

### The Frequency Attack for long messages – Al Kindi

Ι	Τ	Ι	S	Α	Т	R	U	Т	H	
b	V	b	0	t	V	р		V	k	•••

### The Frequency Attack for long messages – Al Kindi

Ι	Т	Ι	S	А	Т	R	U	Т	Η	
b	v	b	0	t	v	р	Ι	v	k	

- The commonest letters in English (decreasing order) are:
- E, T, A, O, I, N, S, H, R, D.

### The Frequency Attack for long messages – Al Kindi

Ι	Т	Ι	S	А	Т	R	U	Т	Н	•••
b	v	b	0	t	v	р	I	v	k	•••

- The commonest letters in English (decreasing order) are:
- E, T, A, O, I, N, S, H, R, D.
- The commonest encrypted letters (in our full message) are:
- u, v, t, x, b, f, o, k, p, q.

### The Frequency Attack for long messages – Al Kindi

Ι	Т	Ι	S	А	Т	R	U	Т	Н	•••
b	v	b	0	t	v	р	I	v	k	•••

- The commonest letters in English (decreasing order) are:
- E, T, A, O, I, N, S, H, R, D.
- The commonest encrypted letters (in our full message) are:
- u, v, t, x, b, f, o, k, p, q.
- Decode: fxvu

### The Frequency Attack for long messages – Al Kindi

Ι	Т	Ι	S	Α	Т	R	U	Т	Η	•••
b	v	b	0	t	v	р	I	V	k	•••

- The commonest letters in English (decreasing order) are:
- E, T, A, O, I, <mark>N</mark>, S, H, R, D.
- The commonest encrypted letters (in our full message) are:
- u, v, t, x, b, f, o, k, p, q.
- Decode: fxvu
- NOTE

### The Frequency Attack for long messages – Al Kindi

Ι	Т	Ι	S	А	Т	R	U	Т	Н	•••
b	v	b	0	t	v	р	I	v	k	•••

- The commonest letters in English (decreasing order) are:
- E, T, A, O, I, N, S, H, R, D.
- The commonest encrypted letters (in our full message) are:
- u, v, t, x, b, f, o, k, p, q.
- Decode: fxvu
- NOTE: This leads us to guess the following decryption:

## Breaking the Permutation Cipher

#### The Frequency Attack for long messages – Al Kindi

Ι	Т	Ι	S	А	Т	R	U	Т	Η	• • •
b	v	b	0	t	v	р	I	v	k	•••

#### Weaknesses

- The commonest letters in English (decreasing order) are:
- E, T, A, O, I, N, S, H, R, D.
- The commonest encrypted letters (in our full message) are:
- u, v, t, x, b, f, o, k, p, q.
- Decode: fxvu
- NOTE: This leads us to guess the following decryption:

• IT IS A TR?TH ?NI?ERSA??? A??NO??ED?ED THAT A SIN??E ?AN IN ?OSSESSION O? A ?OOD ?ORT?NE ??ST ?E IN ?ANT O? A ?I?E.

### Defending against the frequency attack

Michael C. White (Newcastle University)

イロト イポト イヨト イヨ

#### Defending against the frequency attack

• Keep changing your code!

< 17 ▶

. . . . . . .

- Keep changing your code!
- If you change the code with every letter it is hard to break.

- Keep changing your code!
- If you change the code with every letter it is hard to break.
- In the Vigenère Cipher your have a 'Codeword', say "ACE".

- Keep changing your code!
- If you change the code with every letter it is hard to break.
- In the Vigenère Cipher your have a 'Codeword', say "ACE".
- The Codeword tells you how to change codes:

- Keep changing your code!
- If you change the code with every letter it is hard to break.
- In the Vigenère Cipher your have a 'Codeword', say "ACE".
- The Codeword tells you how to change codes:
- "ACE" tells us to move letters on by 1, 3, 5, 1, 3, 5, ...

#### Defending against the frequency attack

- Keep changing your code!
- If you change the code with every letter it is hard to break.
- In the Vigenère Cipher your have a 'Codeword', say "ACE".
- The Codeword tells you how to change codes:
- "ACE" tells us to move letters on by 1, 3, 5, 1, 3, 5, ...

### Vigenère Example

#### Defending against the frequency attack

- Keep changing your code!
- If you change the code with every letter it is hard to break.
- In the Vigenère Cipher your have a 'Codeword', say "ACE".
- The Codeword tells you how to change codes:
- "ACE" tells us to move letters on by 1, 3, 5, 1, 3, 5, ...

### Vigenère Example



#### Defending against the frequency attack

- Keep changing your code!
- If you change the code with every letter it is hard to break.
- In the Vigenère Cipher your have a 'Codeword', say "ACE".
- The Codeword tells you how to change codes:
- "ACE" tells us to move letters on by 1, 3, 5, 1, 3, 5, ...

### Vigenère Example



This is harder to decode. However, in this example we just need 3 frequency tables. One for each letter of the Codeword.

Michael C. White (Newcastle University)

Cryptography

### Use Random Shifts for each letter

イロト イ団ト イヨト イヨト

#### Use Random Shifts for each letter

• If your Codeword is long, then the frequency attack is hard.

