The Storm of the Century! (Statistics of Extremes)

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Who am I?

- **Senior Lecturer** in the School of Maths, Stats & Physics at Newcastle University
- **RSS William Guy Lecturer**, 2018–19
- **Applied statistician**
- **Teach**: 1st/2nd/3rd year students
- **Research**: Statistical models for extreme weather events
- **Other stuff**: Outreach, recruitment, consultancy, schools...

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Background: Statistics of extremes

So many areas of Statistics are interested in what happens **on average**: Mean, median, mode...

One area that is different is the field of **extreme value theory**.

Why might we be interested in **extremes**, rather than averages?

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Earth Scientists – for example, **meteorologists**, **seismologists**, **hydrologists** and **oceanographers** – predict and assess the likelihood of storms, earthquakes, volcanoes and changes in sea level.

We can usually deal with averages here – it is the **extremes** that cause devastation!

Such scientists require the expertise of **mathematicians** to help with **predictions** and **calculations**.

In this talk I will focus on how maths/statistics can be used to help oceanographers and civil engineers to predict extreme **sea-surges**.

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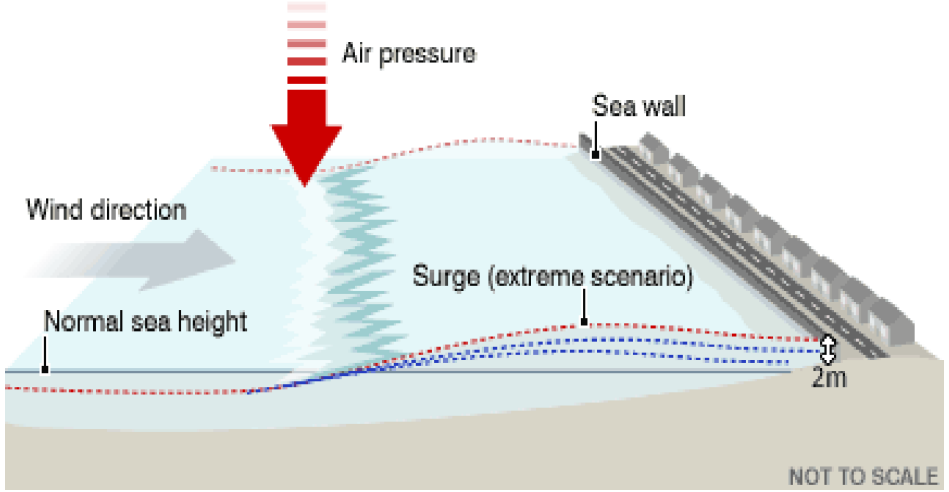
Sea-surge


Air pressure
Sea wall
Wind direction
Surge (extreme scenario)
Normal sea height
2m
NOT TO SCALE

CHICAGO
DAZZLE
MOVIN' OUT
FREE
Washington Mutual

86
Street

FreakingNews.com

Sea-surge: The Great North Sea Flood, 1953

- Killed **2,551 people** in the UK, Holland and Belgium
- Estimated as the "**Storm of the Century**"
- Often referred to as "**Europe's Katrina**"

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Stormy weather in the UK!

- 5 December 2015: **Storm Desmond** – more than a month's rain in parts of Cumbria, flooding in Carlisle
- 22 December 2015: Further flooding in Cumbria
- Christmas Day: **Storm Eva** – 106 flood alerts in UK
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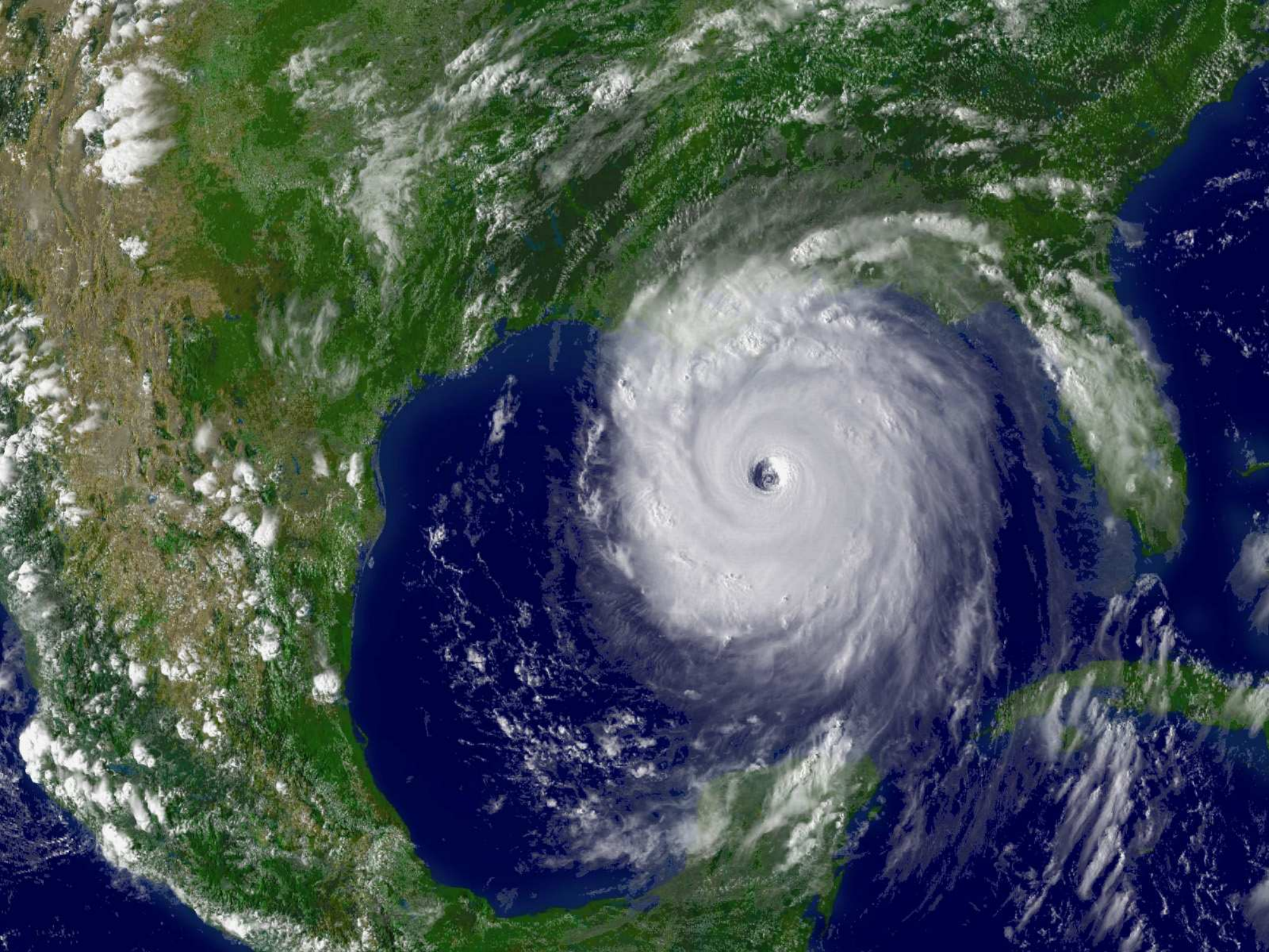

Hurricane Katrina: New Orleans, August 2005

Some facts

- Category 5 Hurricane (Category 3 when it made landfall)
- Killed 1833 people
- Caused $108 bn worth of damage
- Most damage/loss of life caused by **storm surge**
- Storm surge reached 14.4 feet above sea level
- Lowest air pressure 902 mb
- Political controversy
- Billed as the “storm of the century”

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www.KatrinaDestruction.com

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Data application: Sea surges at New Orleans

Below are the annual maximum sea-surge levels observed at New Orleans over a 50 year period *before Hurricane Katrina*:

8.5	8.9	9.1	8.9	8.4	9.7	9.1	9.6	8.7	9.3
9.6	9.3	8.7	9.0	8.8	8.9	8.9	12.2	7.8	7.7
8.3	8.1	7.3	6.8	6.7	7.3	7.6	8.2	8.6	9.8
9.5	7.4	7.3	10.2	10.3	10.4	8.8	9.7	10.0	10.8
11.1	12.7	11.5	11.8	12.6	13.0	10.5	10.5	10.0	9.4

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Data application: Sea surges at New Orleans

Suppose you are the mathematician on the team of scientists working on a new section of flood defence system in New Orleans.

One of the civil engineers asks you to work out some exceedance probabilities for her.

In particular, she wants to know the probability that, this year, the annual maximum sea surge at New Orleans will

1. exceed **8.75 feet**;
2. exceed **11.25 feet**;
3. exceed **14 feet**.

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The relative frequency approach

You will have used this approach to probability in your maths at School.

It helps if the data are re–written in ascending order:

6.7	6.8	7.3	7.3	7.3	7.4	7.6	7.7	7.8	8.1
8.2	8.3	8.4	8.5	8.6	8.7	8.7	8.8	8.8	8.9
8.9	8.9	8.9	9.0	9.1	9.1	9.3	9.3	9.4	9.5
9.6	9.6	9.7	9.7	9.8	10.0	10.0	10.2	10.3	10.4
10.5	10.5	10.8	11.1	11.5	11.8	12.2	12.6	12.7	13.0

Then,

$$P(\text{sea-surge exceeds 8.75 feet}) =$$

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Then,

$$P(\text{sea-surge exceeds 8.75 feet}) = \frac{33}{50} = \frac{66}{100} = 0.66.$$

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Using the relative frequency approach, work out the other two exceedance probabilities:

- P(sea-surge exceeds 11.25 feet)
- P(sea-surge exceeds 14 feet)

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9.6	9.6	9.7	9.7	9.8	10.0	10.0	10.2	10.3	10.4
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$P(\text{sea-surge exceeds 11.25 feet}) =$

The relative frequency approach

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$$P(\text{sea-surge exceeds 11.25 feet}) = \frac{6}{50} = 0.12$$

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10.5	10.5	10.8	11.1	11.5	11.8	12.2	12.6	12.7	13.0

$P(\text{sea-surge exceeds 14 feet}) =$

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8.2	8.3	8.4	8.5	8.6	8.7	8.7	8.8	8.8	8.9
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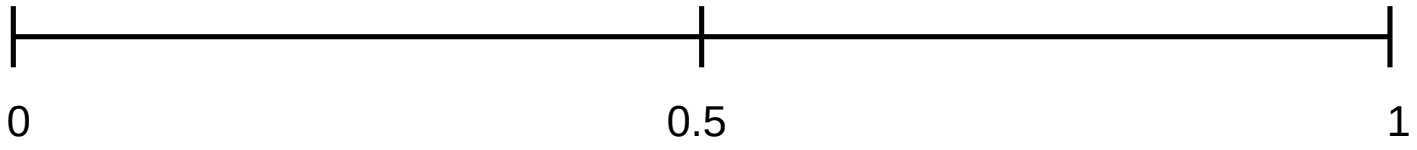
The relative frequency approach

Think about the **probability scale**:



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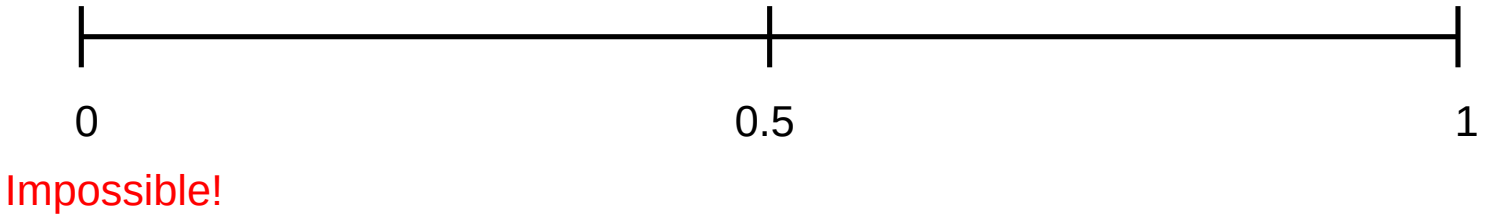

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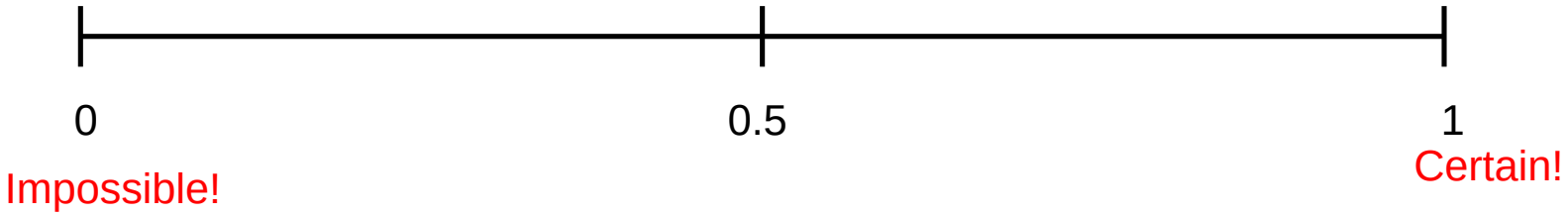

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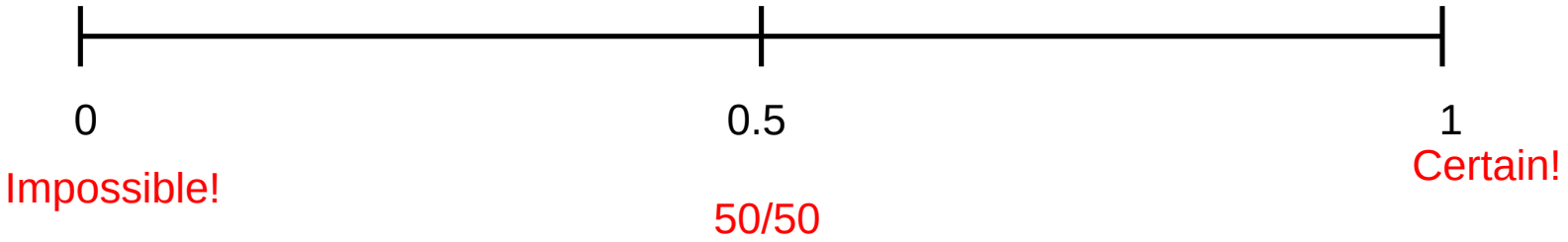

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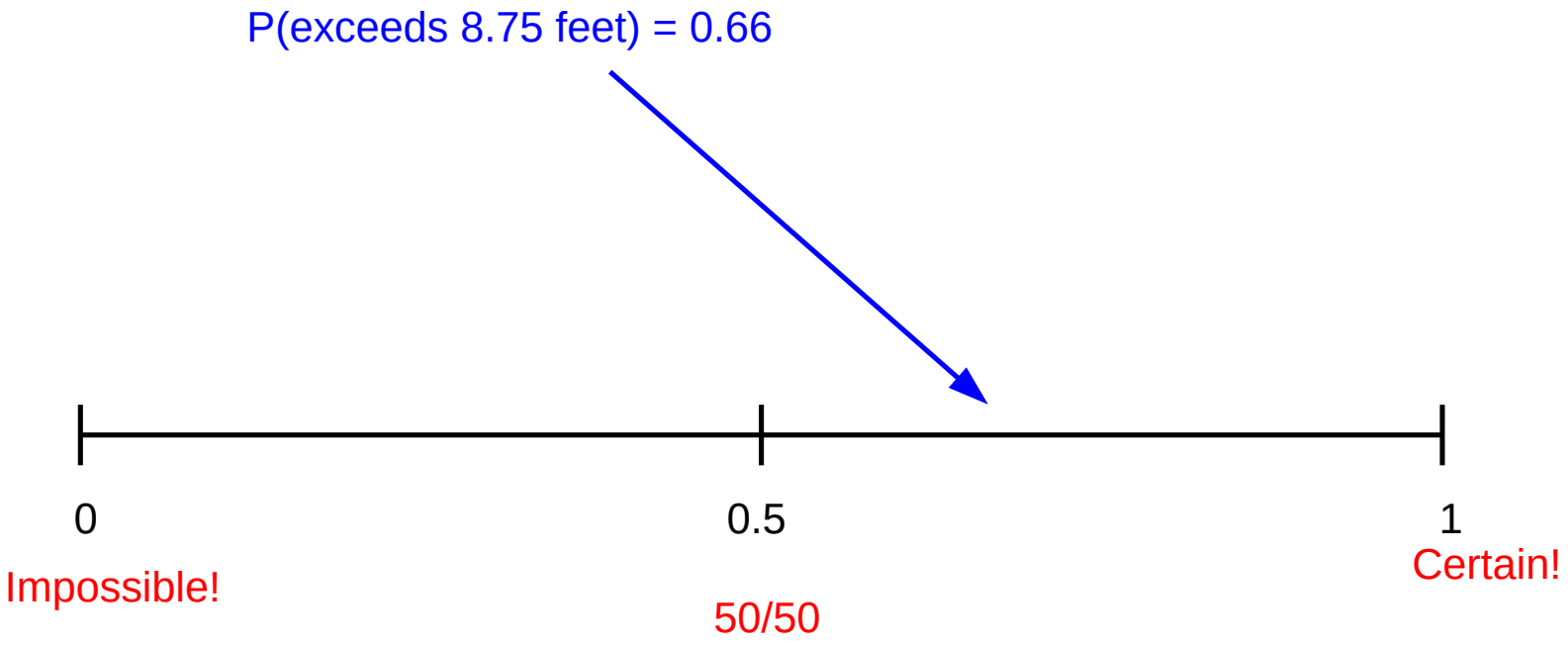

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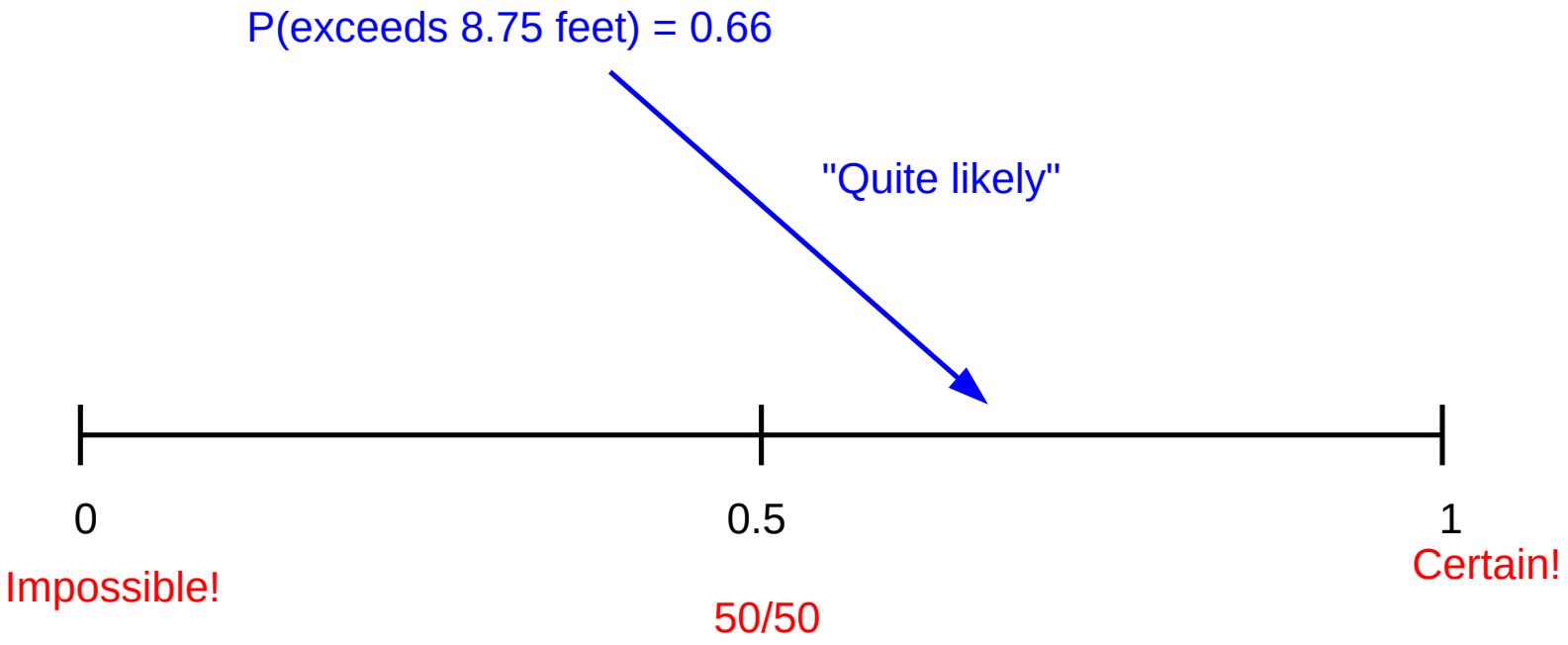

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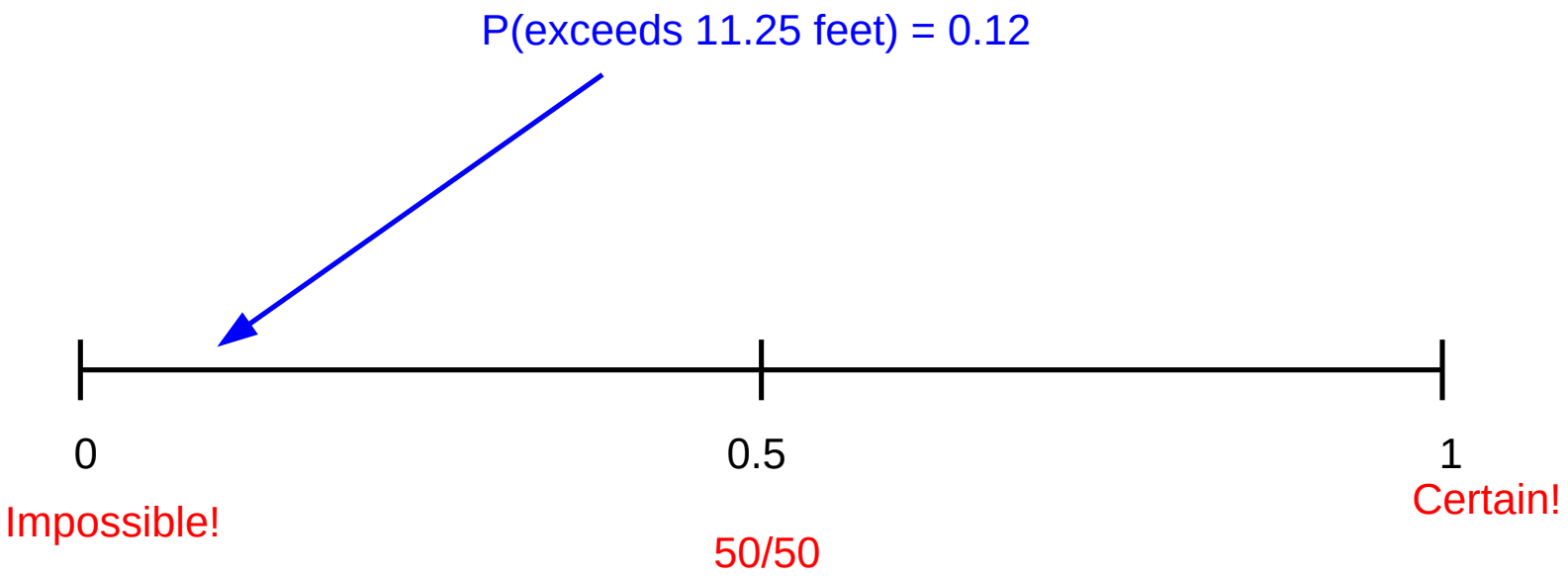

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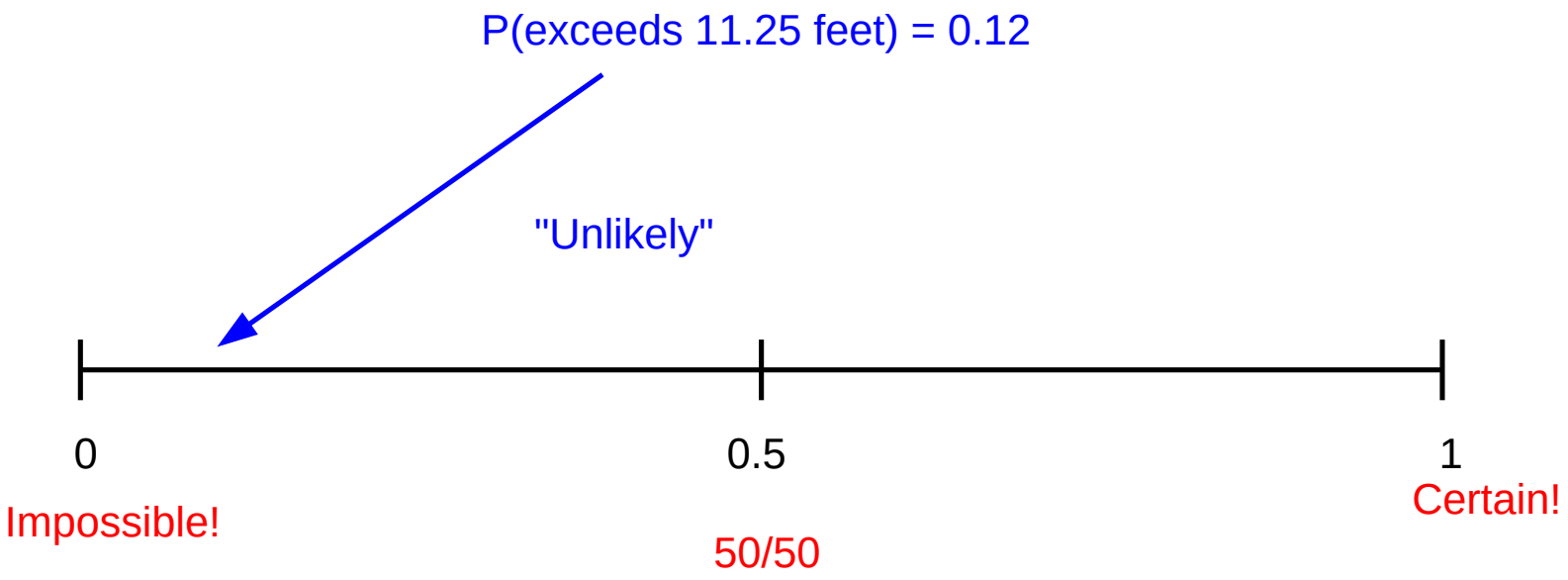

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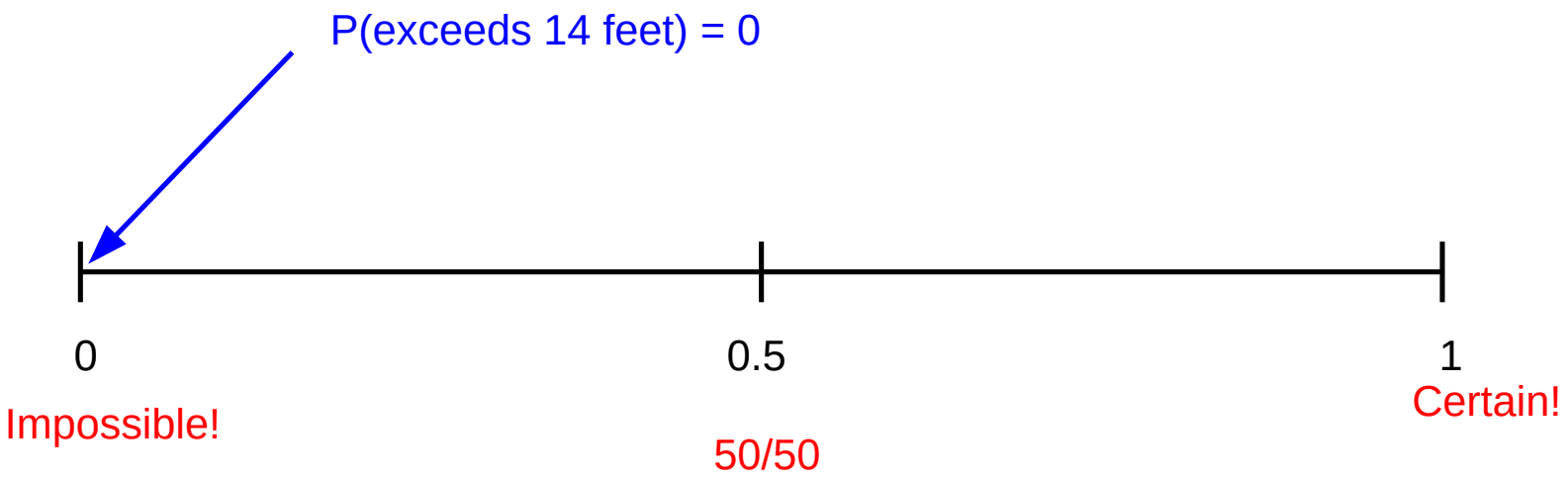

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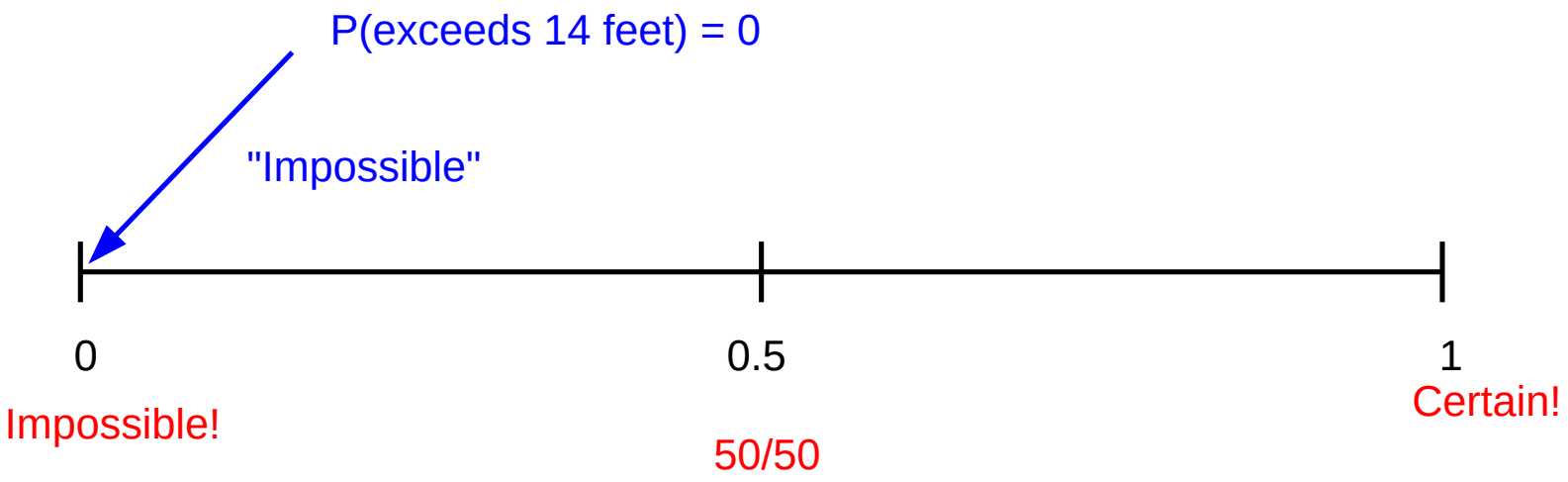

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What is wrong with the exceedance probability associated with a sea-surge of 14 feet?

Our answer suggests a sea-surge of more than 14 feet is impossible – however, this *did* happen during Hurricane Katrina (sea-surges reached 14.4 feet!).

Probability models provide a way forward here.

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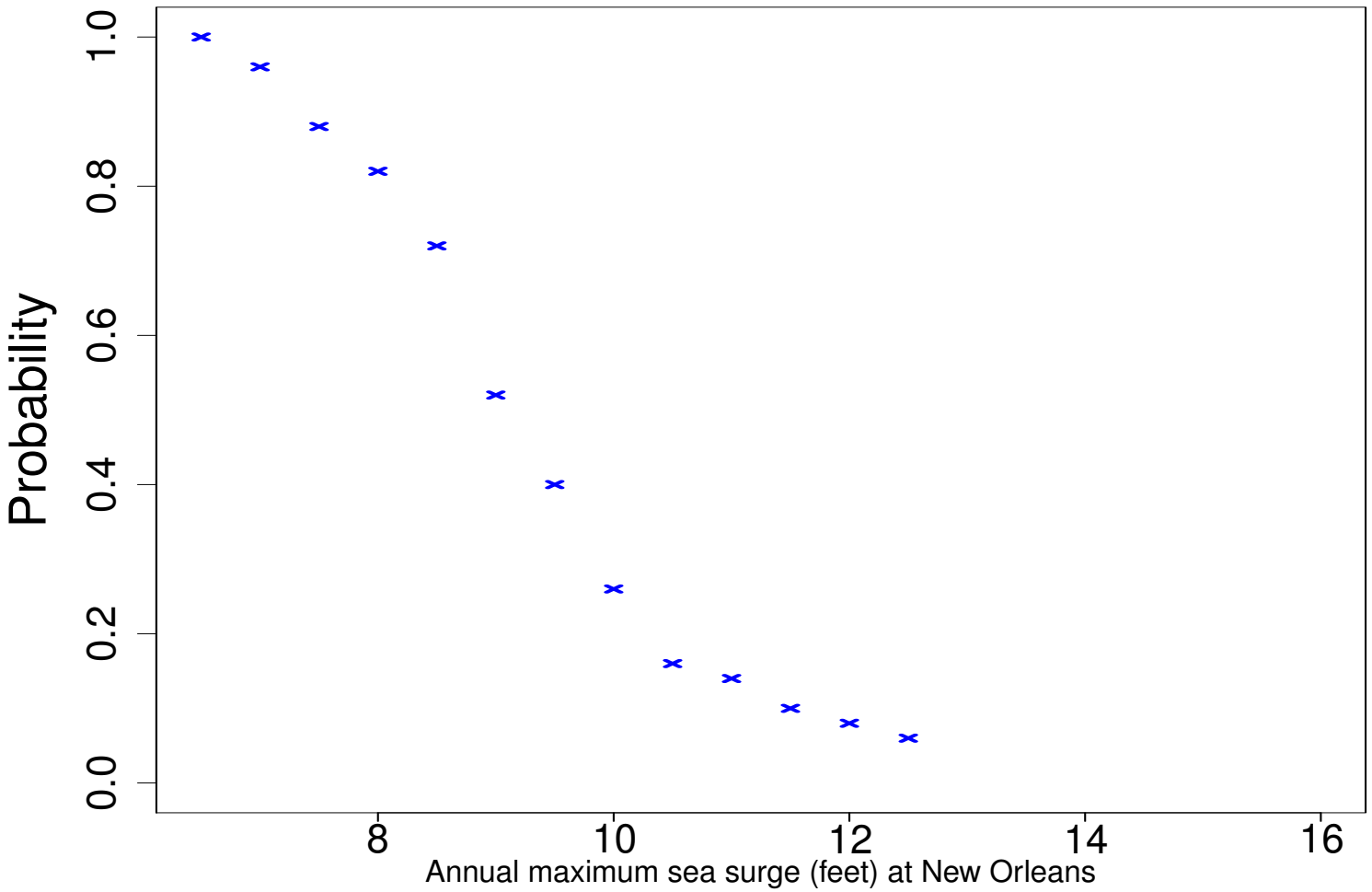

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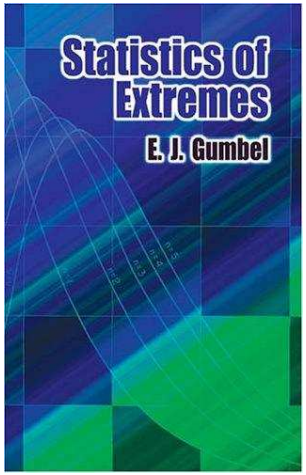

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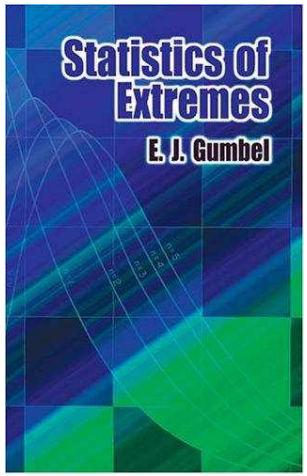

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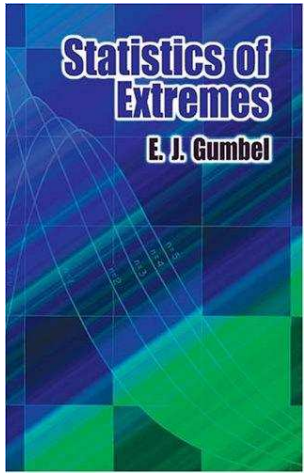

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$$\boldsymbol{P}(X > x) = 1 - \exp\left[-\exp\left\{-\left(\frac{x-\mu}{\sigma}\right)\right\}\right],$$

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Using techniques students learn about at University, we can use the data to estimate the values of the location and scale parameters.

Doing so, gives

$$\mu = 8.536 \qquad \text{and} \qquad \sigma = 1.2$$

for our dataset. Plugging these into Gumbel's formula gives

$$\boldsymbol{P}(X > x) = 1 - \exp\left[-\exp\left\{-\left(\frac{x - 8.536}{1.2}\right)\right\}\right].$$

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$$P(X > 10) = 1 - \exp\left[-\exp\left\{-\left(\frac{10 - 8.536}{1.2}\right)\right\}\right] = \mathbf{0.26}$$

(= 0.26 from the data)

$$P(X > 11.5) = 1 - \exp\left[-\exp\left\{-\left(\frac{11.5 - 8.536}{1.2}\right)\right\}\right] = \mathbf{0.08}$$

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$$P(X > 14) = 1 - \exp\left[-\exp\left\{-\left(\frac{14 - 8.536}{1.2}\right)\right\}\right] = \mathbf{0.01}$$

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We could use Gumbel's formula for lots of different values of x and then plot them on the same graph as the real data.

What do you notice when we do this?

The **Gumbel** model is not the only model to choose from:

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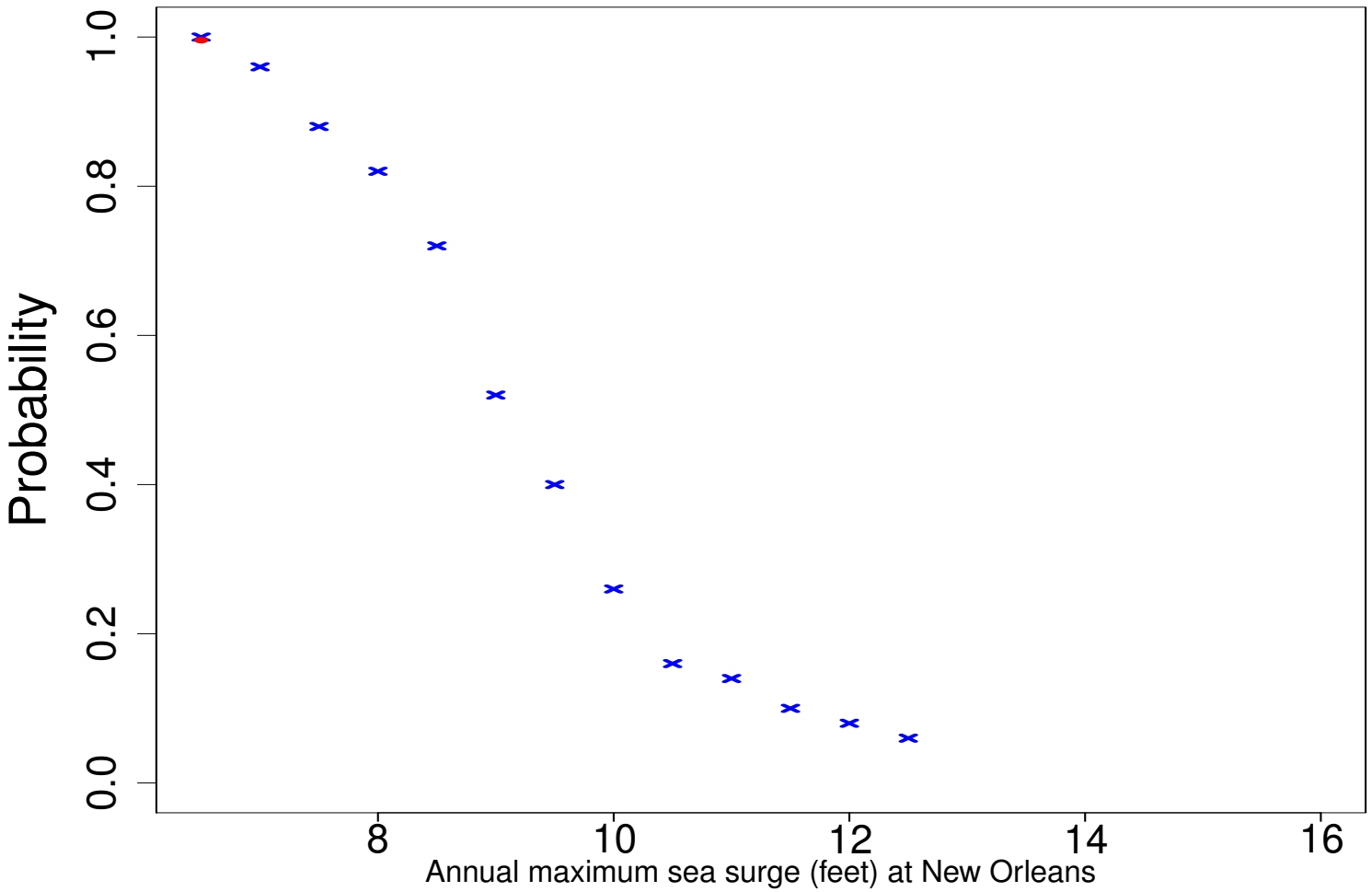

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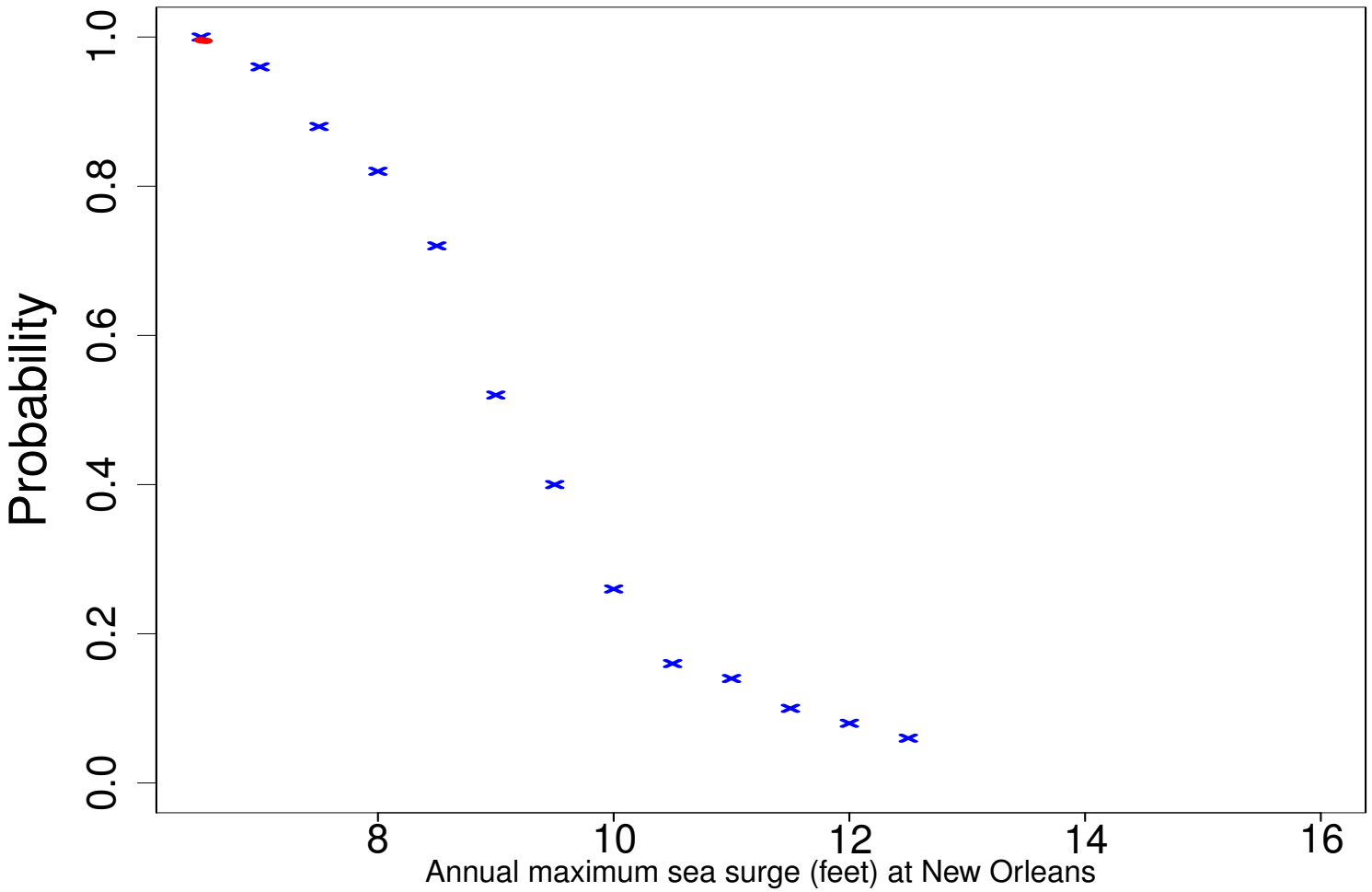

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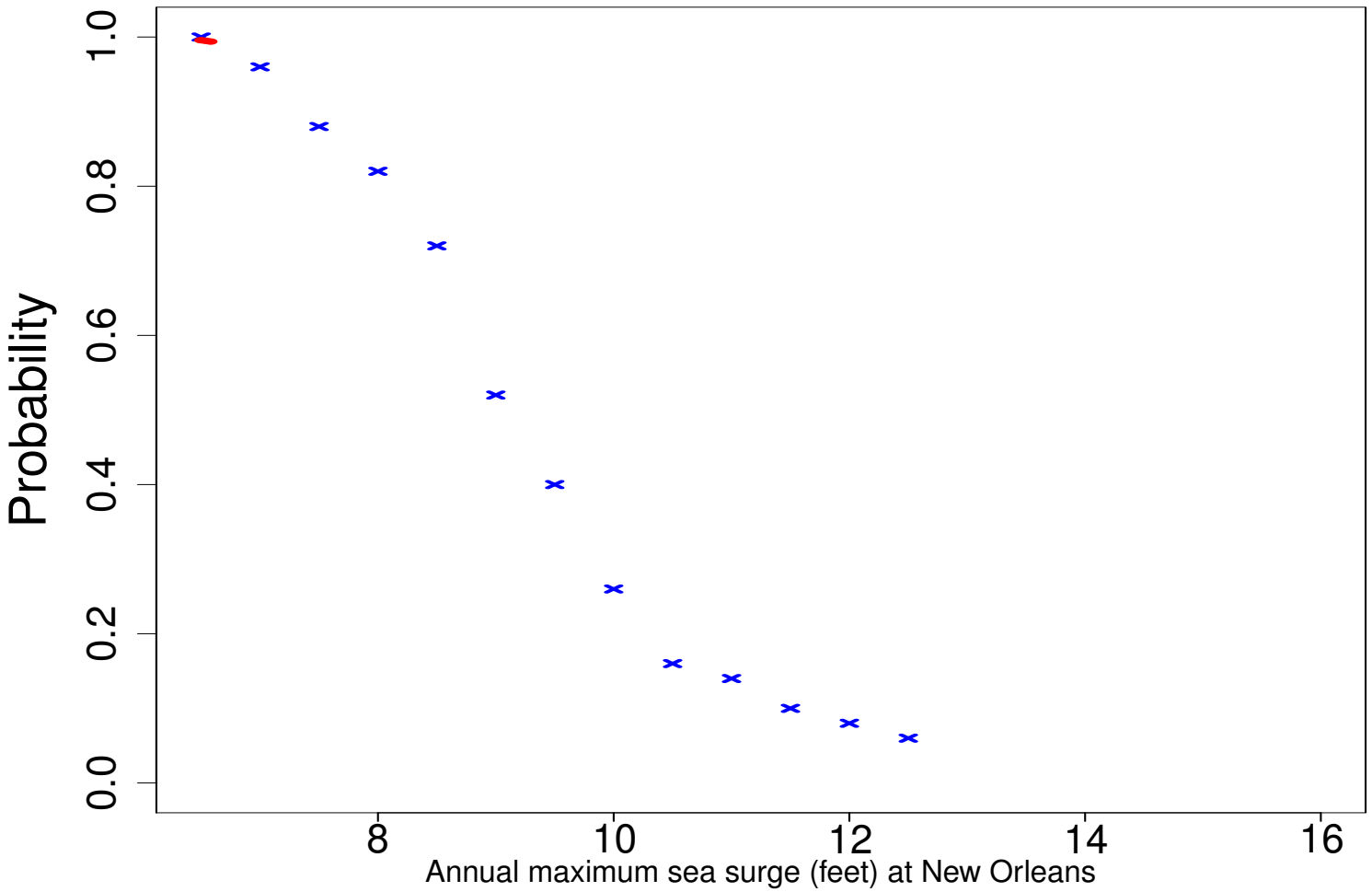

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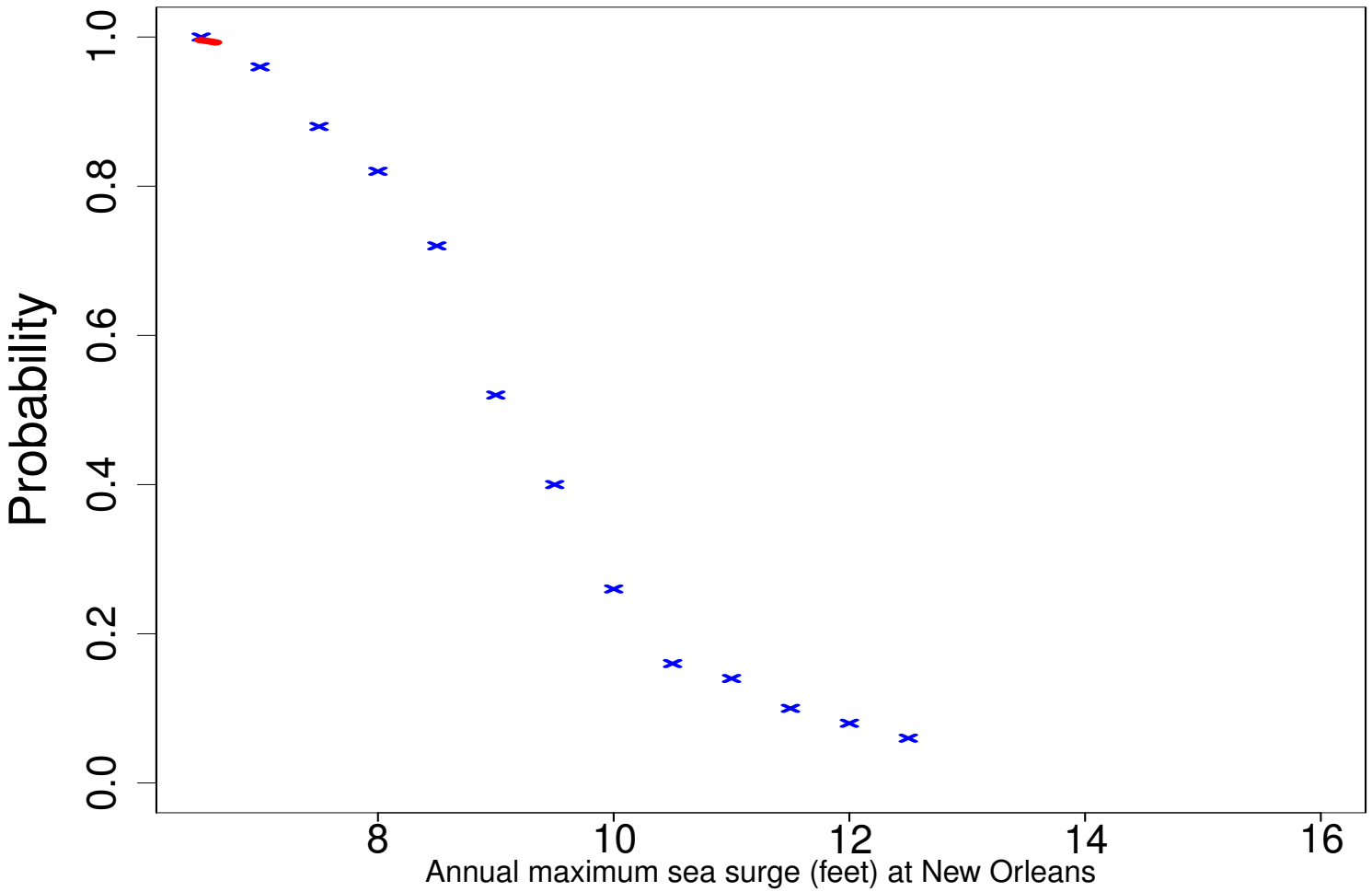

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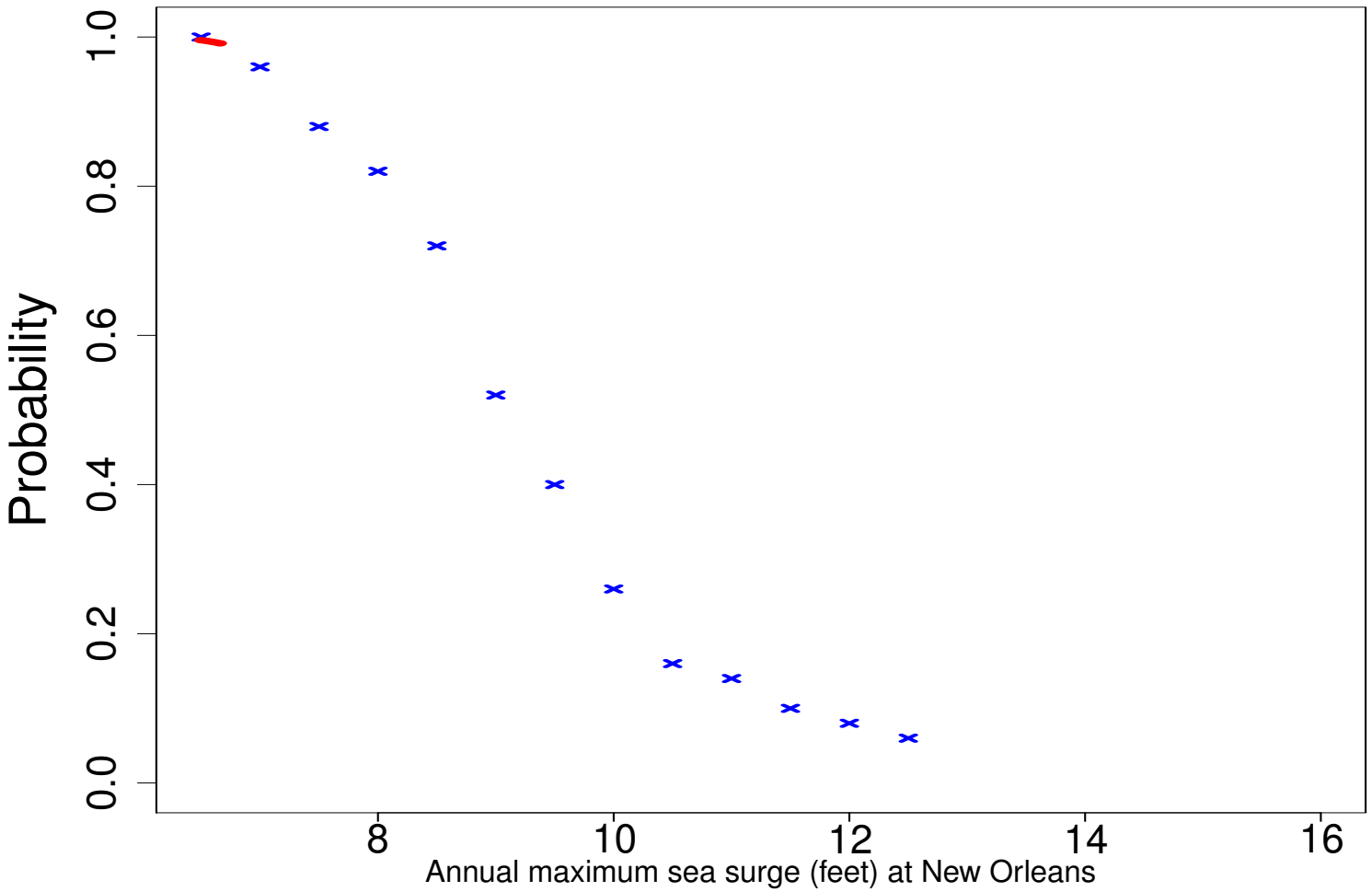

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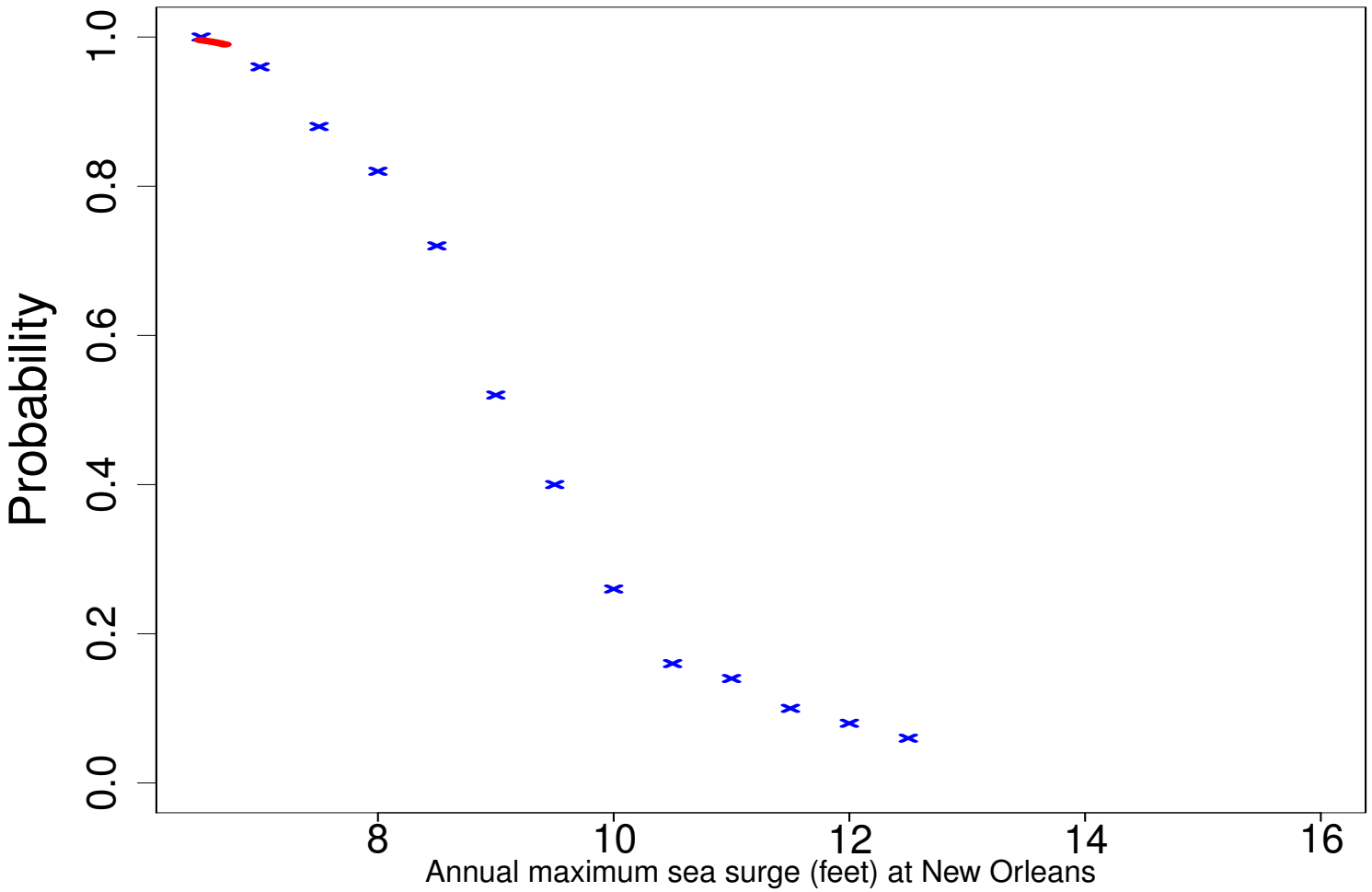