(I) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1))

#### Use Random Shifts for each letter

- If your Codeword is long, then the frequency attack is hard.
- It is tempting to use a long text, like "The Bible", as your Codeword.

< 4 →

. . . . . . .

#### Use Random Shifts for each letter

- If your Codeword is long, then the frequency attack is hard.
- It is tempting to use a long text, like "The Bible", as your Codeword.

#### Vigenère Example

#### Use Random Shifts for each letter

- If your Codeword is long, then the frequency attack is hard.
- It is tempting to use a long text, like "The Bible", as your Codeword.

#### Vigenère Example



#### Use Random Shifts for each letter

- If your Codeword is long, then the frequency attack is hard.
- It is tempting to use a long text, like "The Bible", as your Codeword.

#### Vigenère Example



BUT, someone might guess your CodeText.

#### Use Random Shifts for each letter

- If your Codeword is long, then the frequency attack is hard.
- It is tempting to use a long text, like "The Bible", as your Codeword.

#### Vigenère Example



- BUT, someone might guess your CodeText.
- Better to use random shifts, from a One Time Pad.

#### Use Random Shifts for each letter

- If your Codeword is long, then the frequency attack is hard.
- It is tempting to use a long text, like "The Bible", as your Codeword.

#### Vigenère Example



- BUT, someone might guess your CodeText.
- Better to use random shifts, from a One Time Pad.
- How can I securely send something random to you? (like a number in a One Time Pad)

Michael C. White (Newcastle University)

Cryptography

Sending Secret Numbers

#### Sending Secret Numbers

• How can I tell you how to encrypt a message, without telling you how to decrypt a message?

A D M A A A M M

. . . . . . .

#### Sending Secret Numbers

• How can I tell you how to encrypt a message, without telling you how to decrypt a message?

#### Sending Secret Numbers

• How can I tell you how to encrypt a message, without telling you how to decrypt a message?

### The (**3**, 100) Cipher

• To send 17 send the last two digits of  $17^3 = 17 \times 17 \times 17 = 49113$ ;

#### Sending Secret Numbers

• How can I tell you how to encrypt a message, without telling you how to decrypt a message?

- To send 17 send the last two digits of  $17^3 = 17 \times 17 \times 17 = 49113$ ;
- Encode: 17 as the number 13.

#### Sending Secret Numbers

 How can I tell you how to encrypt a message, without telling you how to decrypt a message?

- To send 17 send the last two digits of  $17^3 = 17 \times 17 \times 17 = 49113$ ;
- Encode: 17 as the number 13.
- The Public Key is (3, 100).

#### Sending Secret Numbers

 How can I tell you how to encrypt a message, without telling you how to decrypt a message?

- To send 17 send the last two digits of  $17^3 = 17 \times 17 \times 17 = 49113$ ;
- Encode: 17 as the number 13.
- The Public Key is (3, 100).
- It is hard to see how to decode this message.

#### Sending Secret Numbers

 How can I tell you how to encrypt a message, without telling you how to decrypt a message?

- To send 17 send the last two digits of  $17^3 = 17 \times 17 \times 17 = 49113$ ;
- Encode: 17 as the number 13.
- The Public Key is (3, 100).
- It is hard to see how to decode this message.
- To decode: I compute the last two digits of 13<sup>27</sup>, [27 is my Secret!]

#### Sending Secret Numbers

 How can I tell you how to encrypt a message, without telling you how to decrypt a message?

- To send 17 send the last two digits of  $17^3 = 17 \times 17 \times 17 = 49113$ ;
- Encode: 17 as the number 13.
- The Public Key is (3, 100).
- It is hard to see how to decode this message.
- To decode: I compute the last two digits of 13<sup>27</sup>, [27 is my Secret!]

• 
$$13^{27} = (((13)^3)^3)^3) \equiv ((97)^3)^3 \equiv 73^3 \equiv 17.$$

#### Sending Secret Numbers

 How can I tell you how to encrypt a message, without telling you how to decrypt a message?

#### The (**3**, 100) Cipher

- To send 17 send the last two digits of  $17^3 = 17 \times 17 \times 17 = 49113$ ;
- Encode: 17 as the number 13.
- The Public Key is (3, 100).
- It is hard to see how to decode this message.
- To decode: I compute the last two digits of 13<sup>27</sup>, [27 is my Secret!]

• 
$$13^{27} = (((13)^3)^3)^3) \equiv ((97)^3)^3 \equiv 73^3 \equiv 17.$$

• Why 27?

#### Sending Secret Numbers

 How can I tell you how to encrypt a message, without telling you how to decrypt a message?

- To send 17 send the last two digits of  $17^3 = 17 \times 17 \times 17 = 49113$ ;
- Encode: 17 as the number 13.
- The Public Key is (3, 100).
- It is hard to see how to decode this message.
- To decode: I compute the last two digits of 13<sup>27</sup>, [27 is my Secret!]