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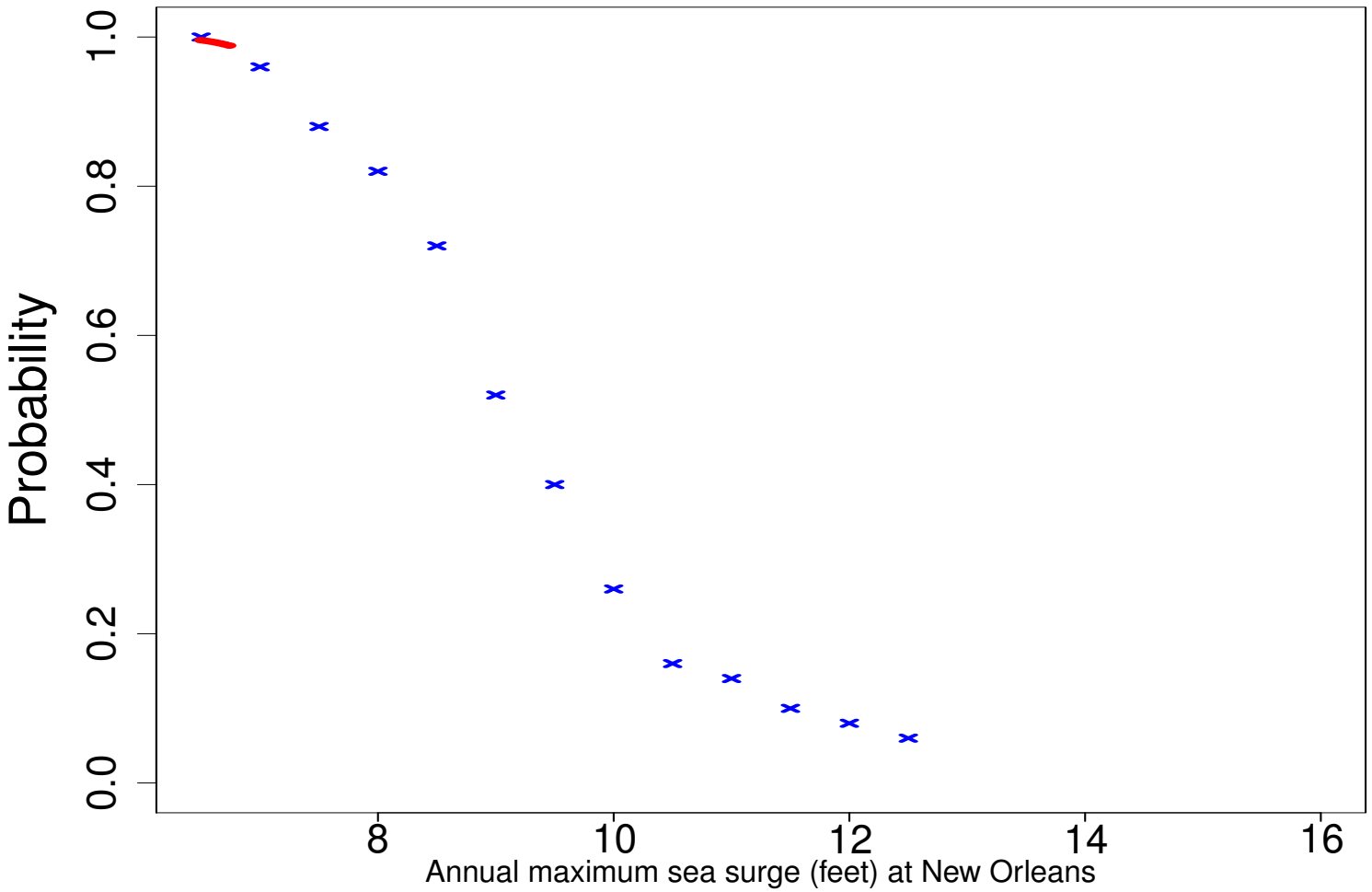

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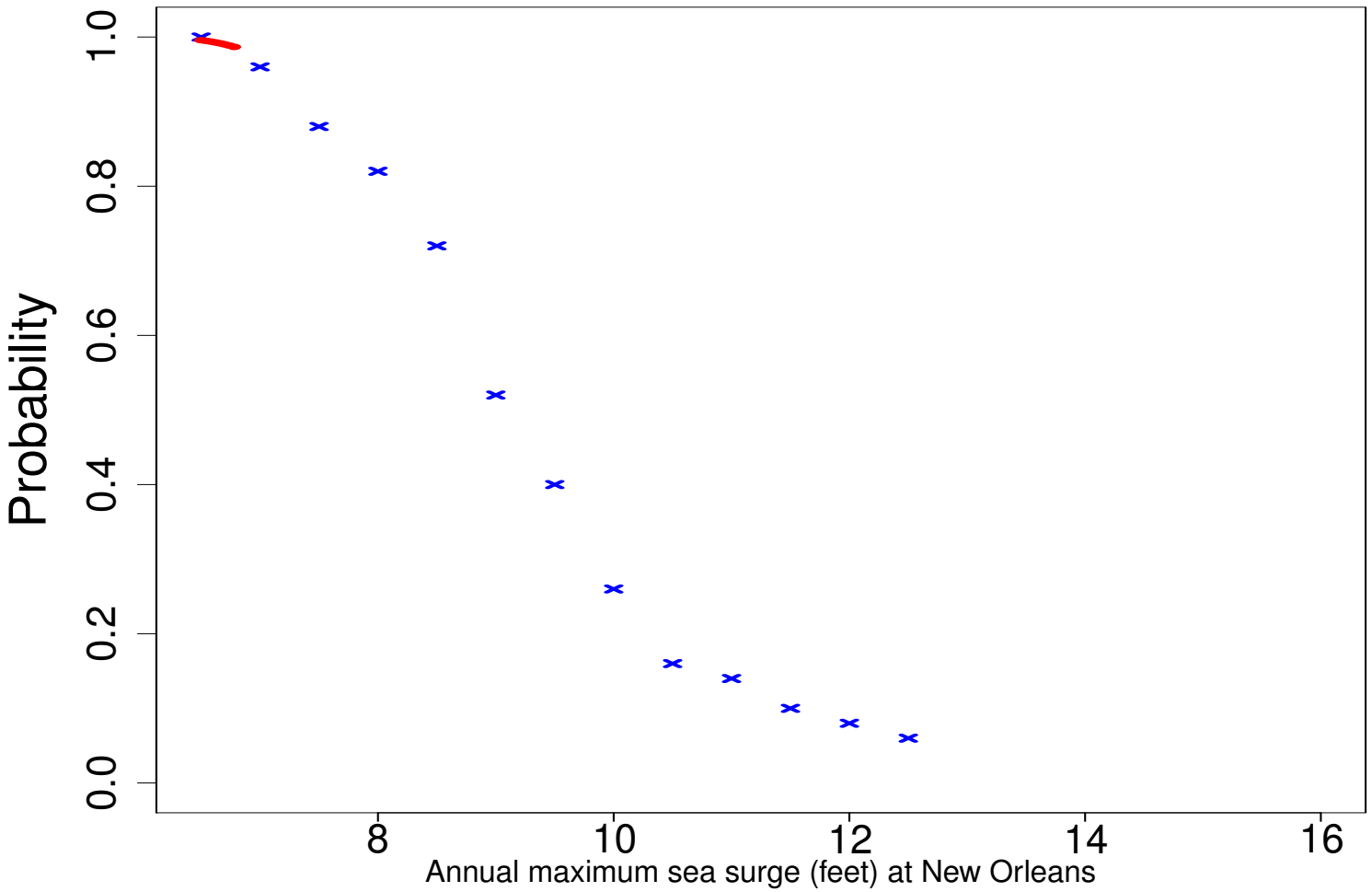

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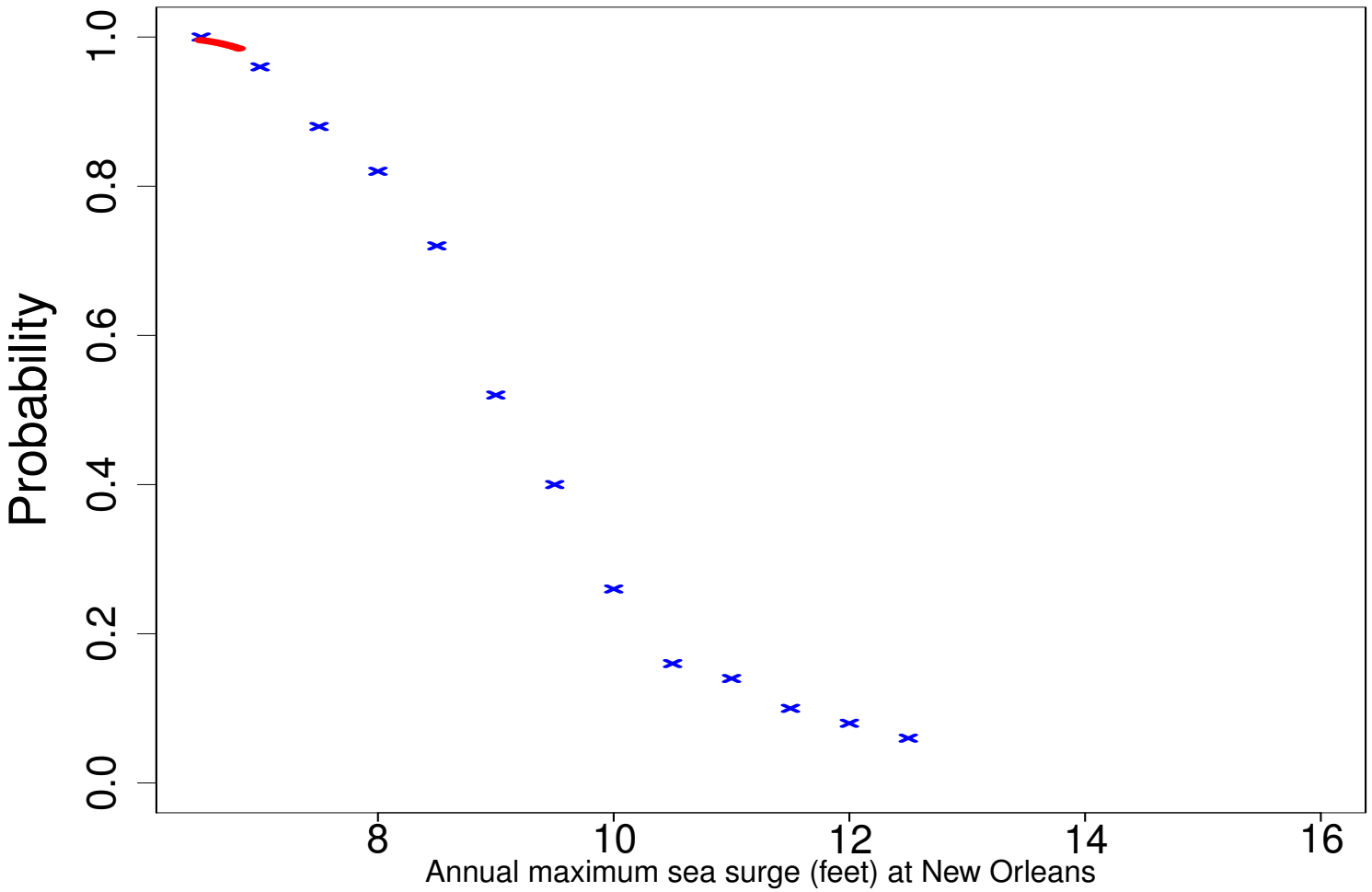

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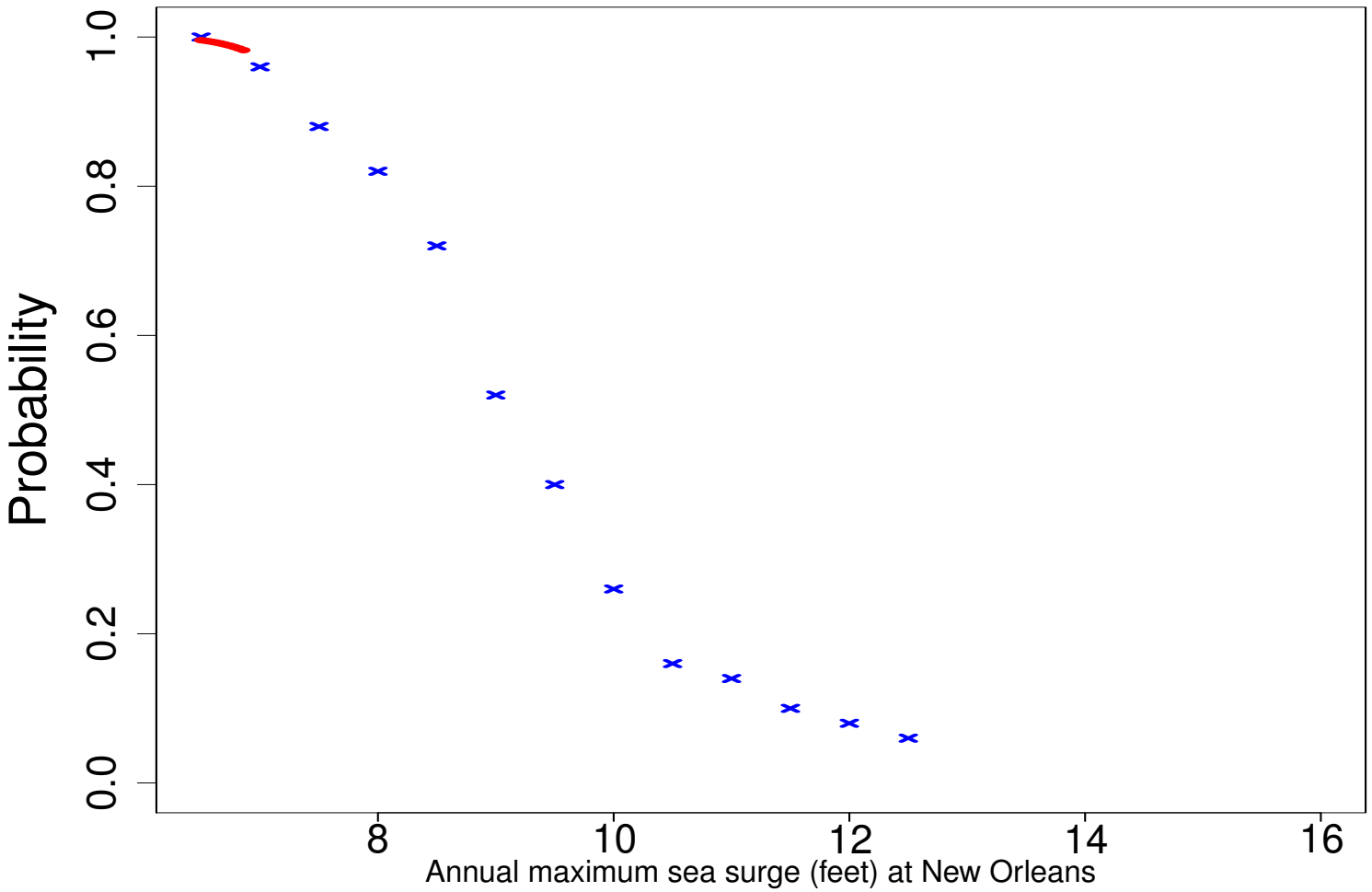

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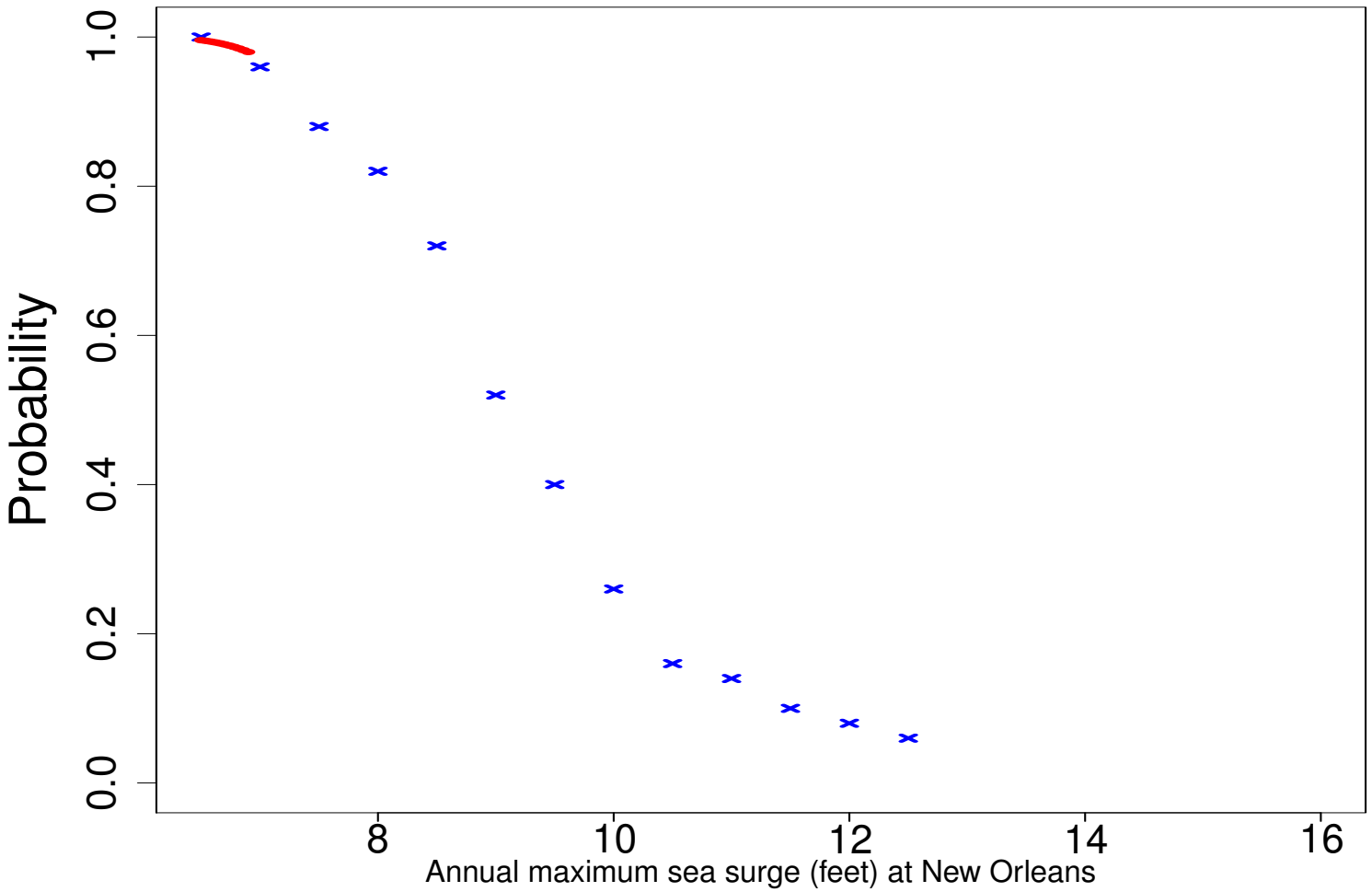

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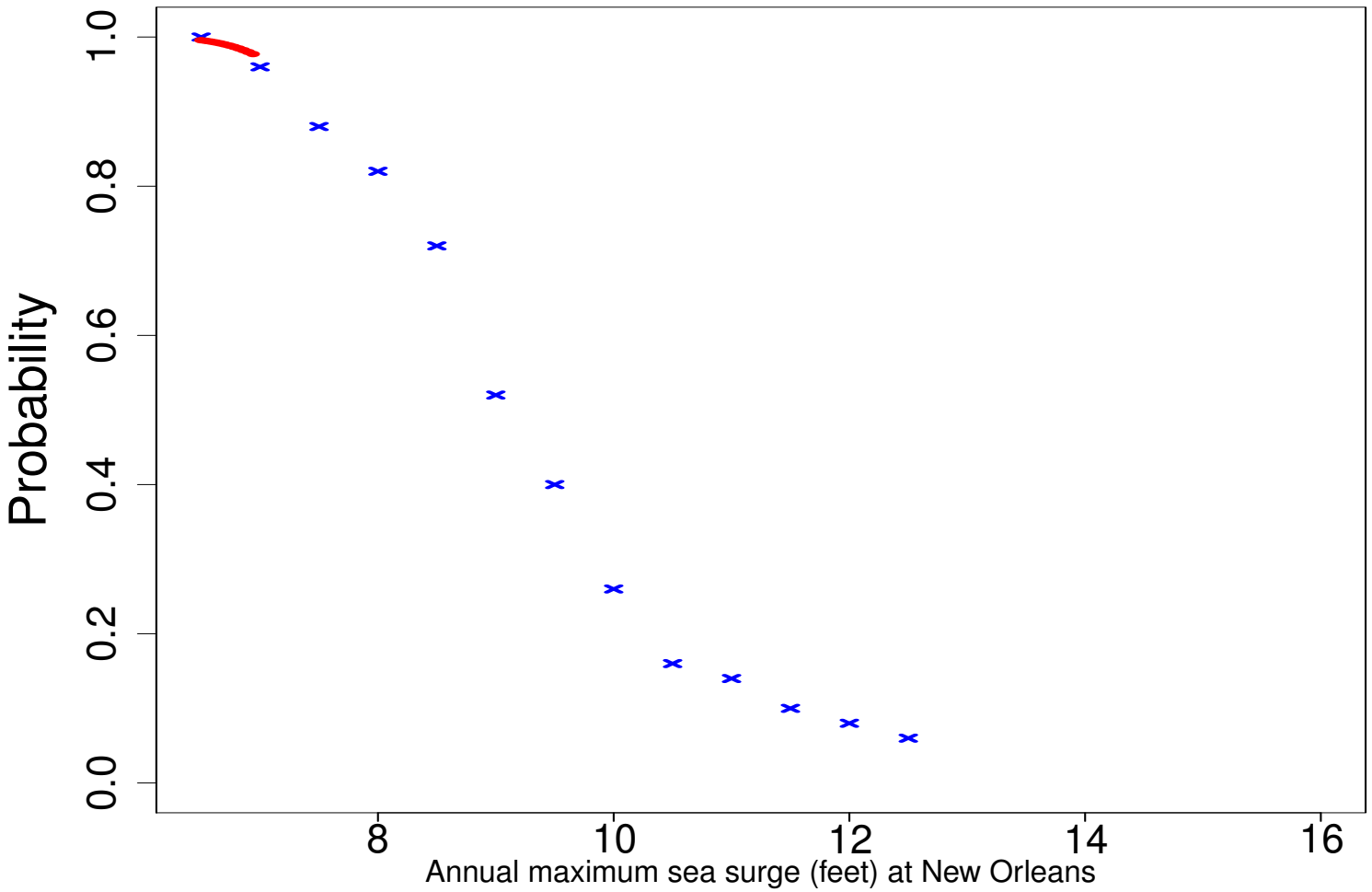

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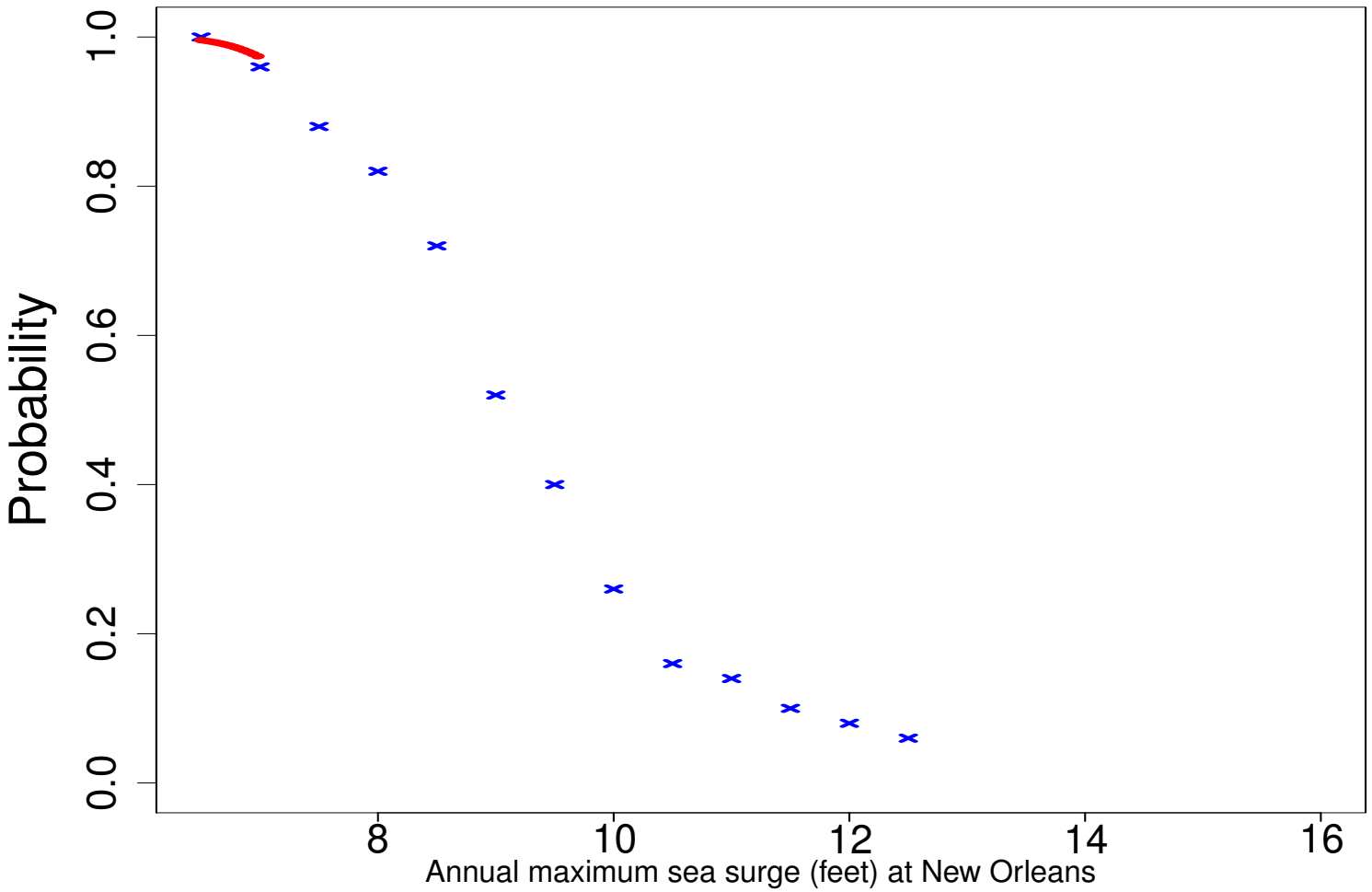

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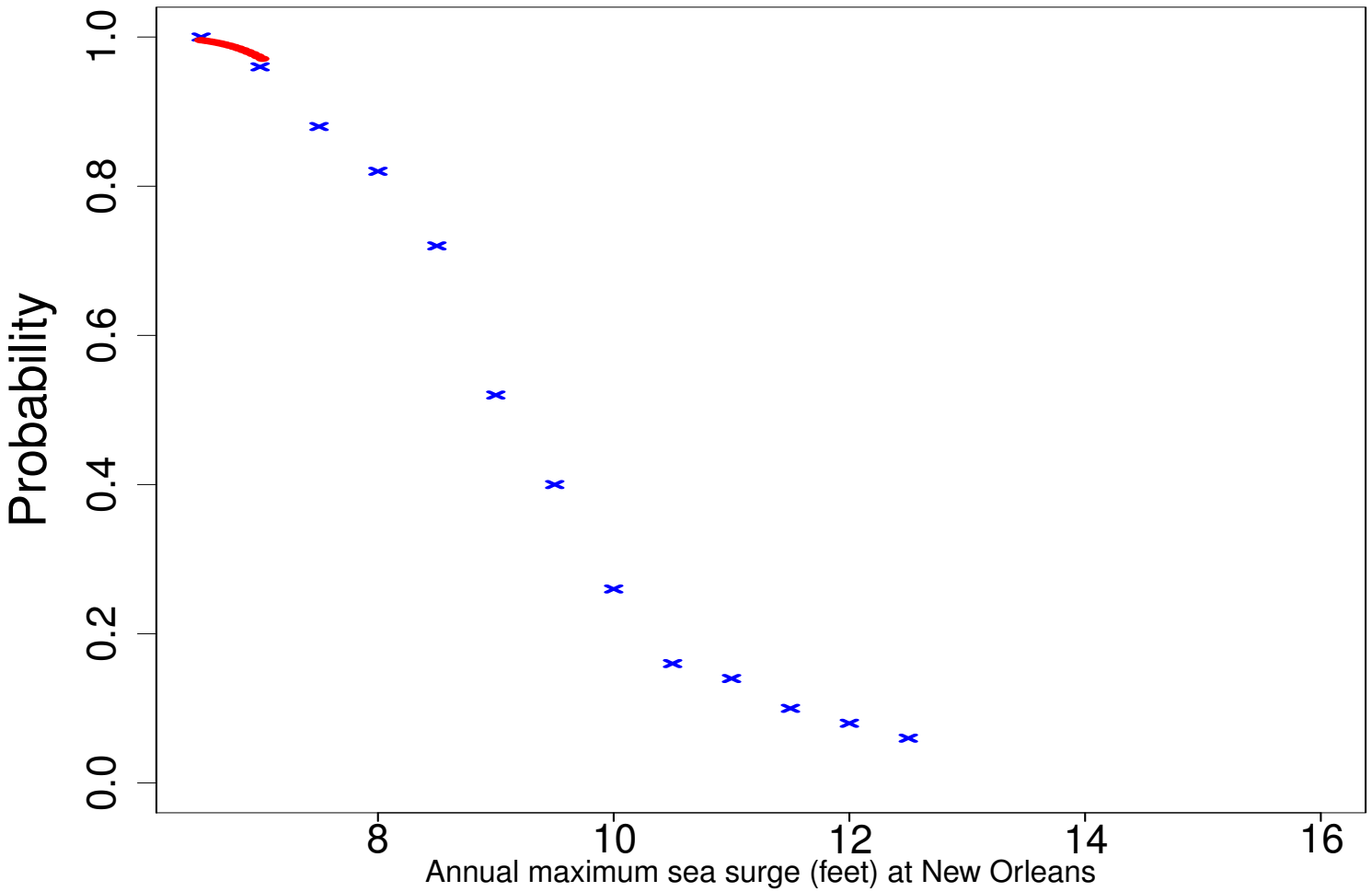

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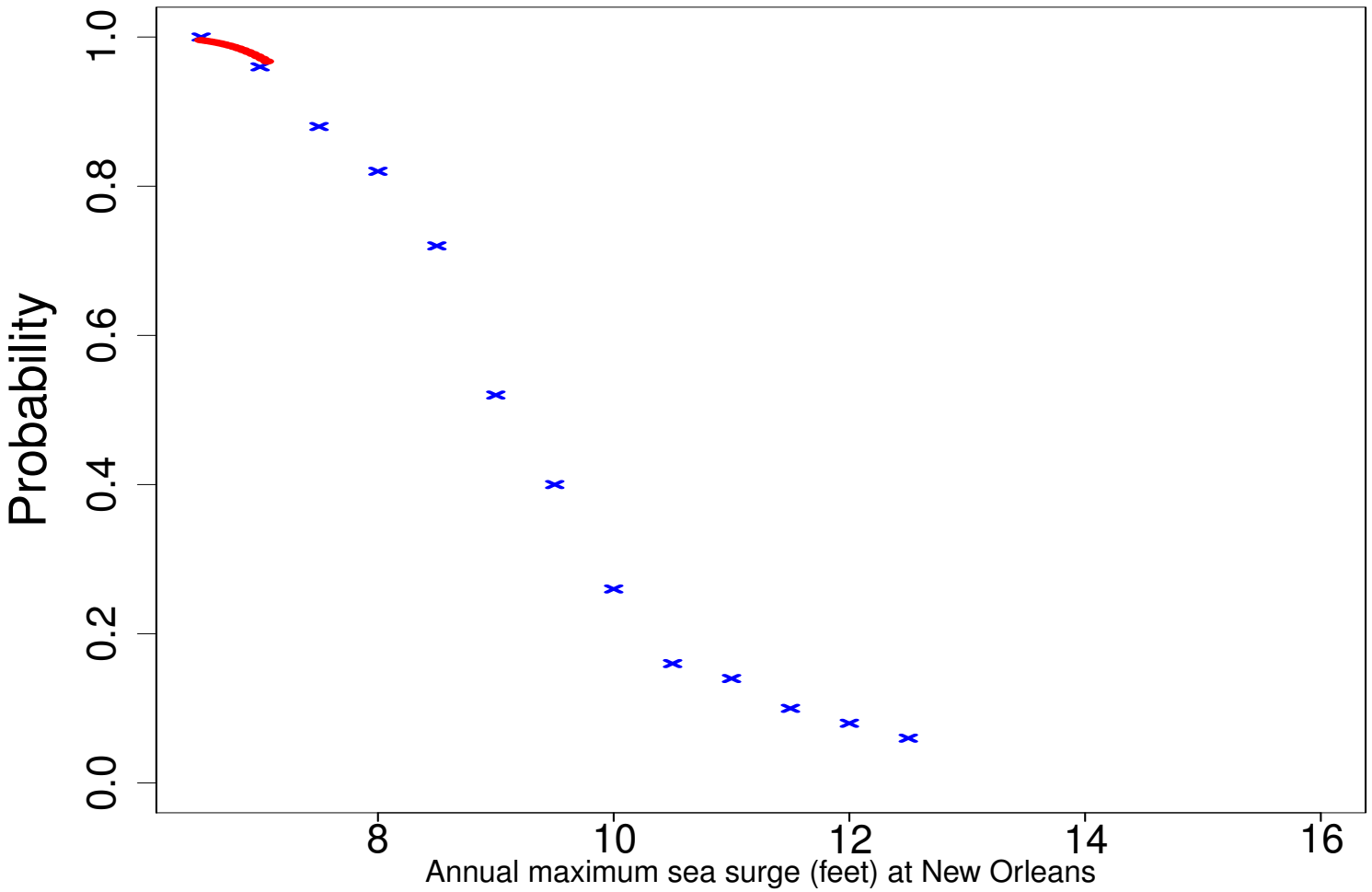

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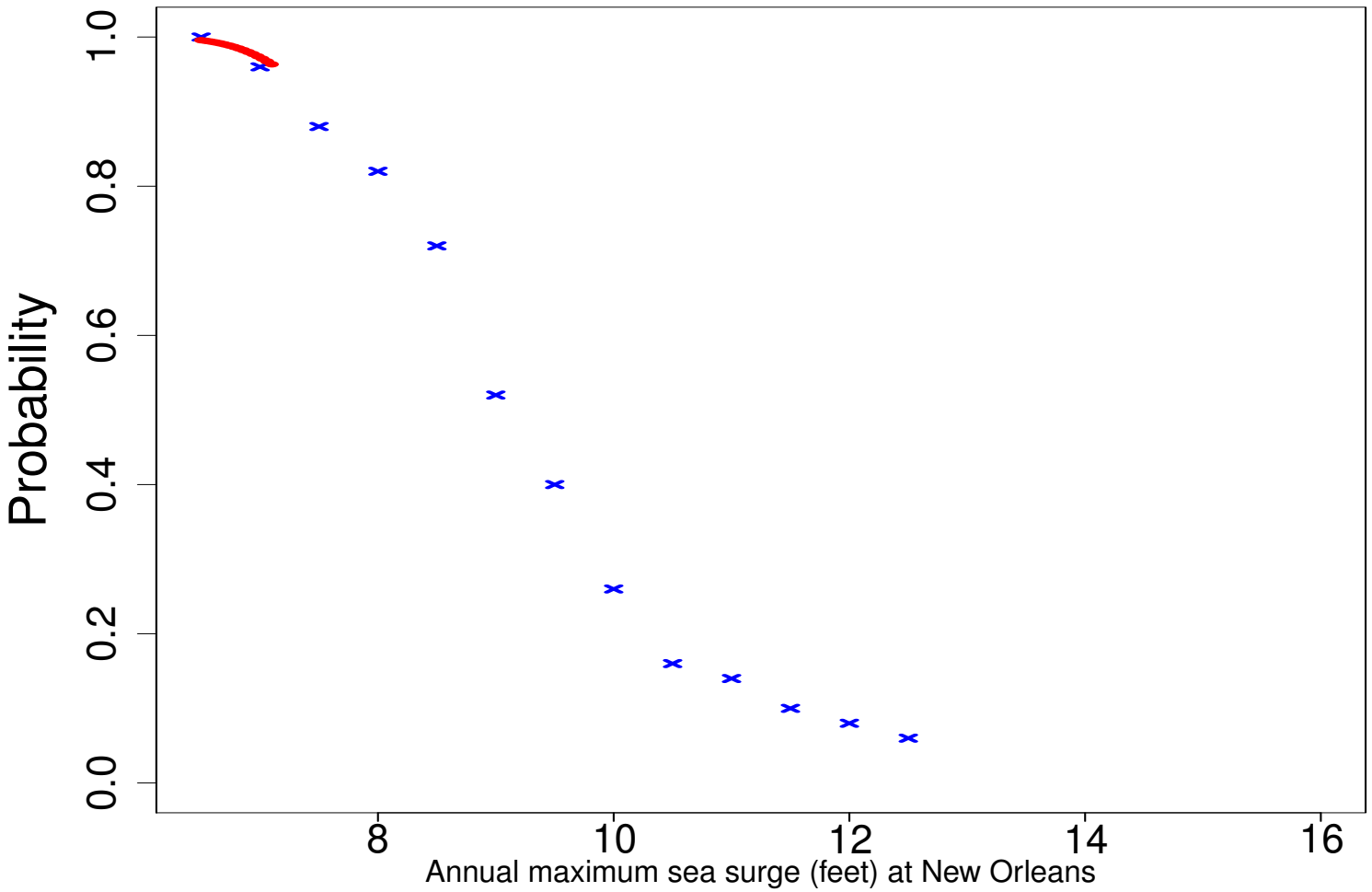

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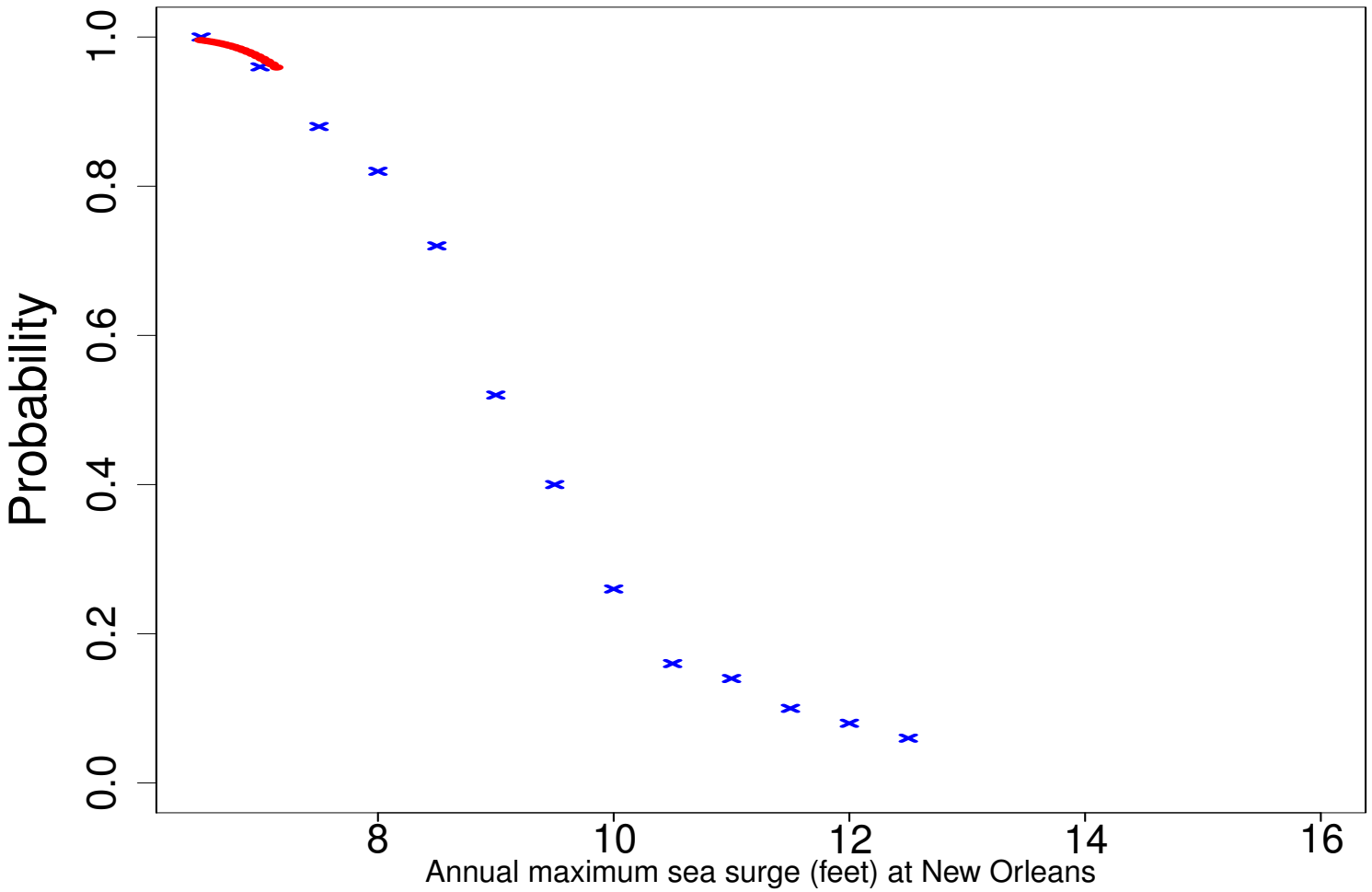

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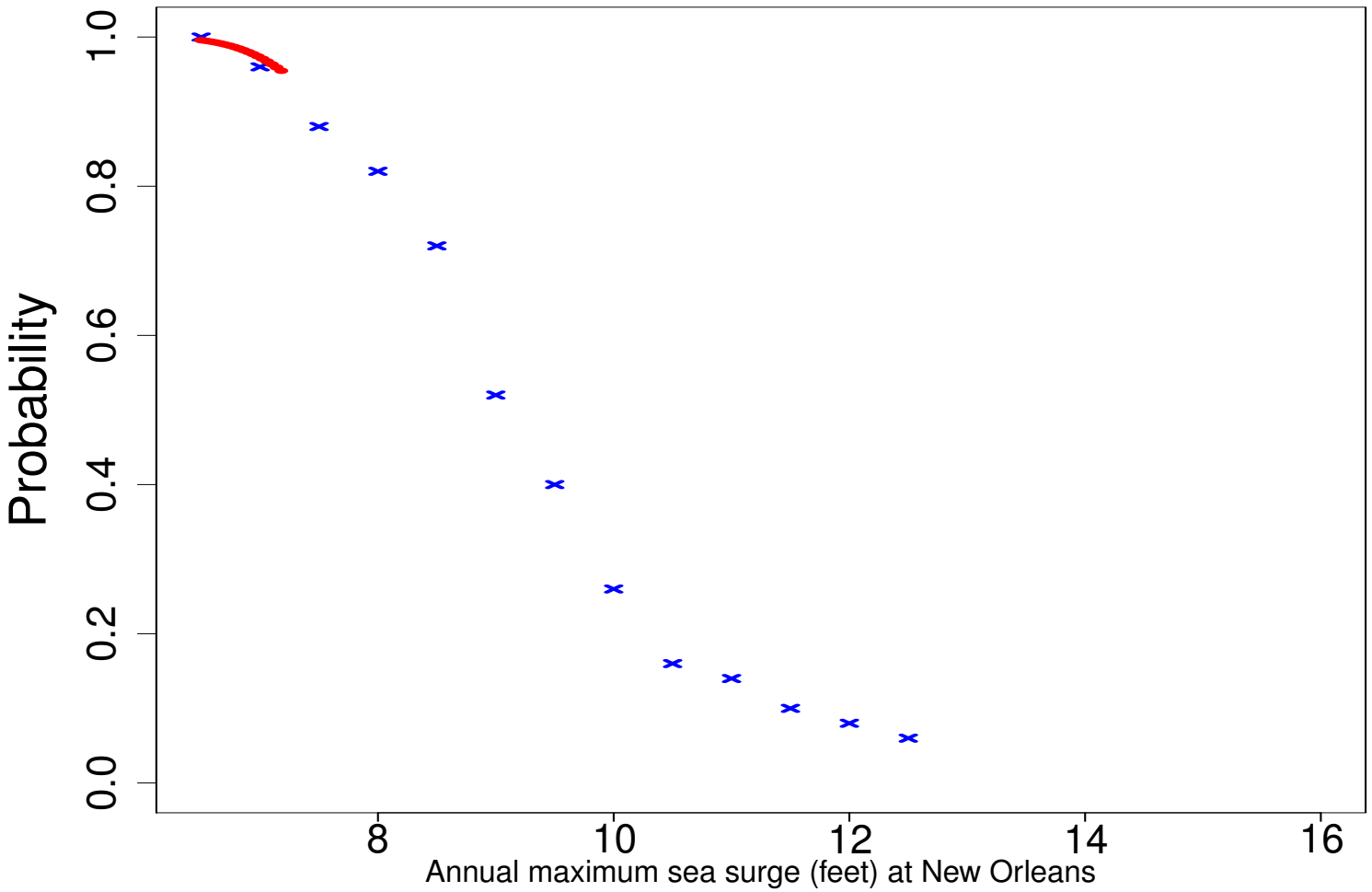

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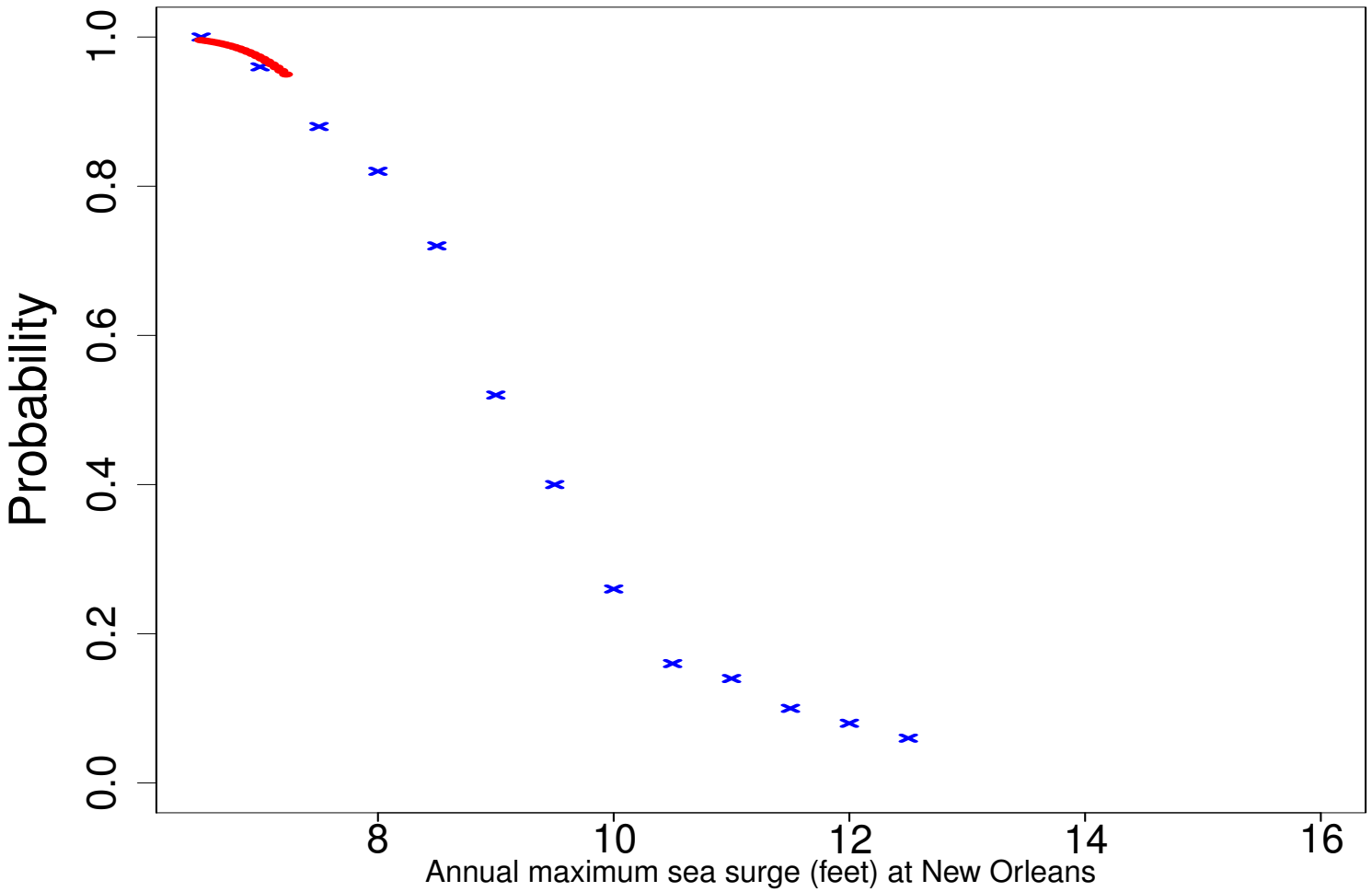

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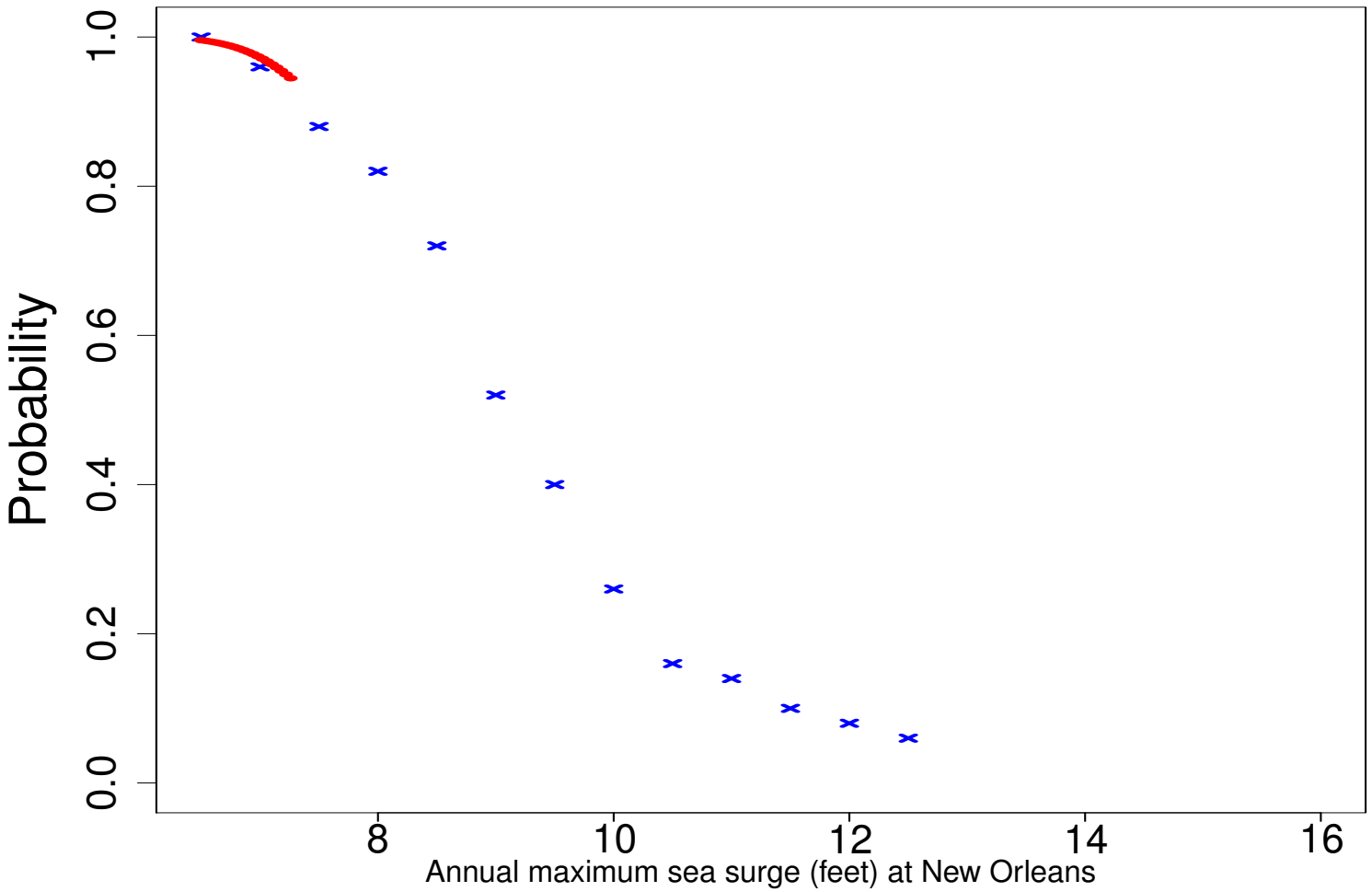

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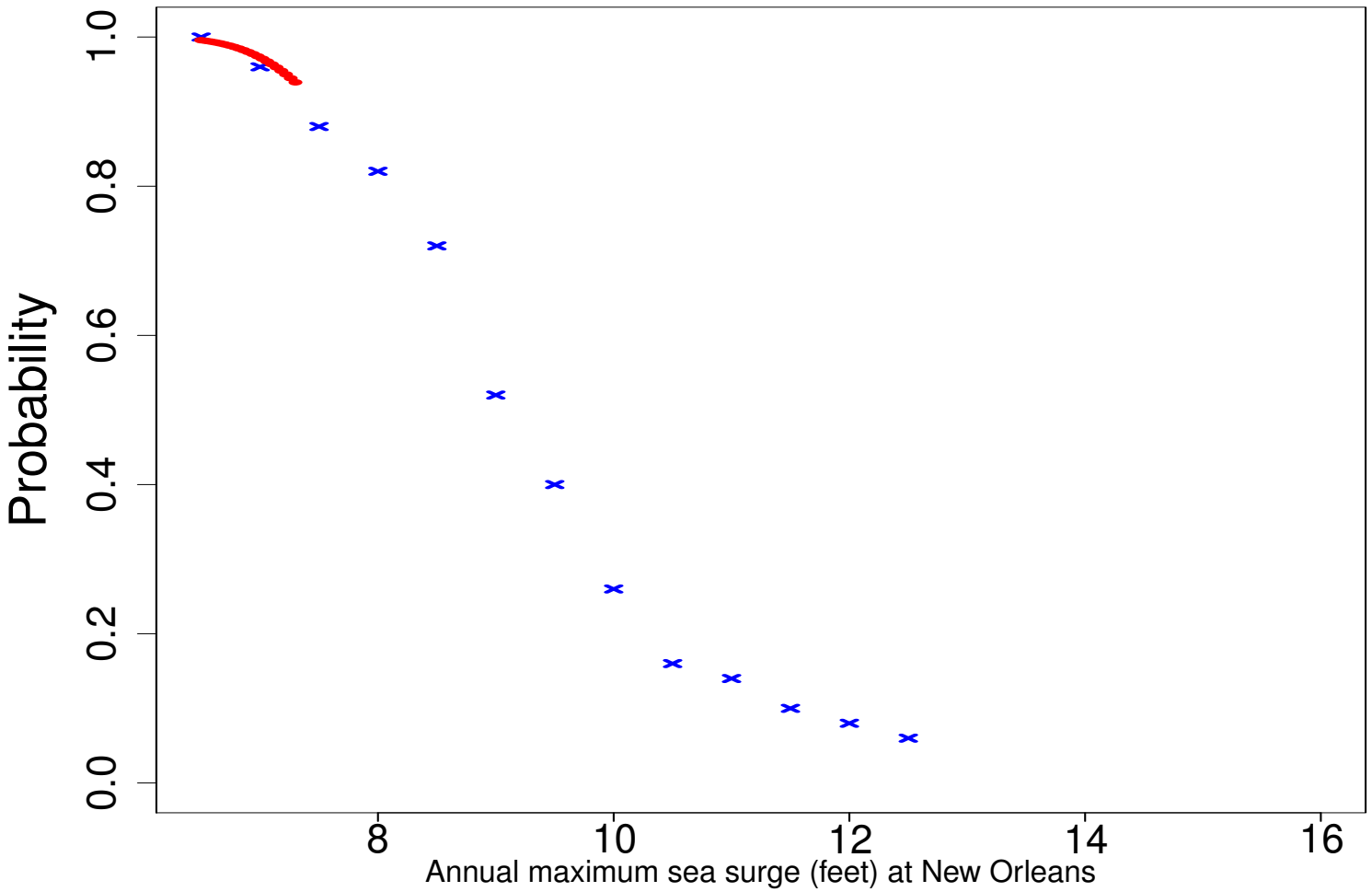

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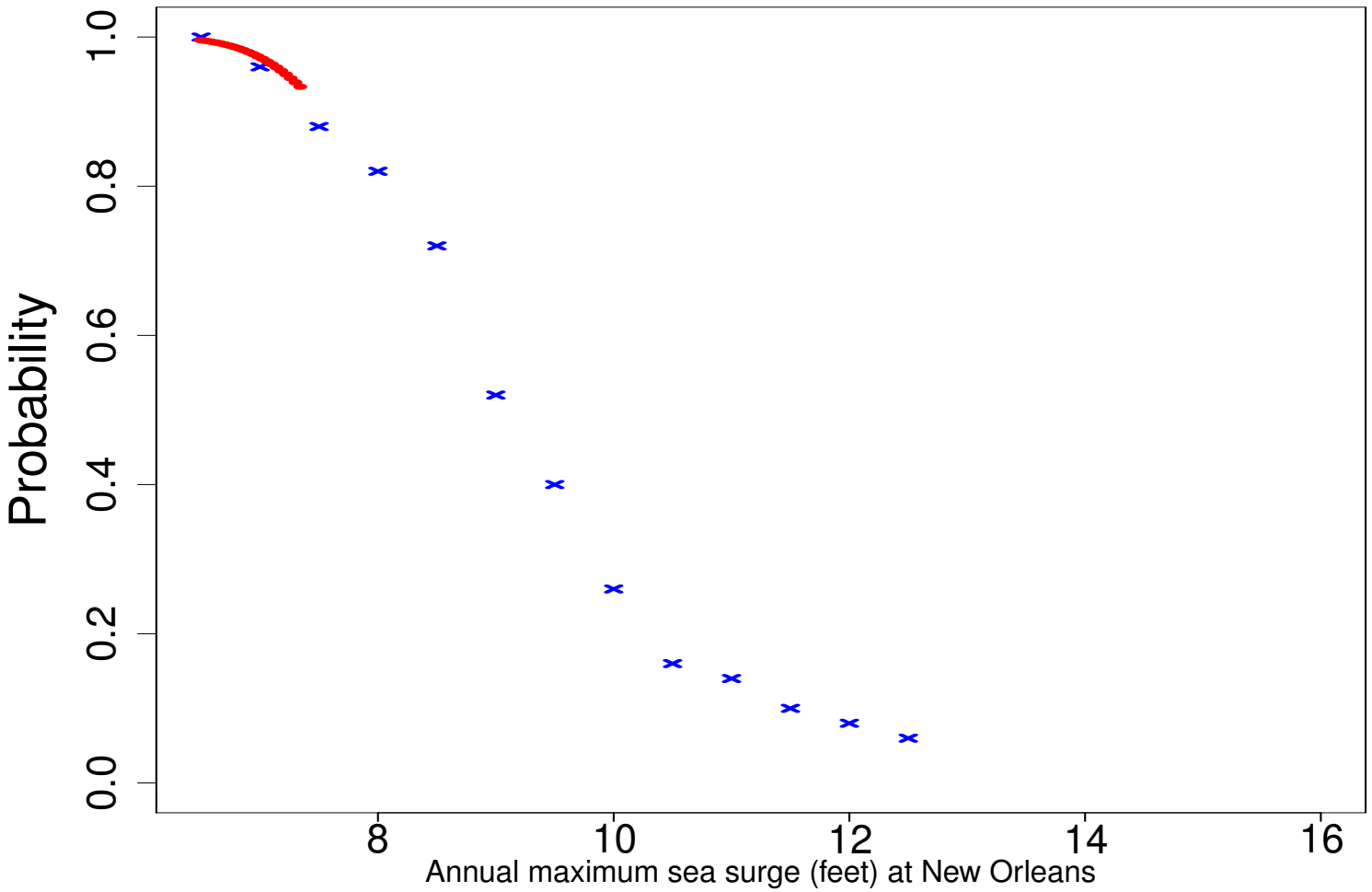