• 
$$13^{27} = (((13)^3)^3)^3) \equiv ((97)^3)^3 \equiv 73^3 \equiv 17.$$

- Why 27? "Because"  $100 = 2^2 \times 5^2$  and
  - ${\color{black}{3\times27}=1+2^1(2-1)5^1(5-1)\times2}.$

### Sending Secret Numbers

• How can I tell you how to encrypt a message, without telling you how to decrypt a message?

- To send 17 send the last two digits of  $17^3 = 17 \times 17 \times 17 = 49113$ ;
- Encode: 17 as the number 13.
- The Public Key is (3, 100).
- It is hard to see how to decode this message.
- To decode: I compute the last two digits of 13<sup>27</sup>, [27 is my Secret!]
- $13^{27} = (((13)^3)^3)^3) \equiv ((97)^3)^3 \equiv 73^3 \equiv 17.$
- Why 27? "Because"  $100=2^2\times 5^2$  and
  - $3 \times 27 = 1 + 2^{1}(2 1)5^{1}(5 1) \times 2.$
- In practice we use big numbers, which are hard to factor.

### Summary

Michael C. White (Newcastle University)

2

イロト イヨト イヨト イヨト

#### Summary

• The Caesar Cipher quickly hides a message from plain sight.

< ロ > < 同 > < 回 > < 回 >

- The Caesar Cipher quickly hides a message from plain sight.
- The Permutation Cipher shows that even complicated methods have a weakness whenever there is a pattern.

- The Caesar Cipher quickly hides a message from plain sight.
- The Permutation Cipher shows that even complicated methods have a weakness whenever there is a pattern.
- The Vigenère Cipher built a better cipher, based on Caesar.

- The Caesar Cipher quickly hides a message from plain sight.
- The Permutation Cipher shows that even complicated methods have a weakness whenever there is a pattern.
- The Vigenère Cipher built a better cipher, based on Caesar.
- The One Time Pad is ideal, but hard to put into practice.

- The Caesar Cipher quickly hides a message from plain sight.
- The Permutation Cipher shows that even complicated methods have a weakness whenever there is a pattern.
- The Vigenère Cipher built a better cipher, based on Caesar.
- The One Time Pad is ideal, but hard to put into practice.
- The Public Key Cryptography system allows secure transmission, but really needs a computer to implement.

#### Summary

- The Caesar Cipher quickly hides a message from plain sight.
- The Permutation Cipher shows that even complicated methods have a weakness whenever there is a pattern.
- The Vigenère Cipher built a better cipher, based on Caesar.
- The One Time Pad is ideal, but hard to put into practice.
- The Public Key Cryptography system allows secure transmission, but really needs a computer to implement.

#### Summary

- The Caesar Cipher quickly hides a message from plain sight.
- The Permutation Cipher shows that even complicated methods have a weakness whenever there is a pattern.
- The Vigenère Cipher built a better cipher, based on Caesar.
- The One Time Pad is ideal, but hard to put into practice.
- The Public Key Cryptography system allows secure transmission, but really needs a computer to implement.

#### **Further Reading**

Come and join me in MAS3214!

#### Summary

- The Caesar Cipher quickly hides a message from plain sight.
- The Permutation Cipher shows that even complicated methods have a weakness whenever there is a pattern.
- The Vigenère Cipher built a better cipher, based on Caesar.
- The One Time Pad is ideal, but hard to put into practice.
- The Public Key Cryptography system allows secure transmission, but really needs a computer to implement.

- Come and join me in MAS3214!
- More materials are on Blackboard: Example Sheet, and this talk.

#### Summary

- The Caesar Cipher quickly hides a message from plain sight.
- The Permutation Cipher shows that even complicated methods have a weakness whenever there is a pattern.
- The Vigenère Cipher built a better cipher, based on Caesar.
- The One Time Pad is ideal, but hard to put into practice.
- The Public Key Cryptography system allows secure transmission, but really needs a computer to implement.

- Come and join me in MAS3214!
- More materials are on Blackboard: Example Sheet, and this talk.
- At Newcastle you begin to study Number Theory in your first term.

#### Summary

- The Caesar Cipher quickly hides a message from plain sight.
- The Permutation Cipher shows that even complicated methods have a weakness whenever there is a pattern.
- The Vigenère Cipher built a better cipher, based on Caesar.
- The One Time Pad is ideal, but hard to put into practice.
- The Public Key Cryptography system allows secure transmission, but really needs a computer to implement.

- Come and join me in MAS3214!
- More materials are on Blackboard: Example Sheet, and this talk.
- At Newcastle you begin to study Number Theory in your first term.
- sgzmj xnt!

#### Summary

- The Caesar Cipher quickly hides a message from plain sight.
- The Permutation Cipher shows that even complicated methods have a weakness whenever there is a pattern.
- The Vigenère Cipher built a better cipher, based on Caesar.
- The One Time Pad is ideal, but hard to put into practice.
- The Public Key Cryptography system allows secure transmission, but really needs a computer to implement.

- Come and join me in MAS3214!
- More materials are on Blackboard: Example Sheet, and this talk.
- At Newcastle you begin to study Number Theory in your first term.
- sgzmj xnt! ... I mean ... THANK YOU!