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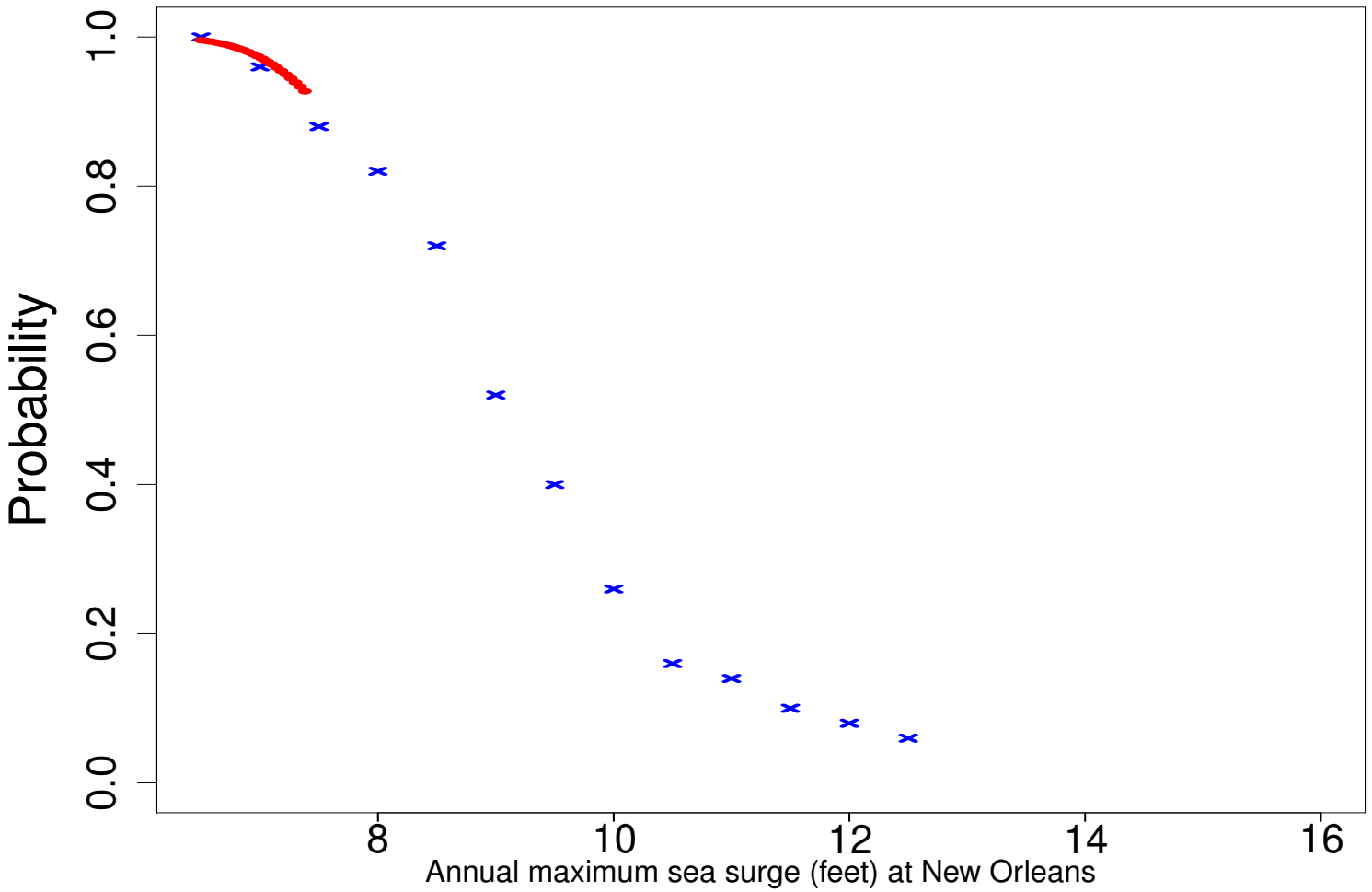

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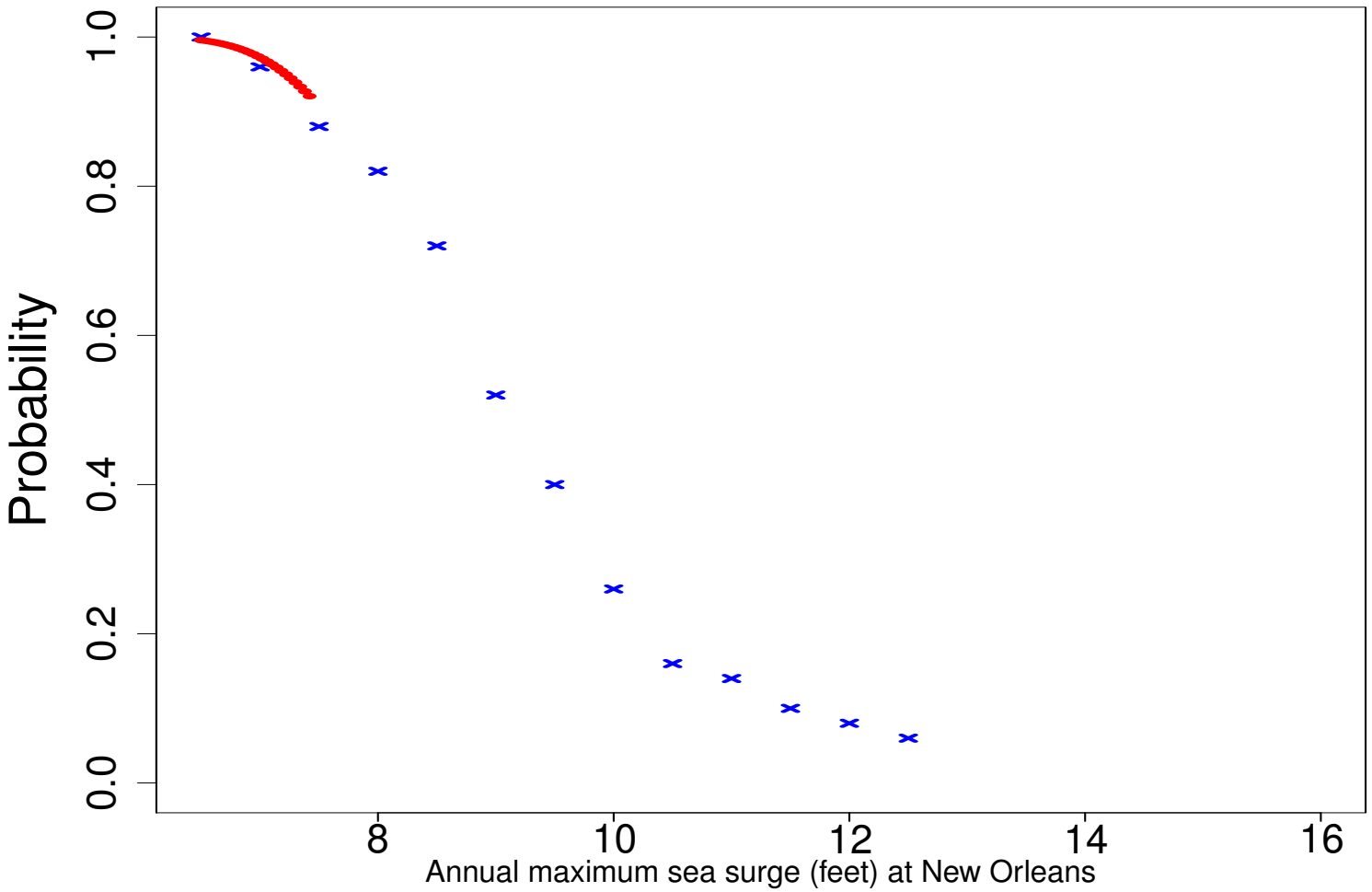

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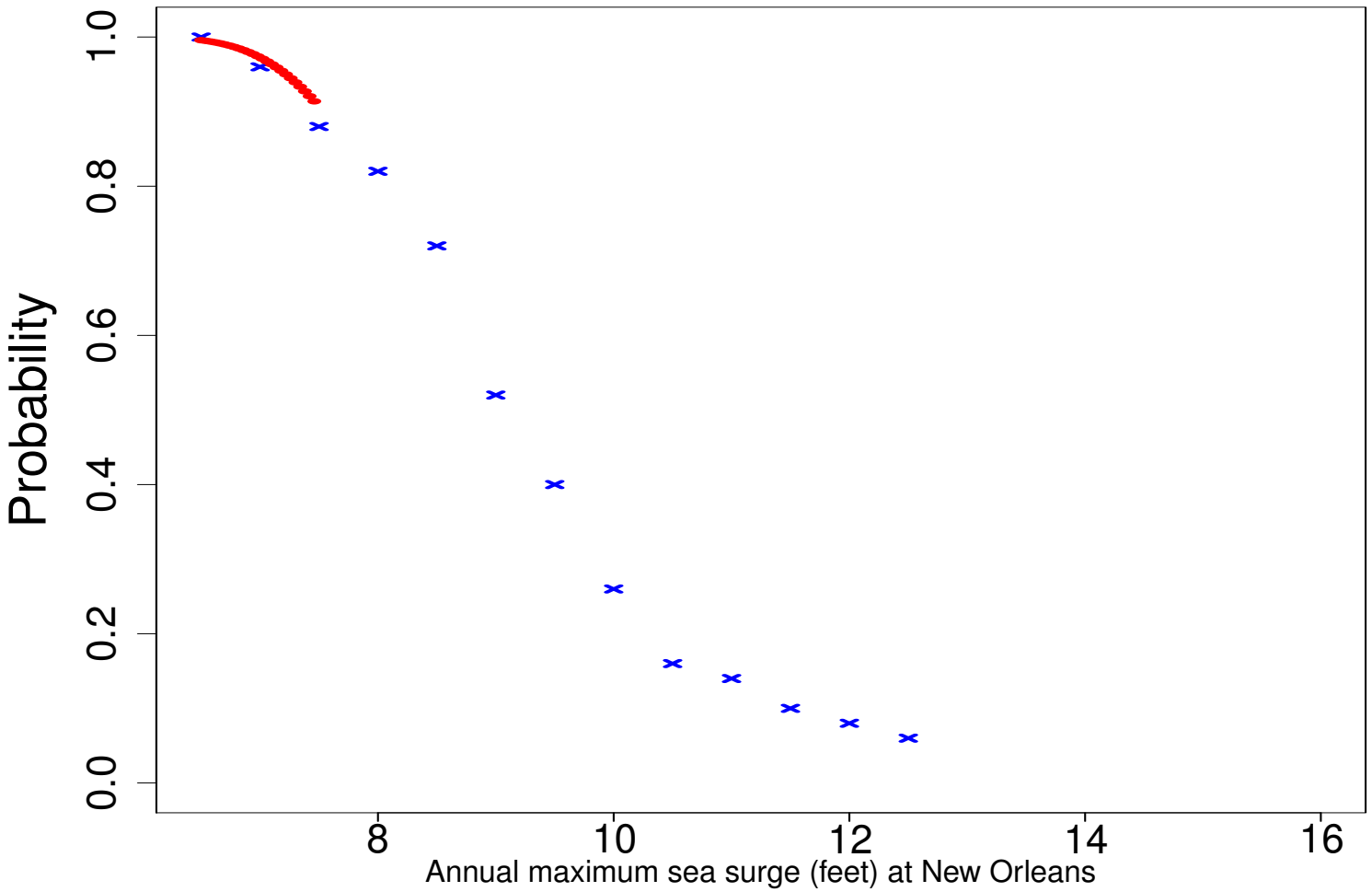

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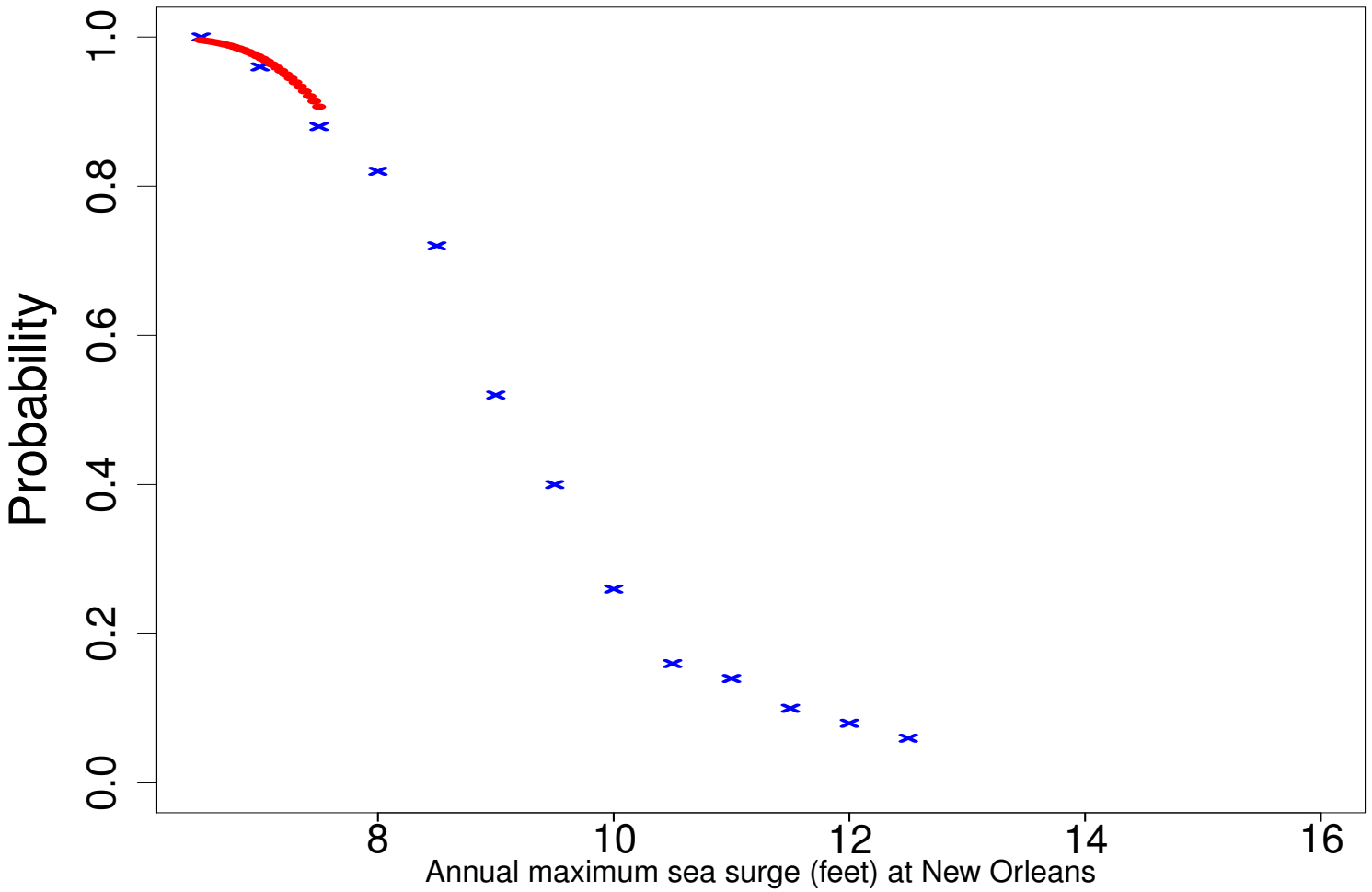

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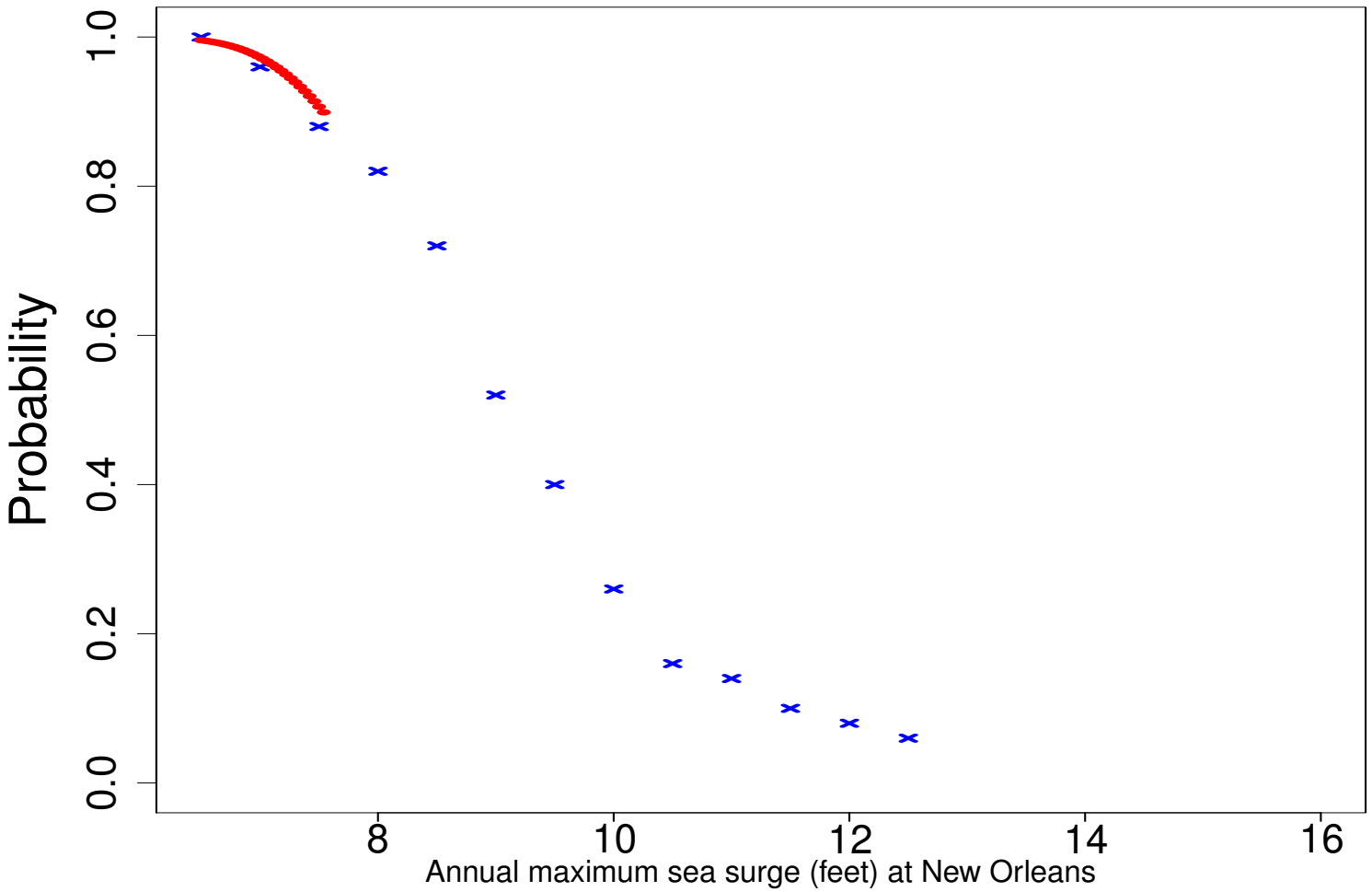

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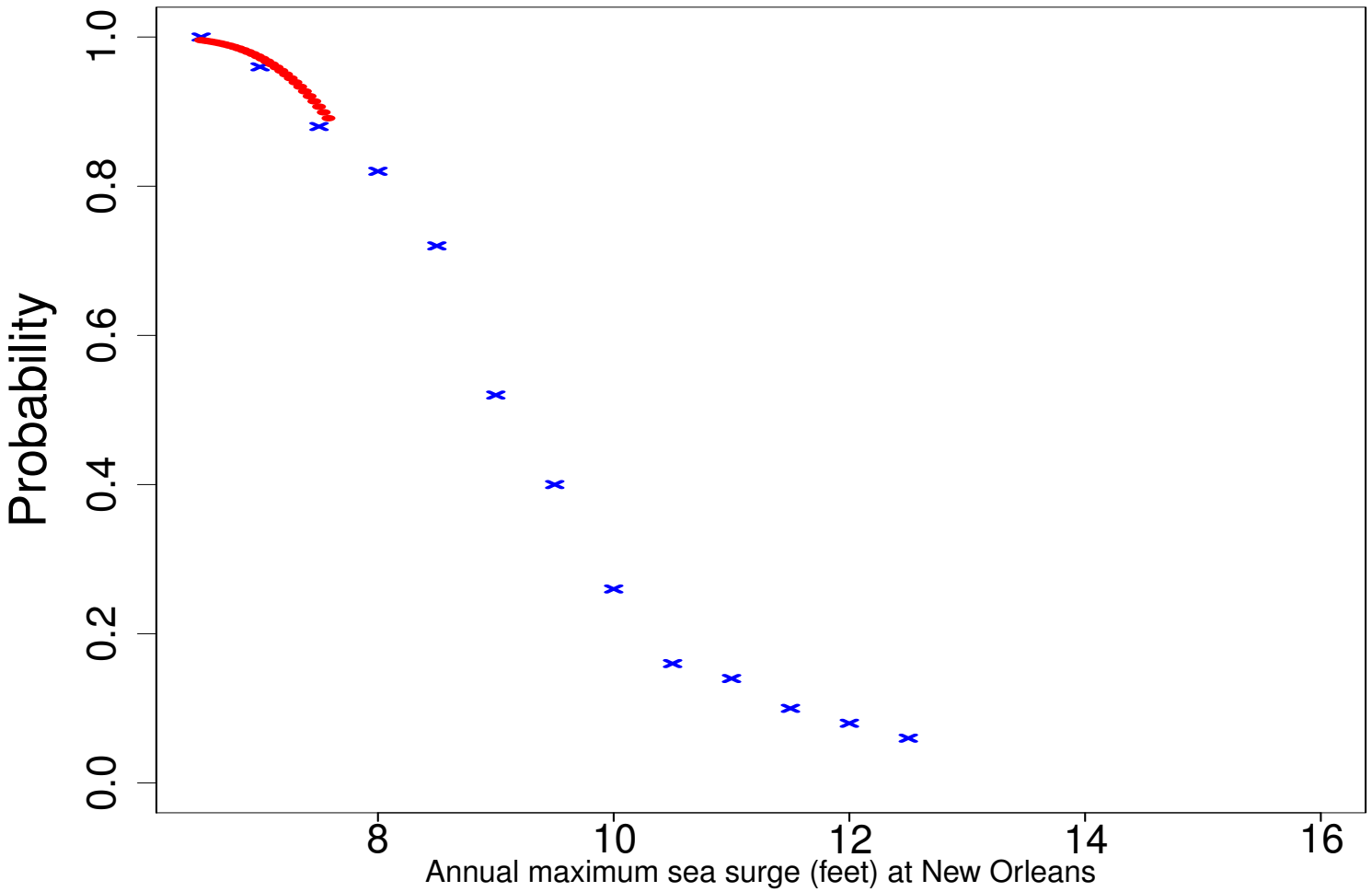

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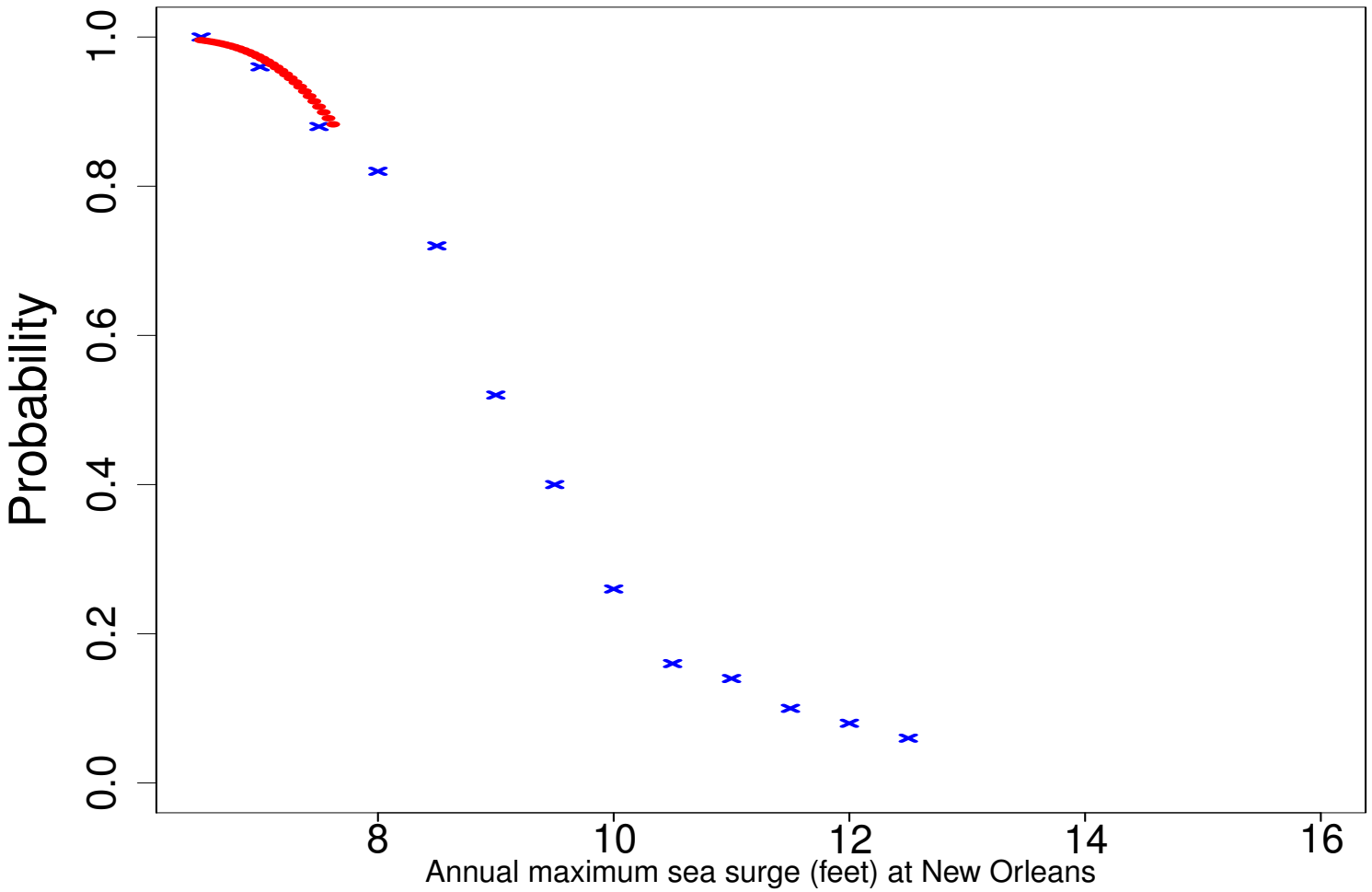

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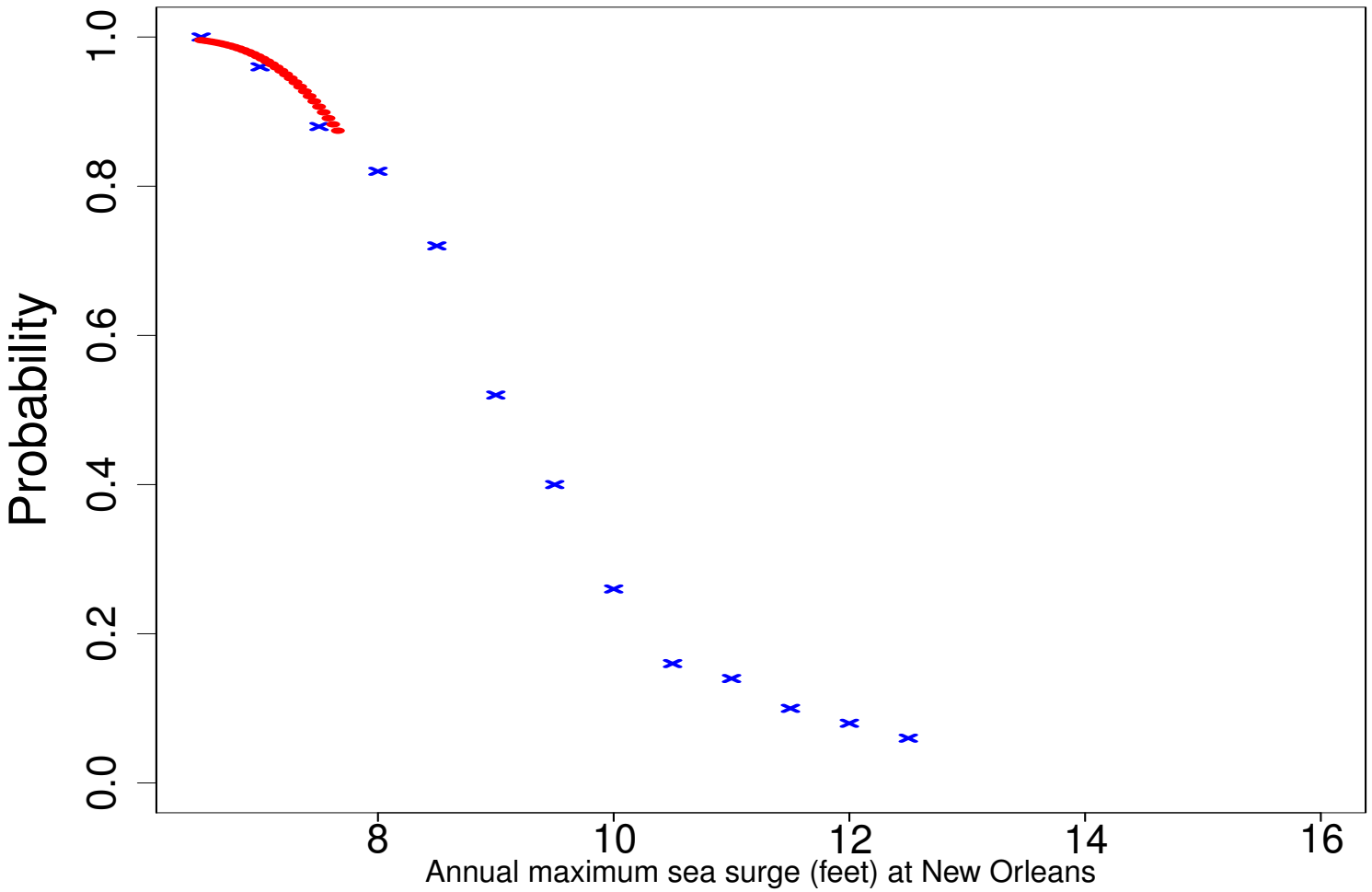

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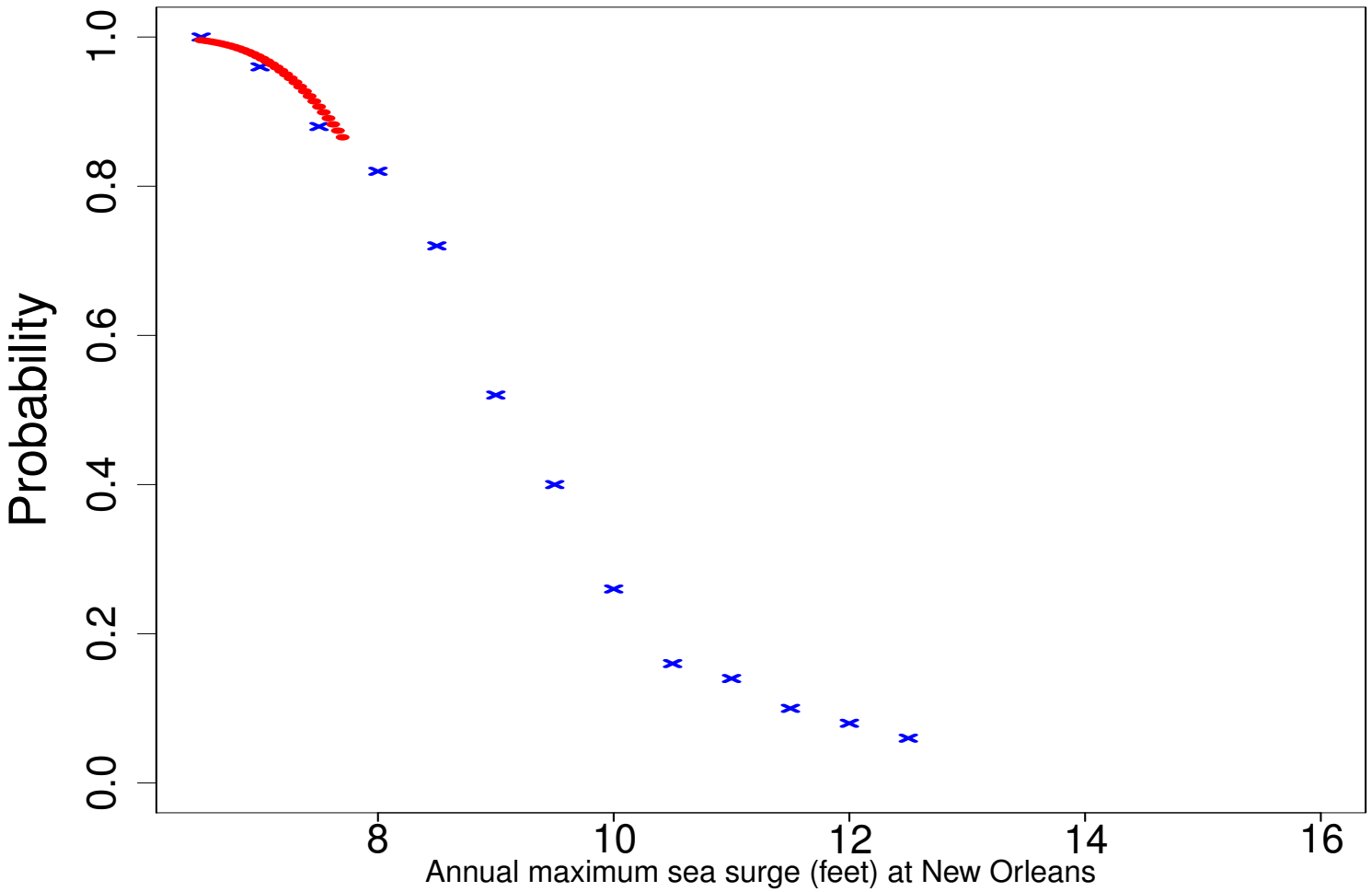

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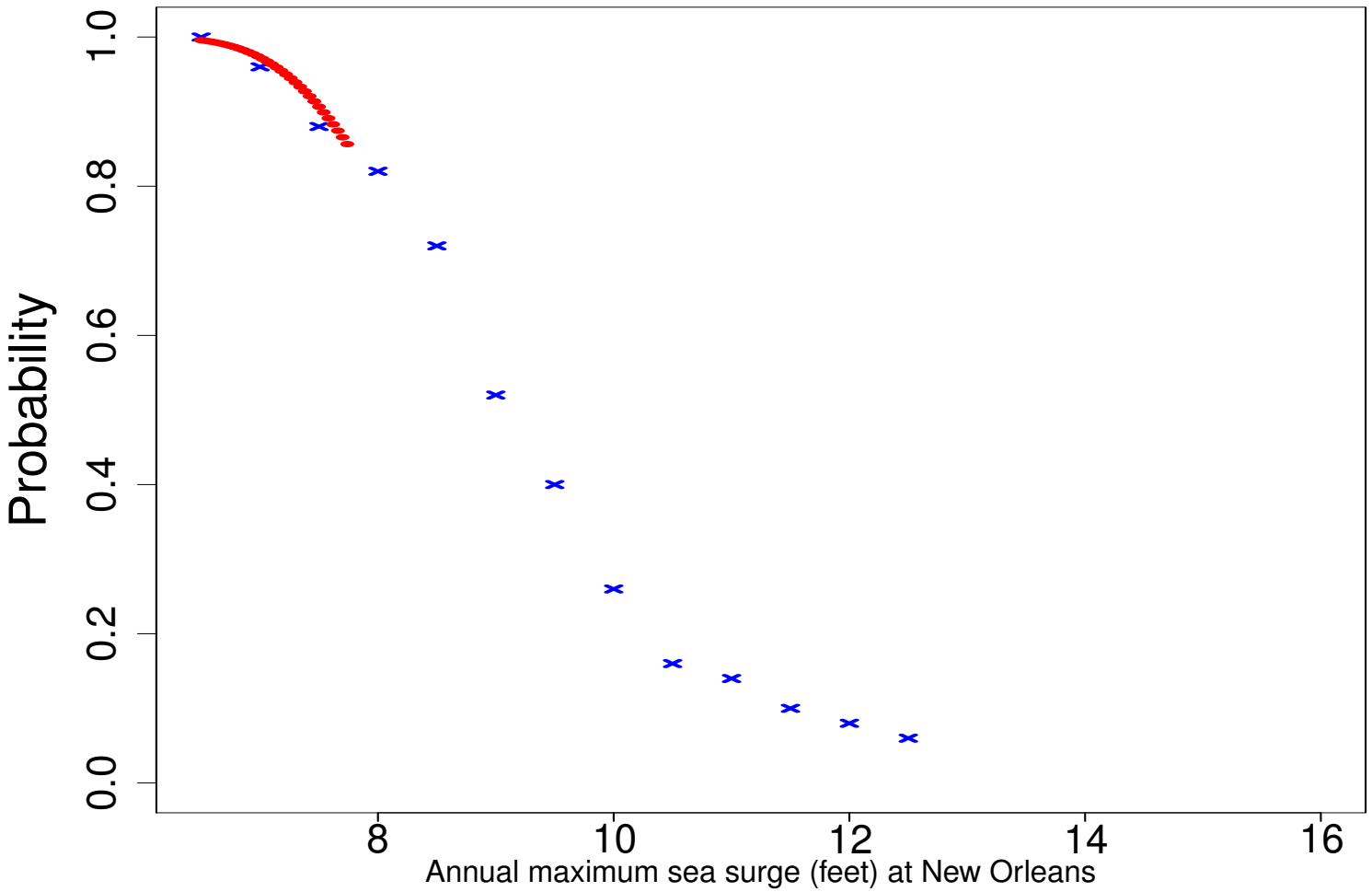

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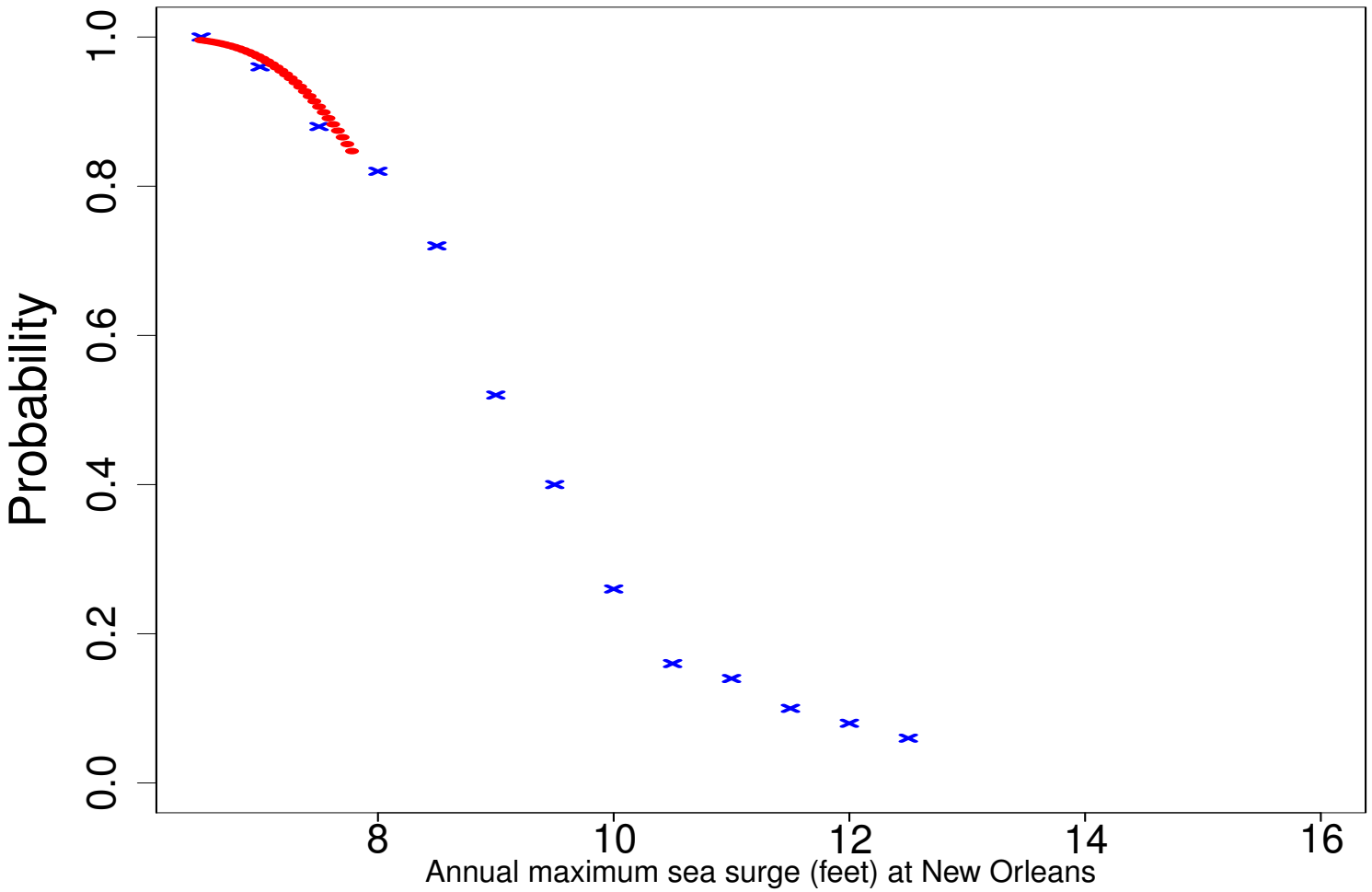

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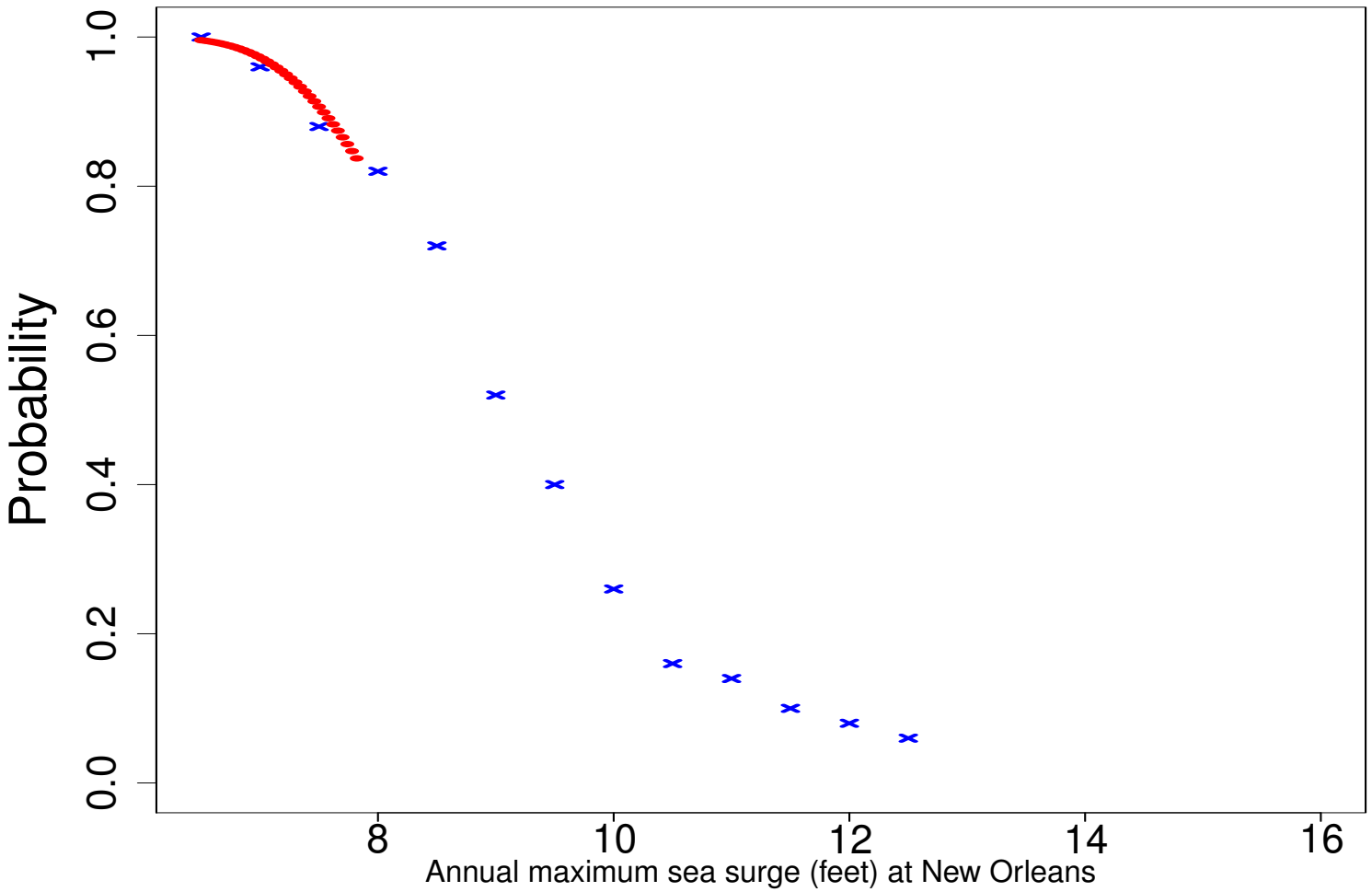

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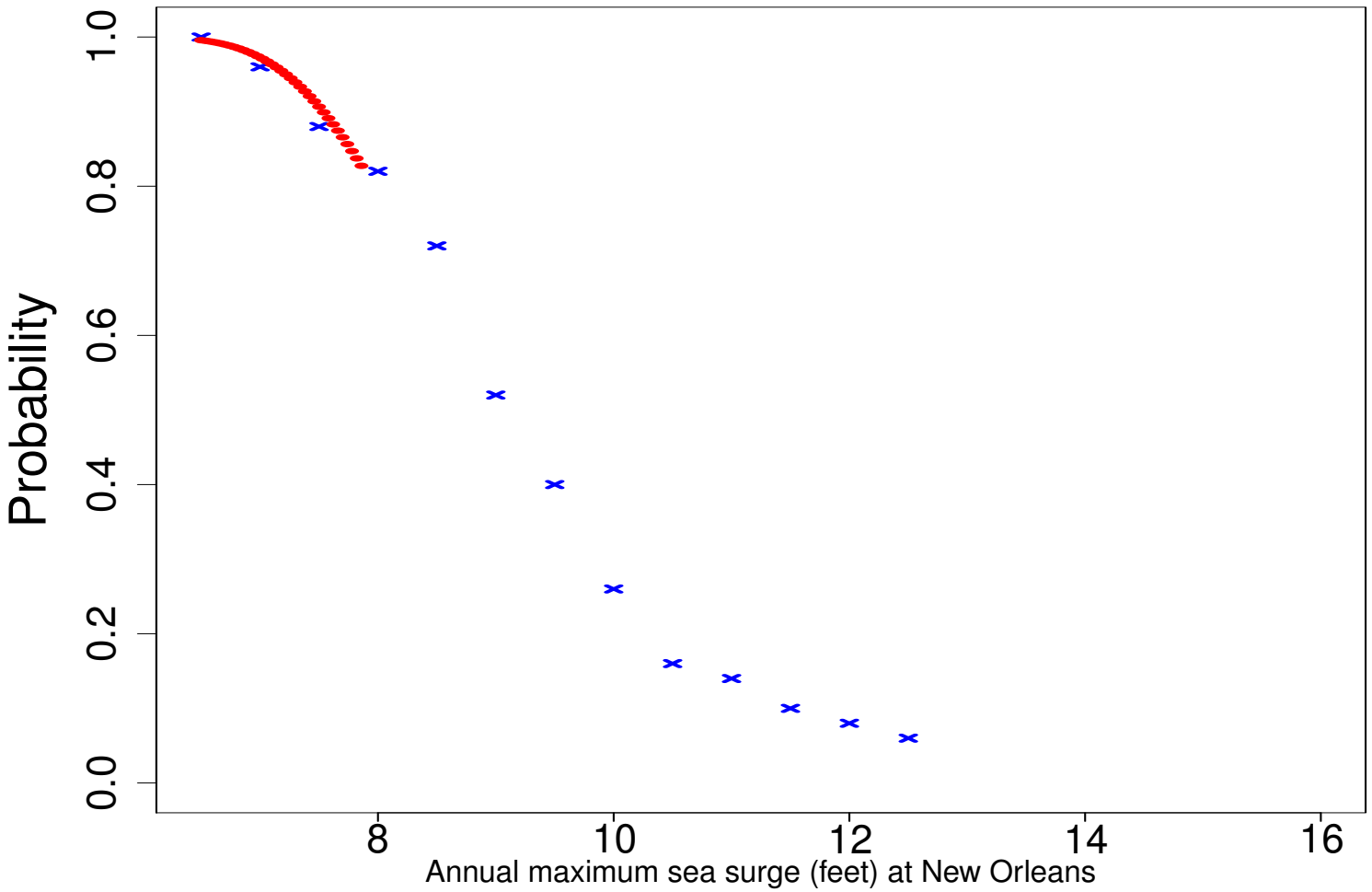

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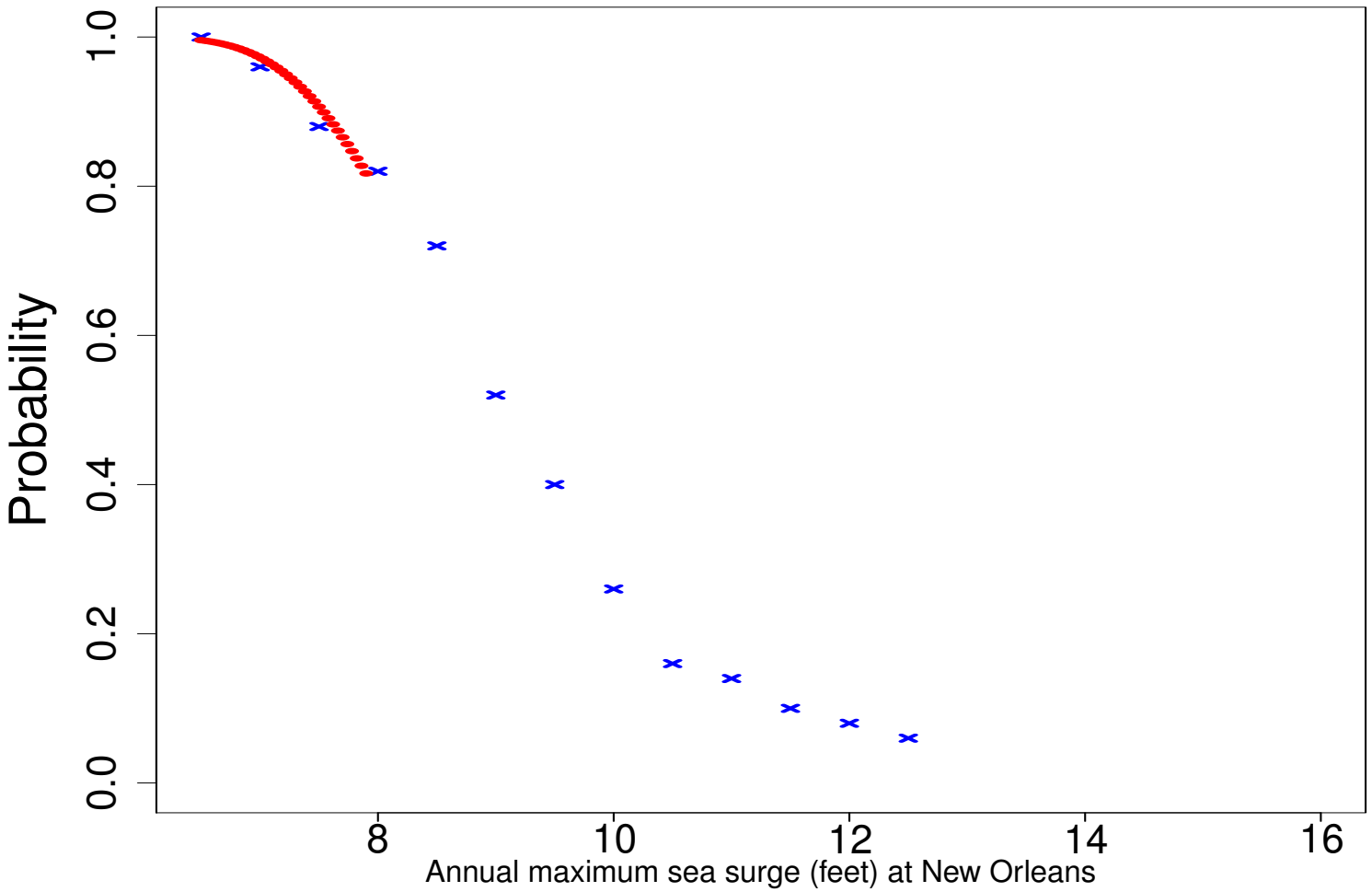

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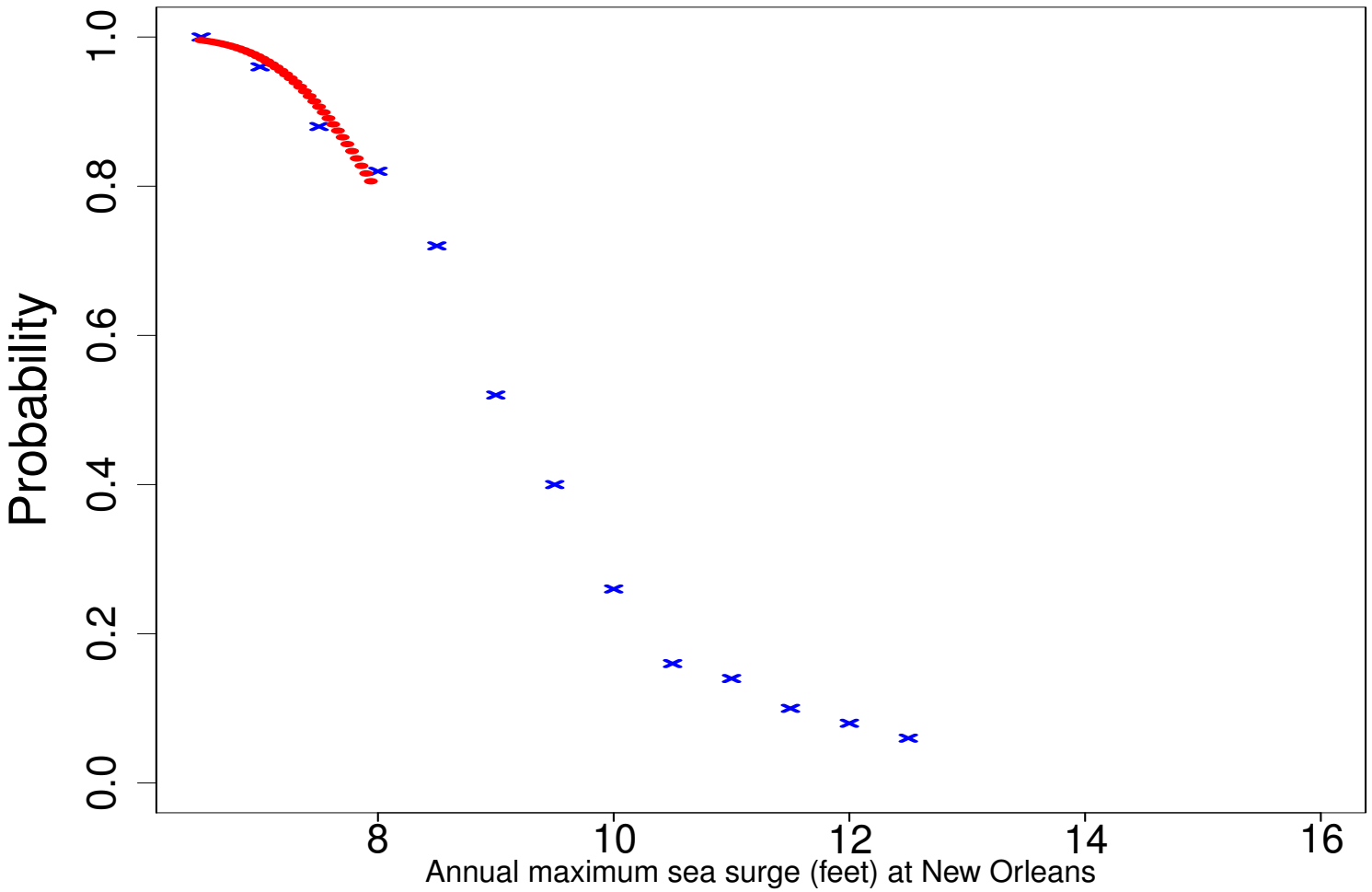

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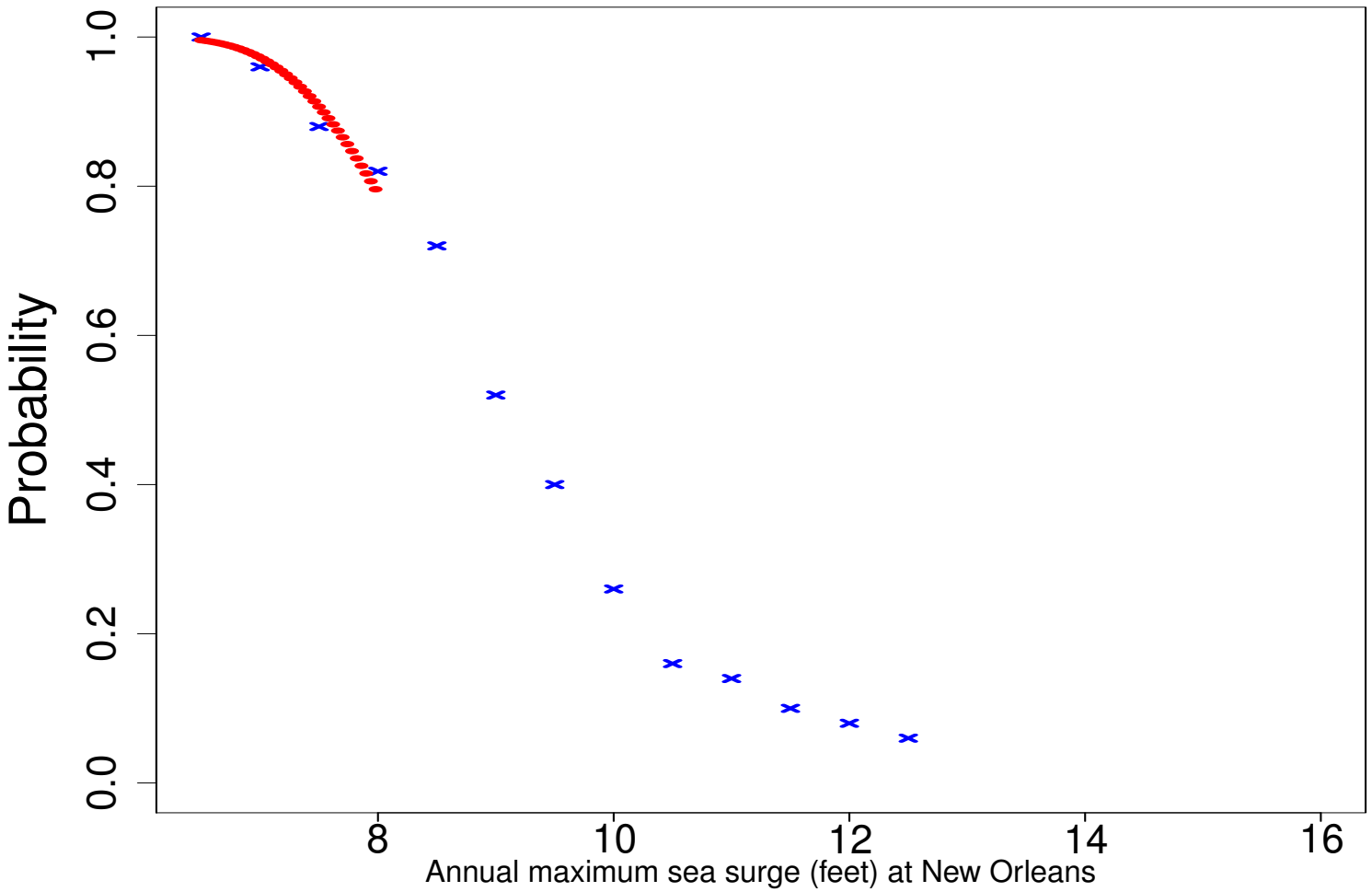

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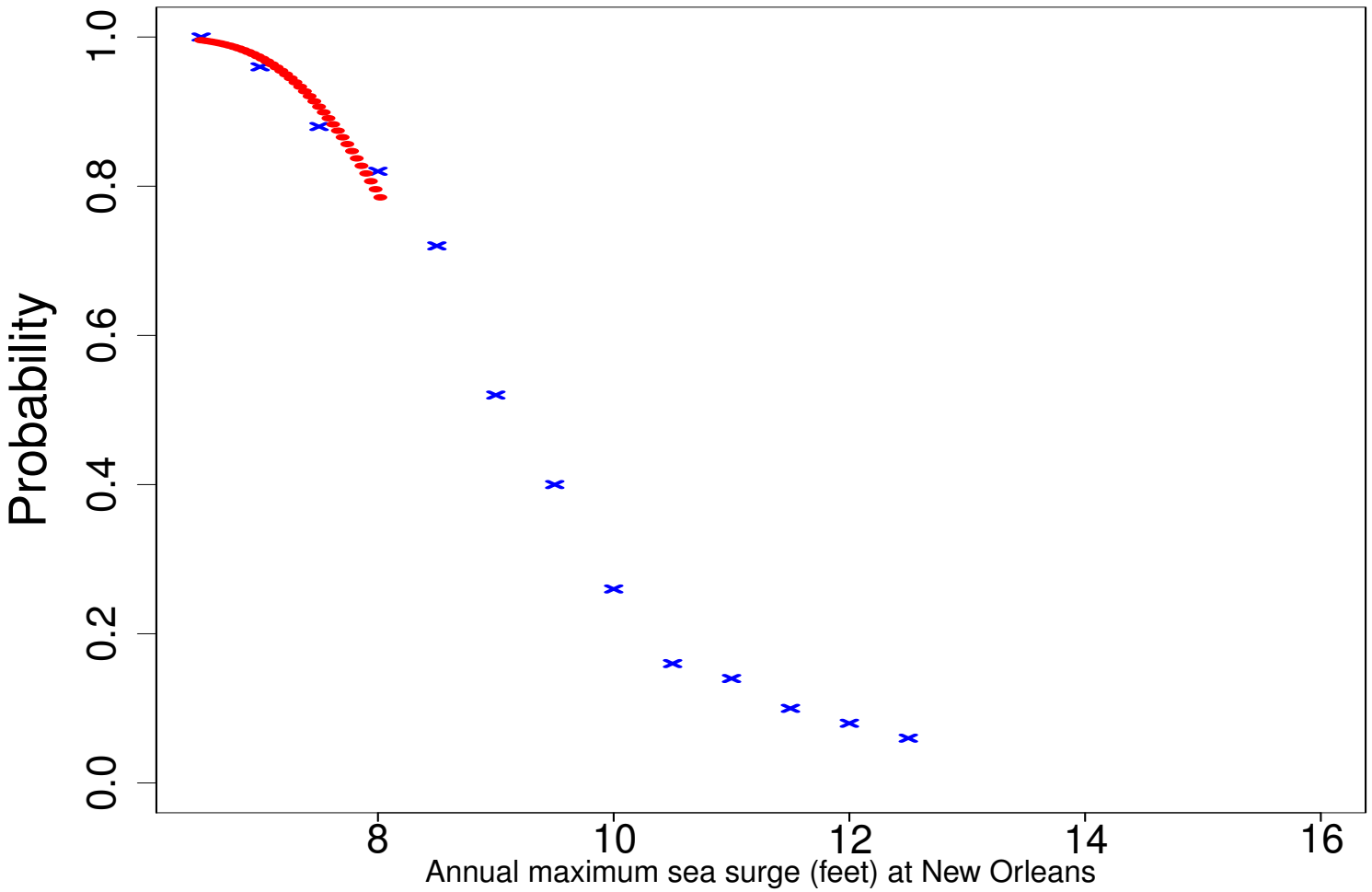

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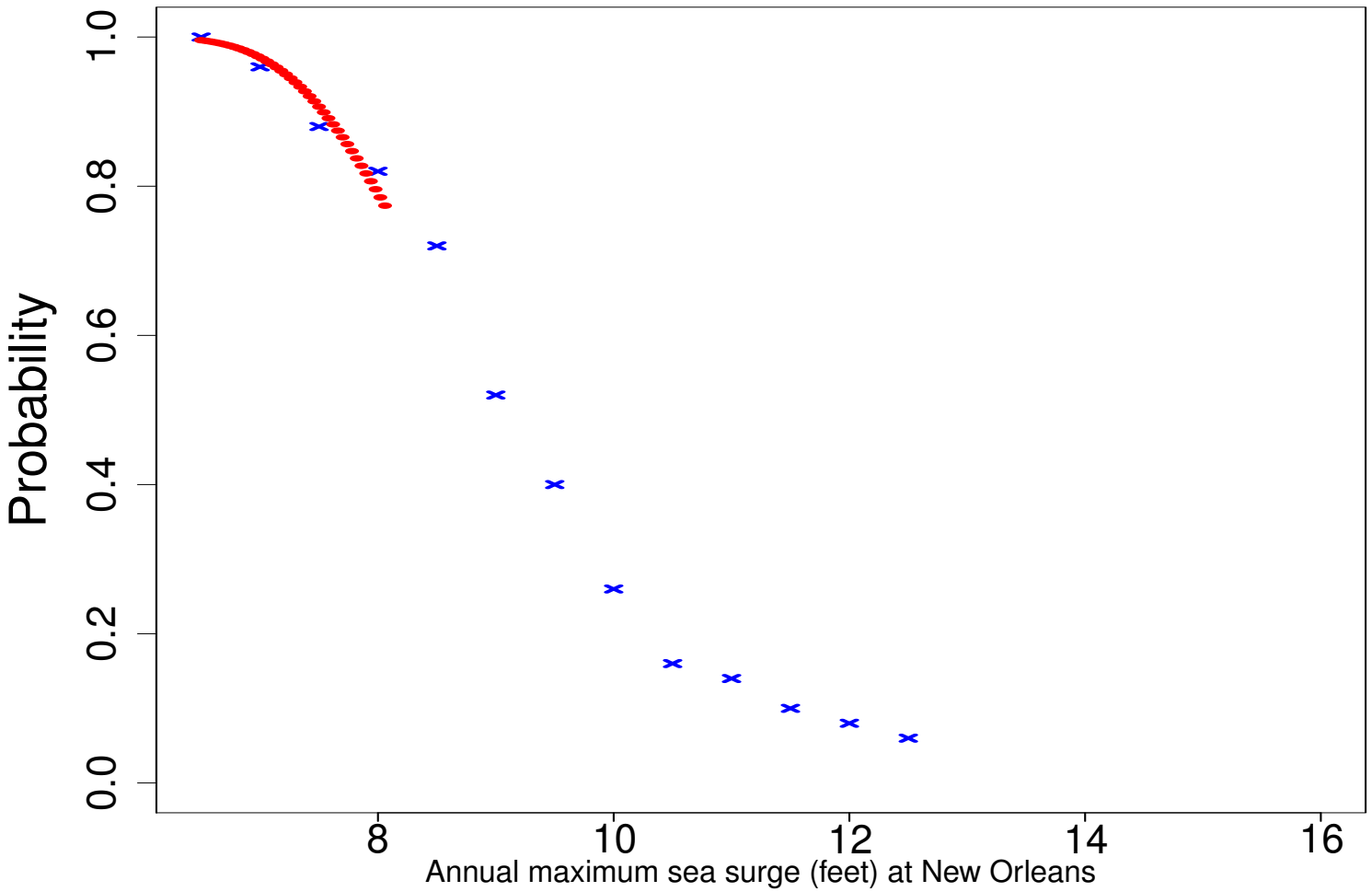

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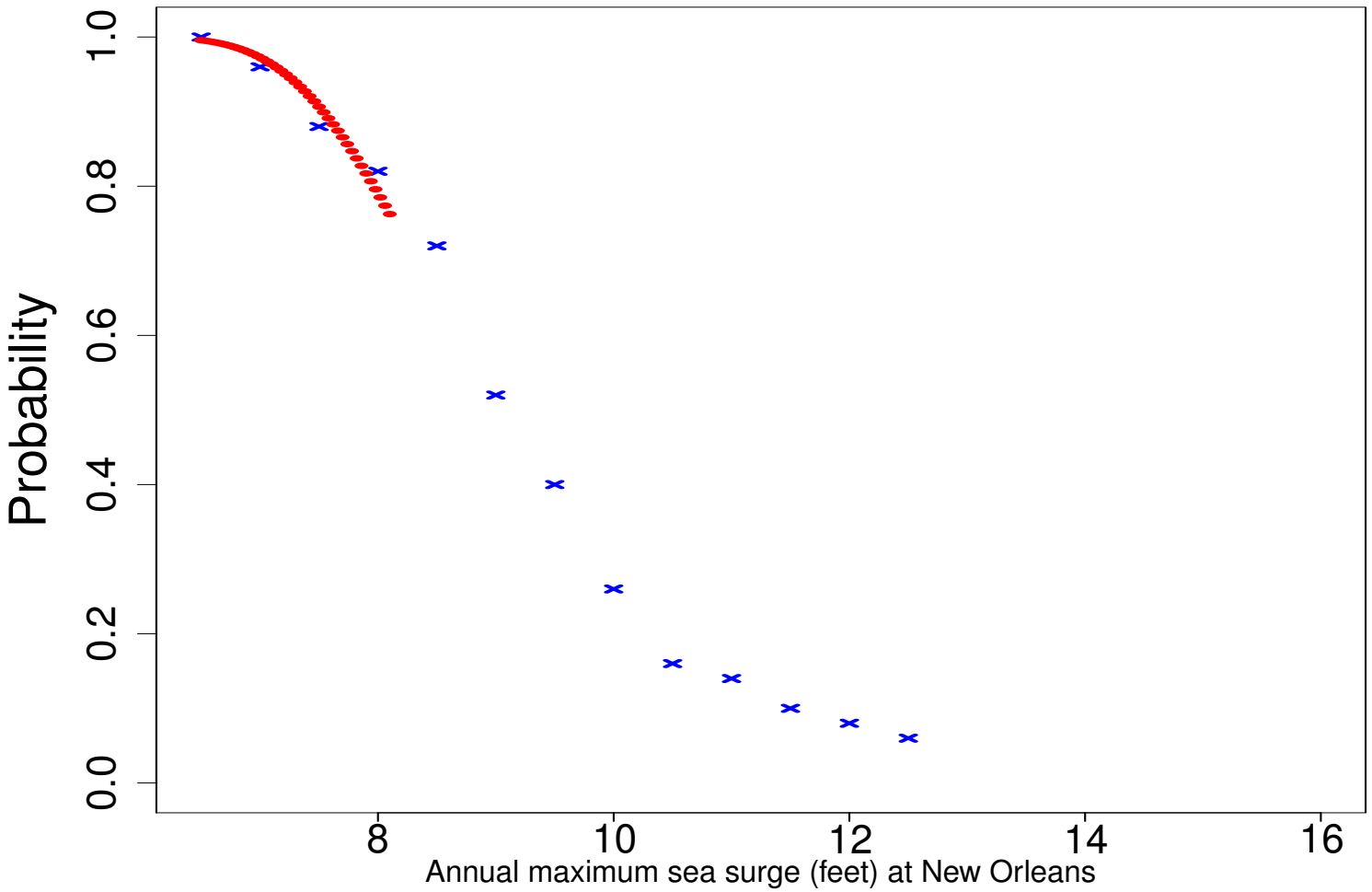

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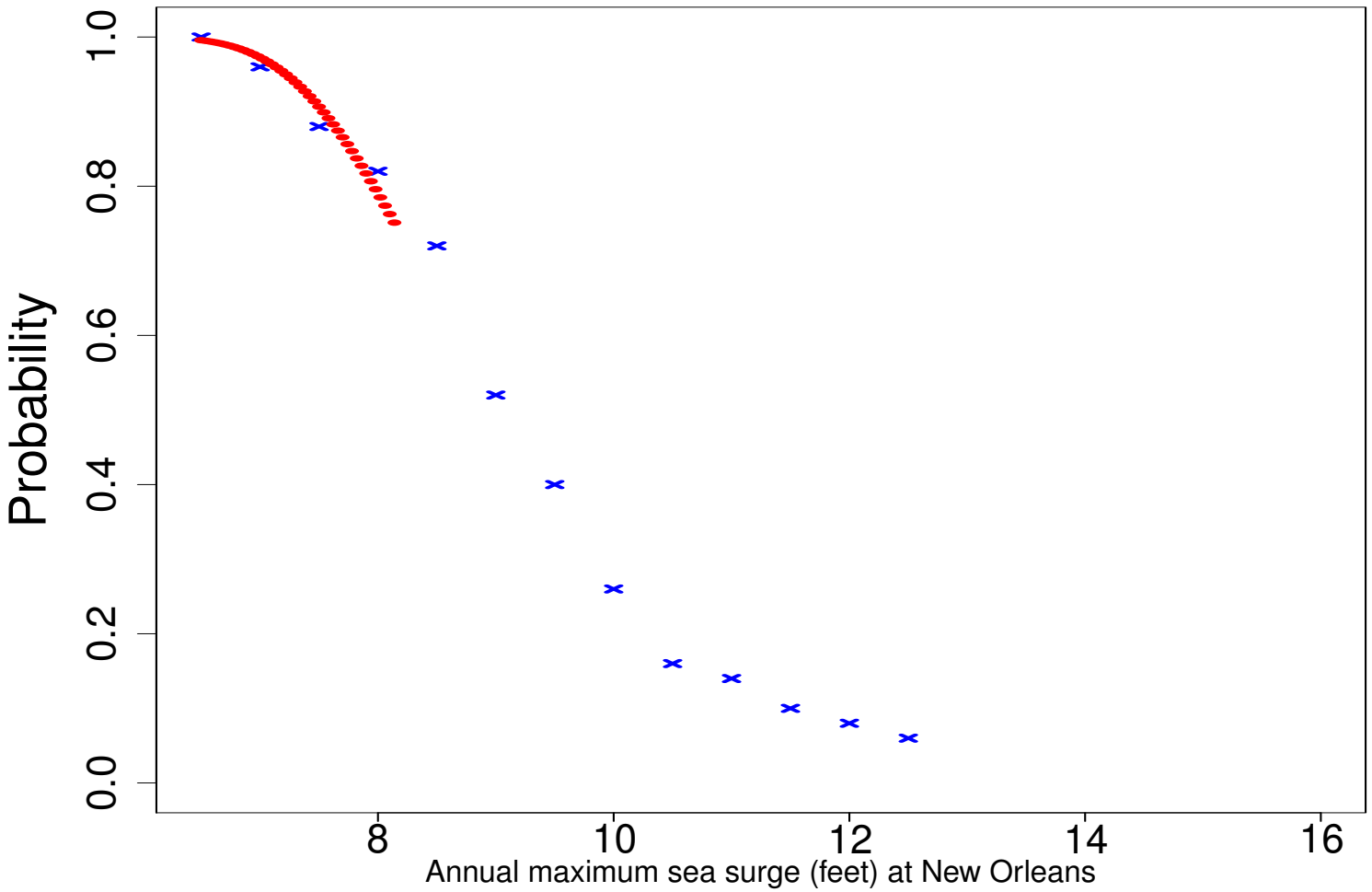

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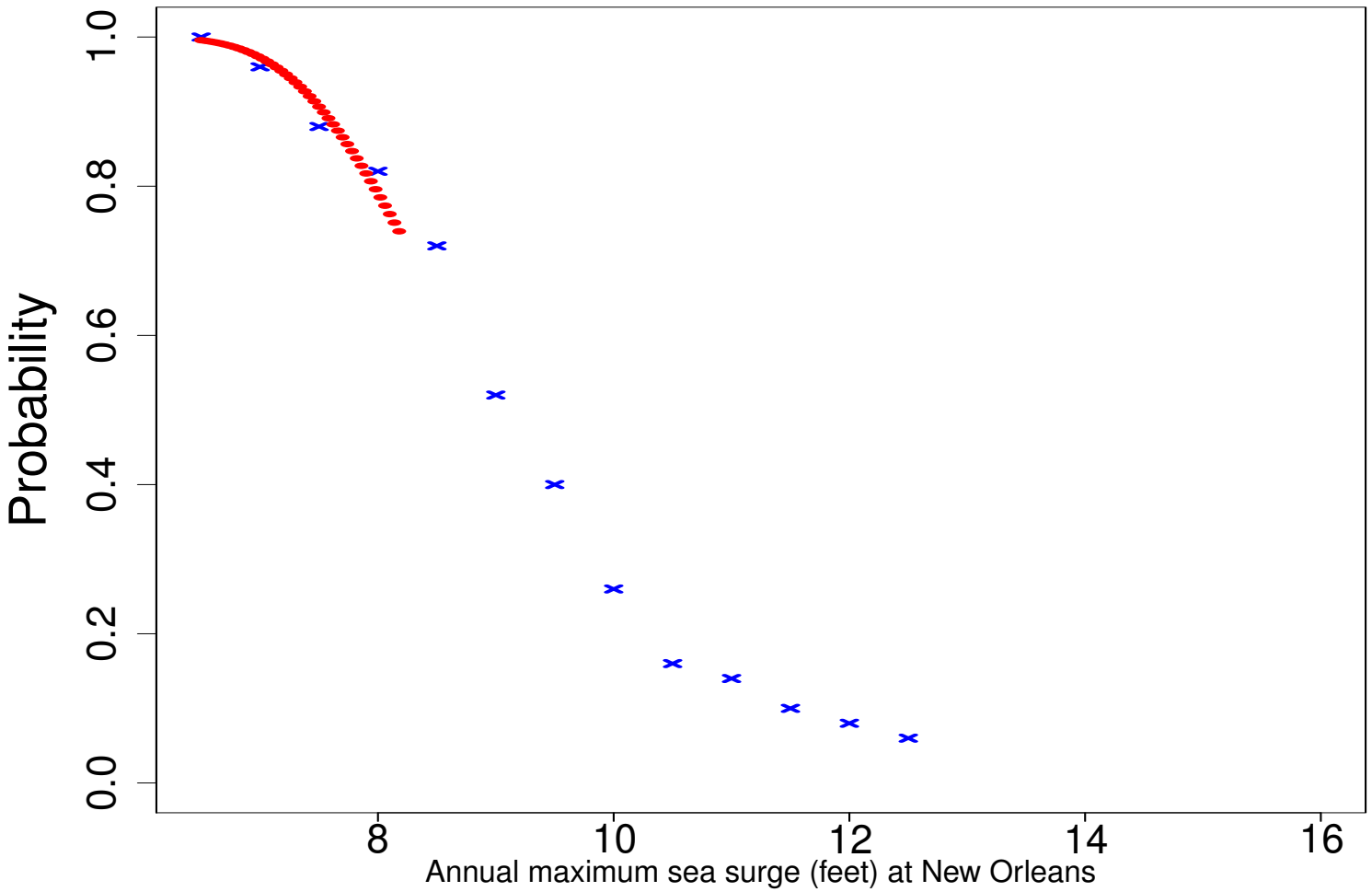

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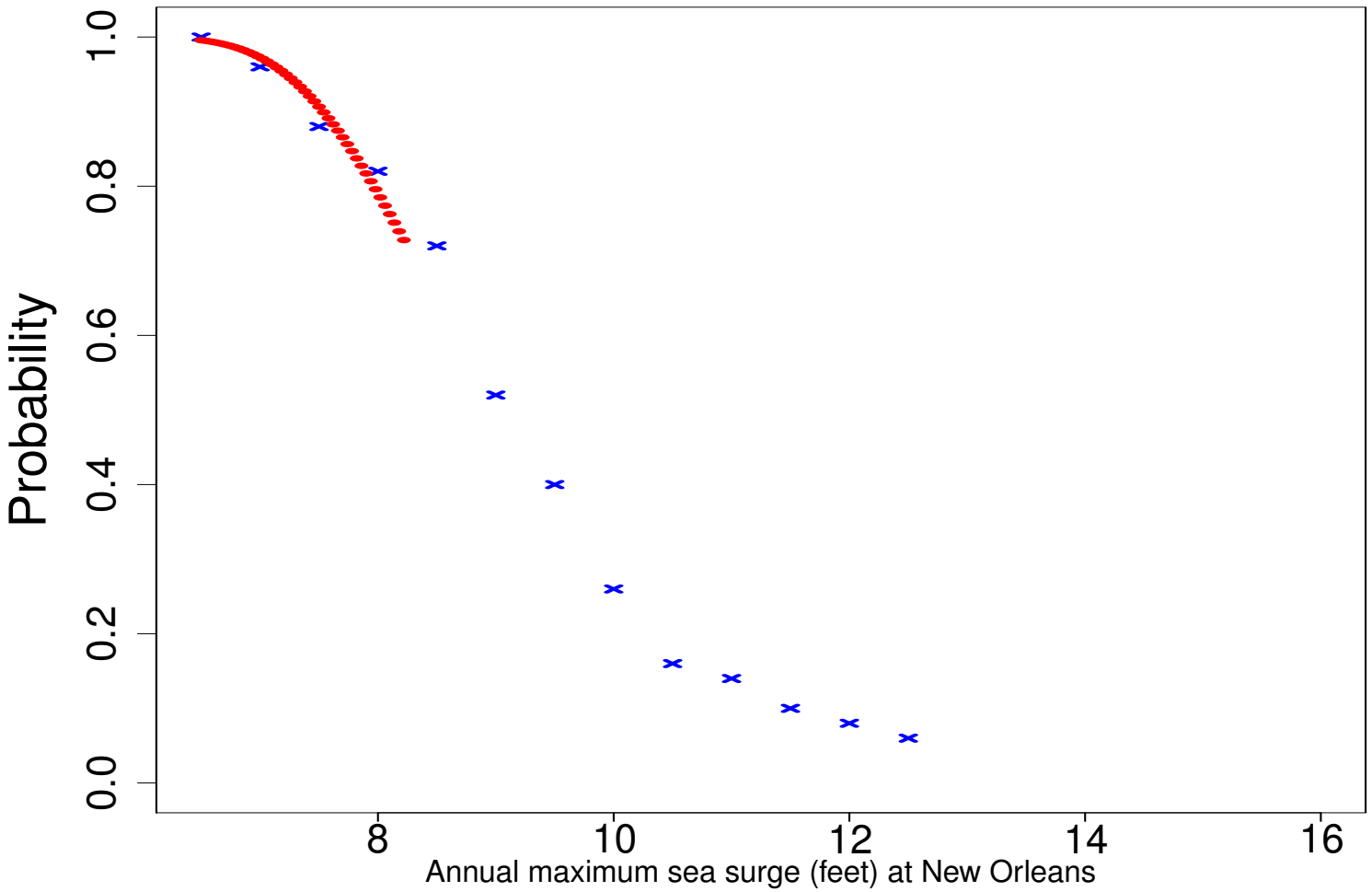

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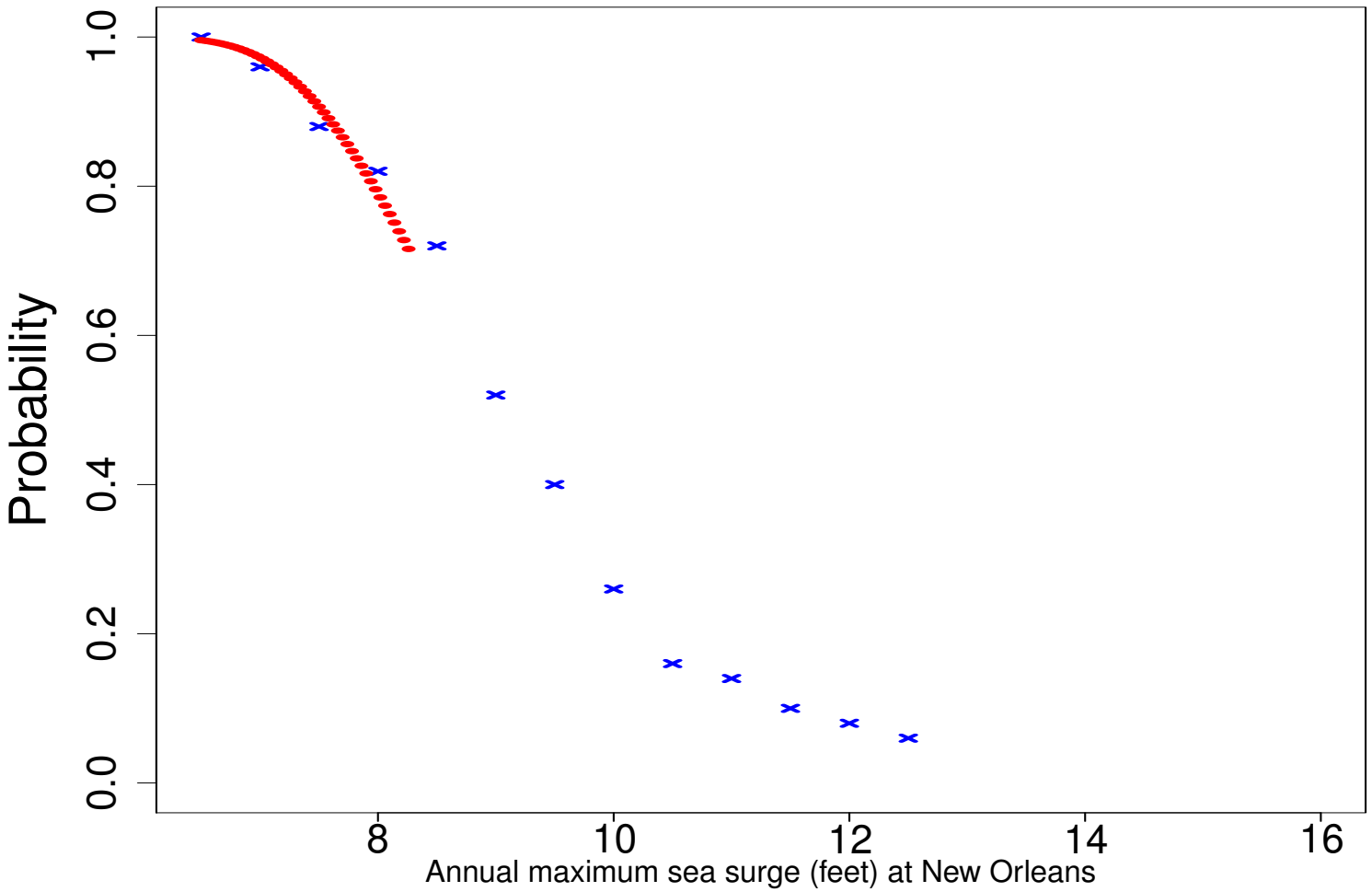

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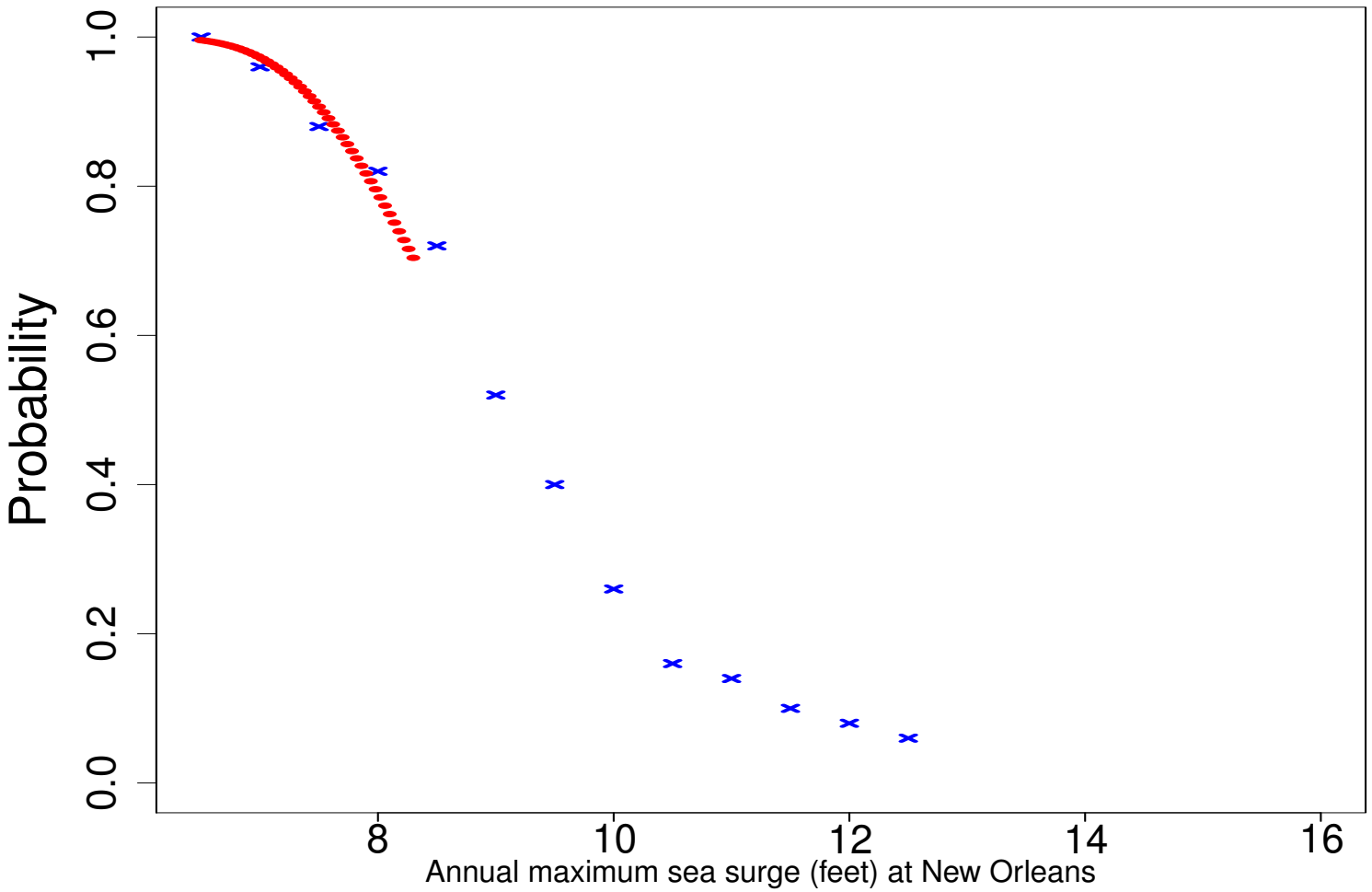

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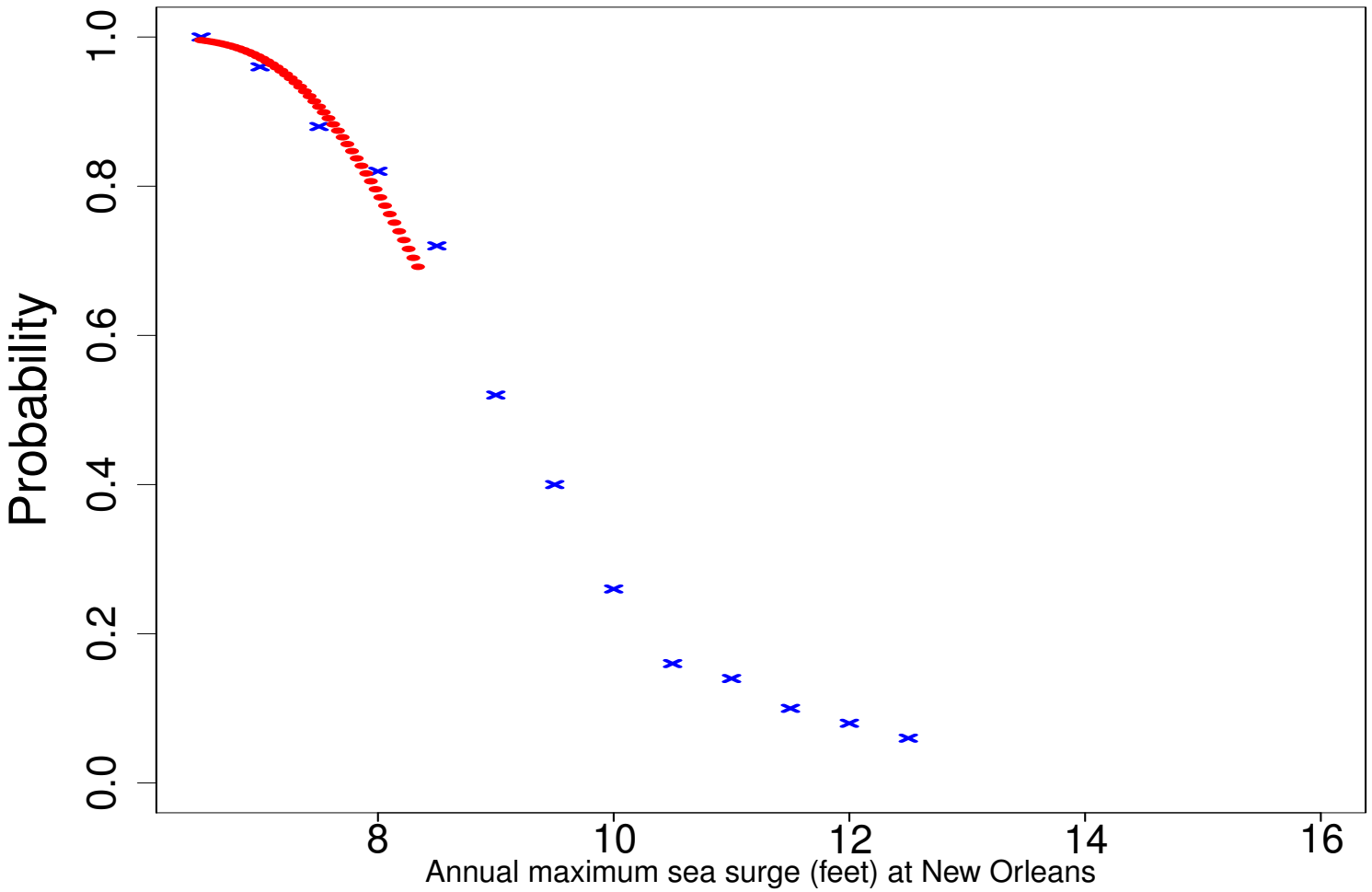

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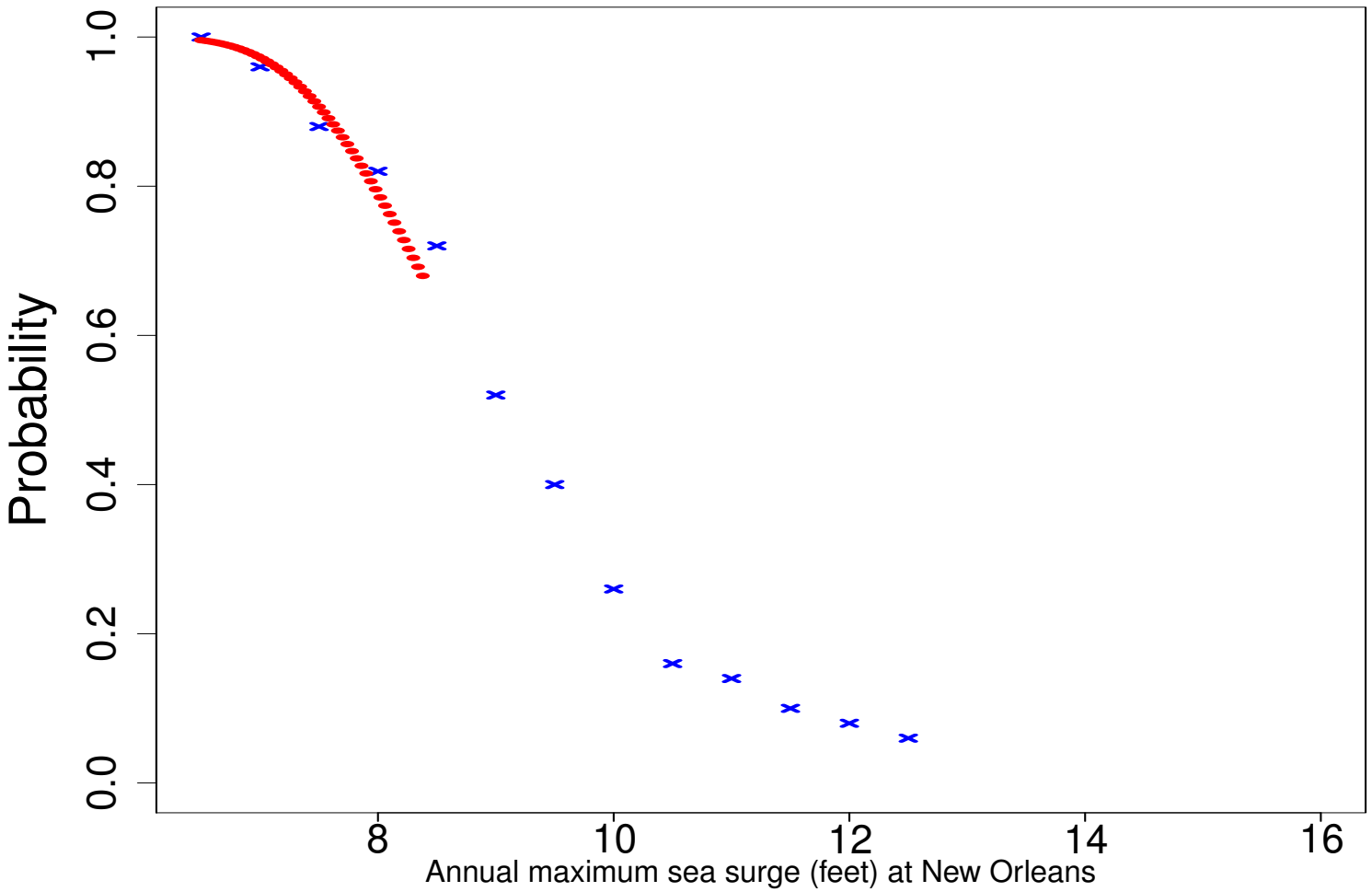

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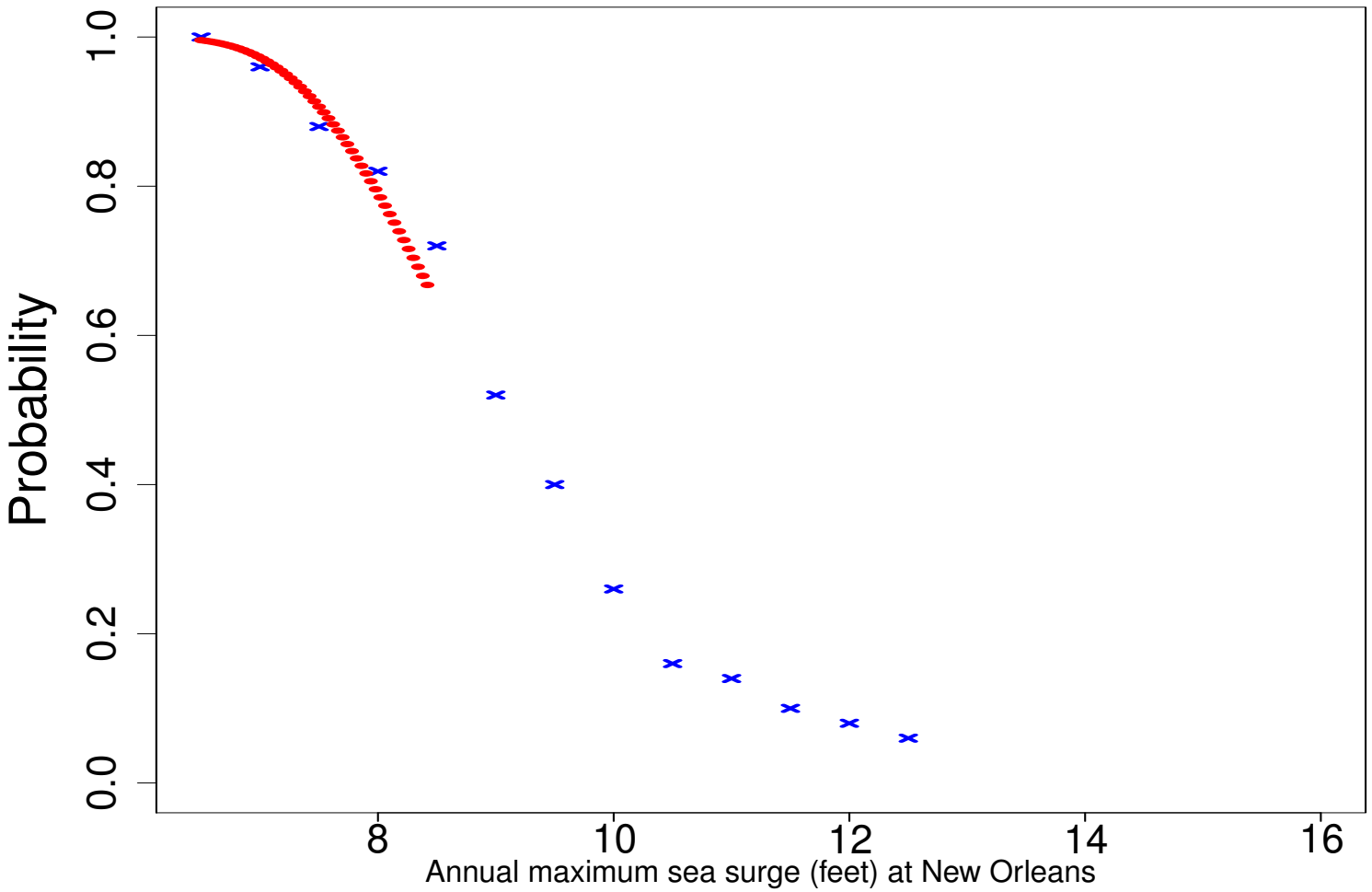

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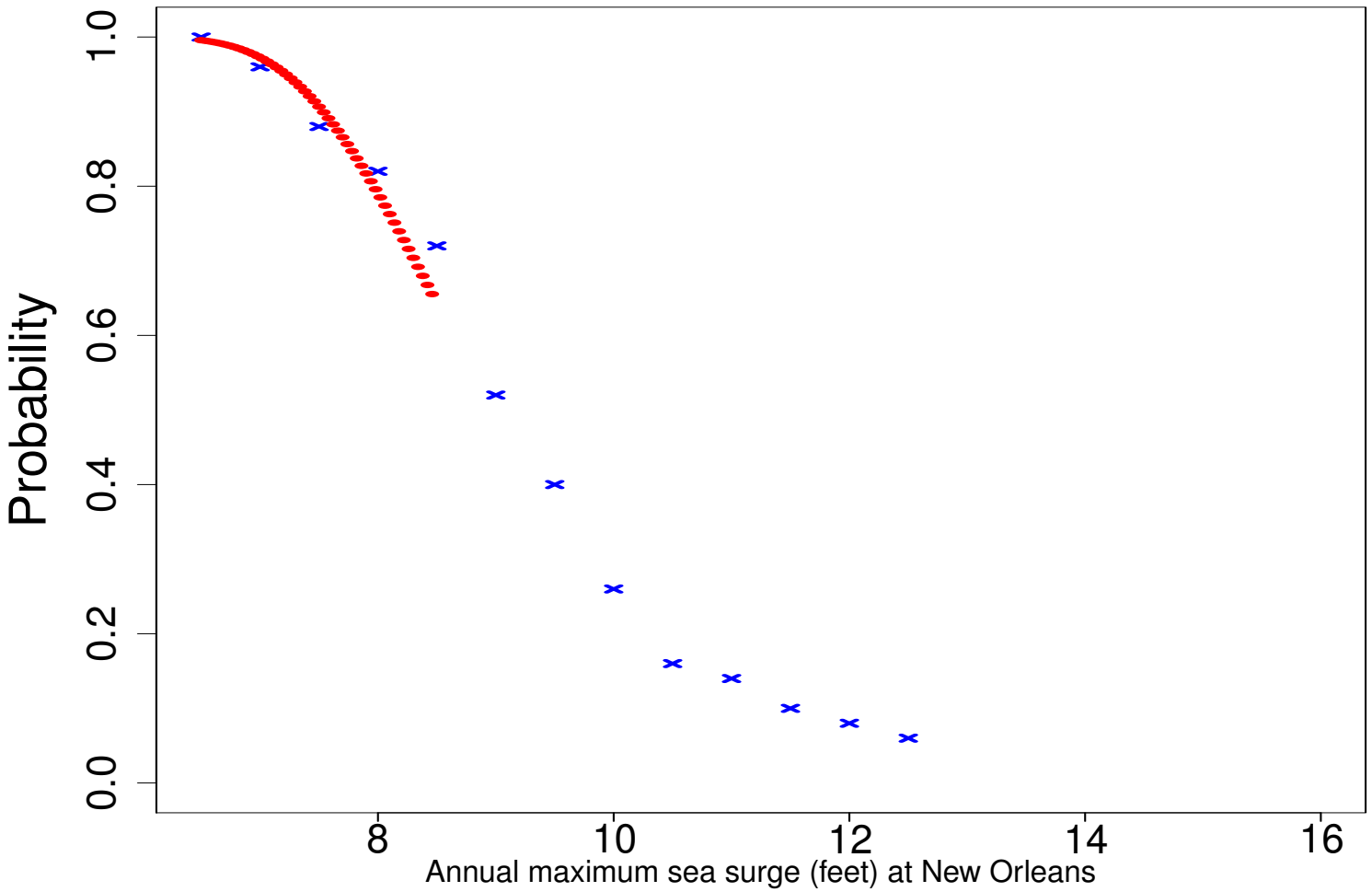

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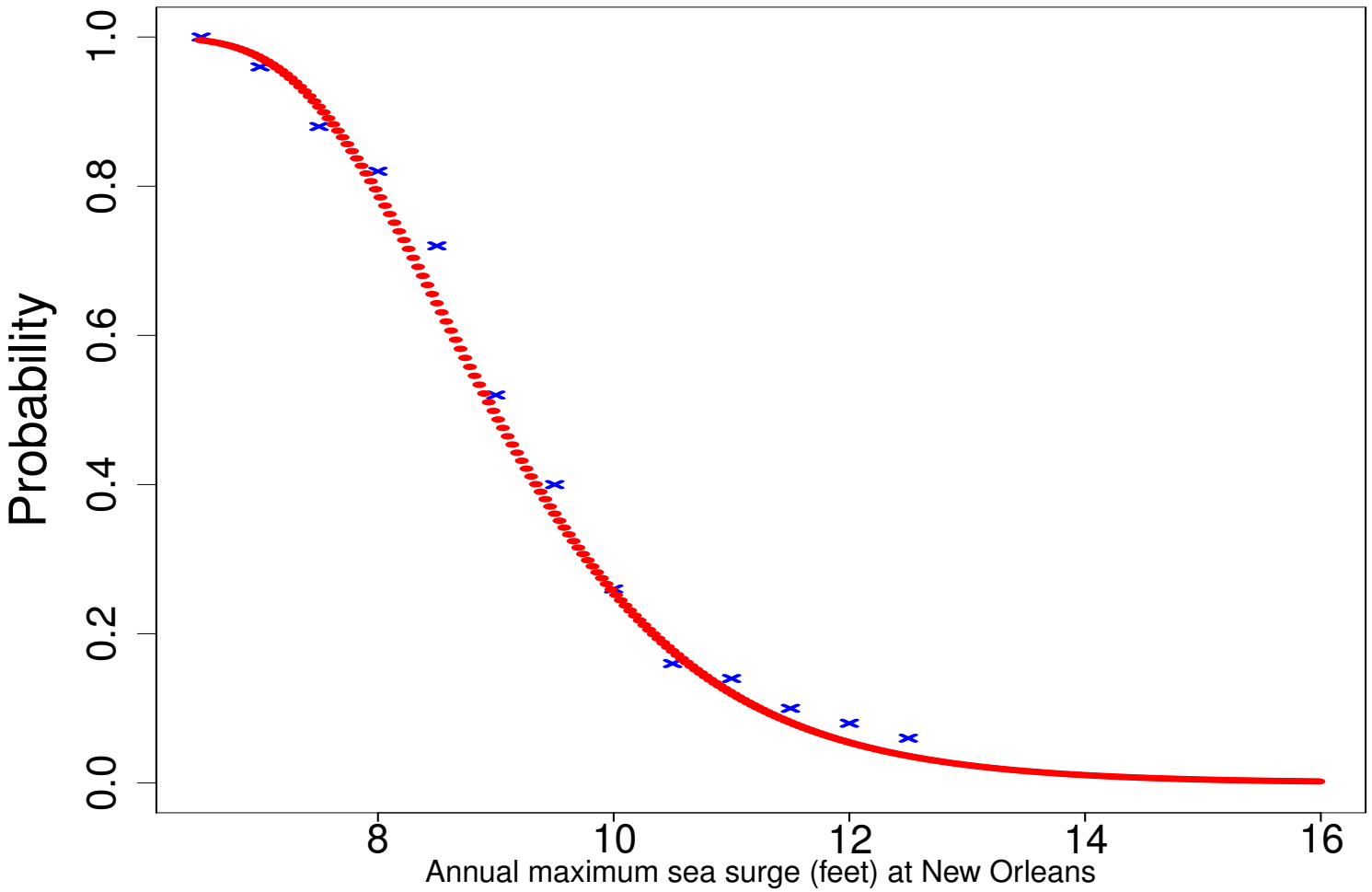

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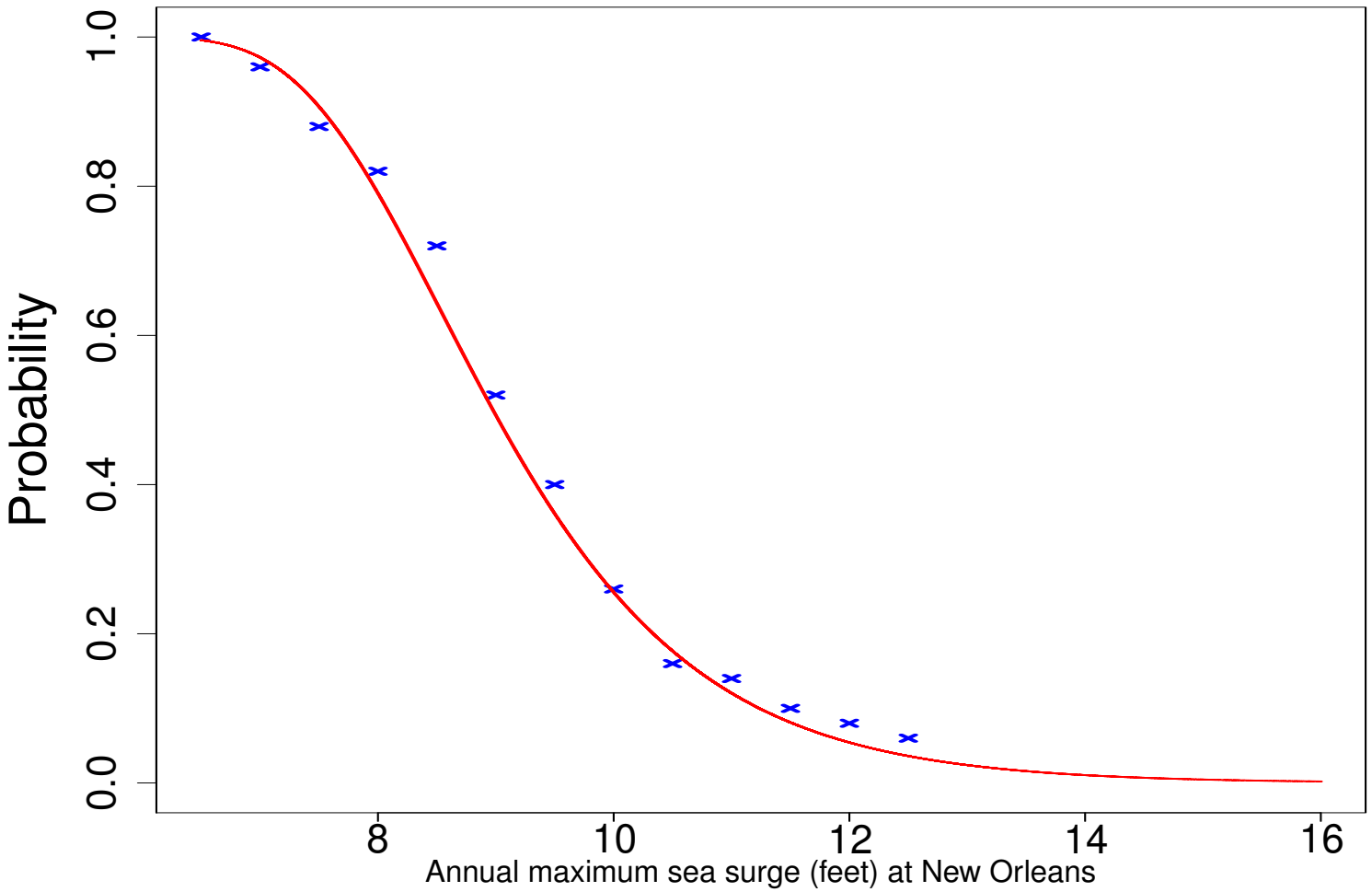

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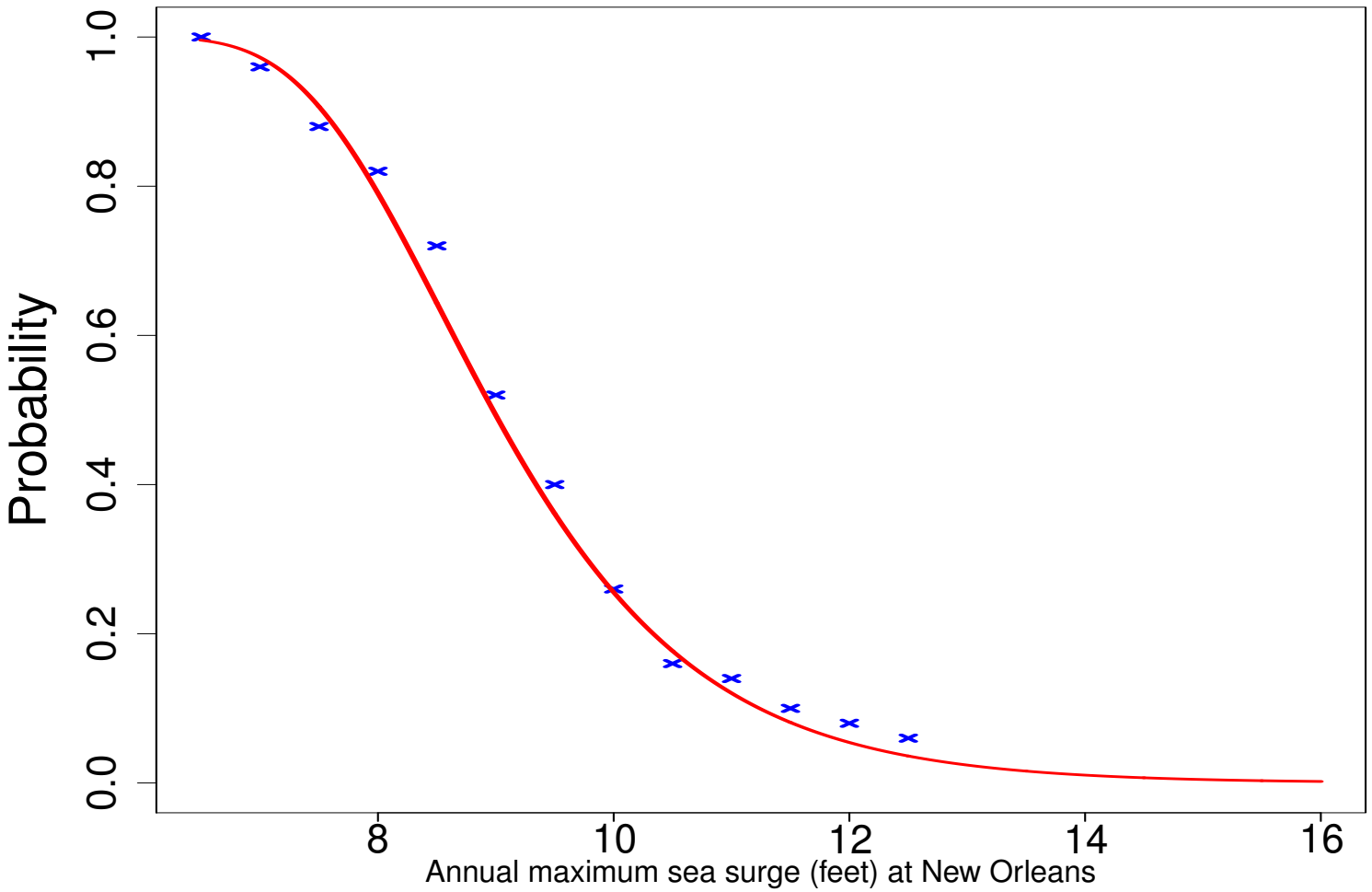

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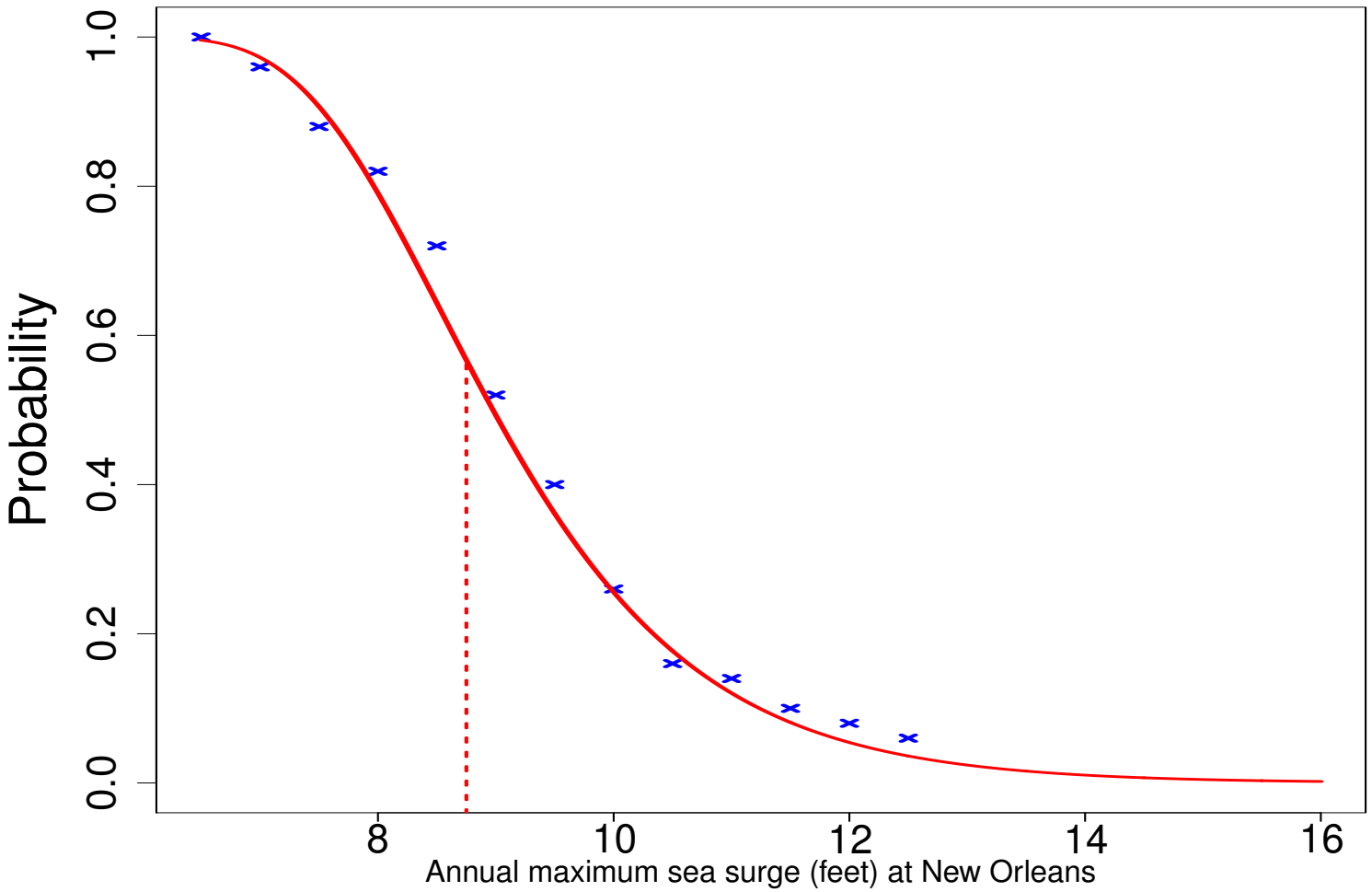

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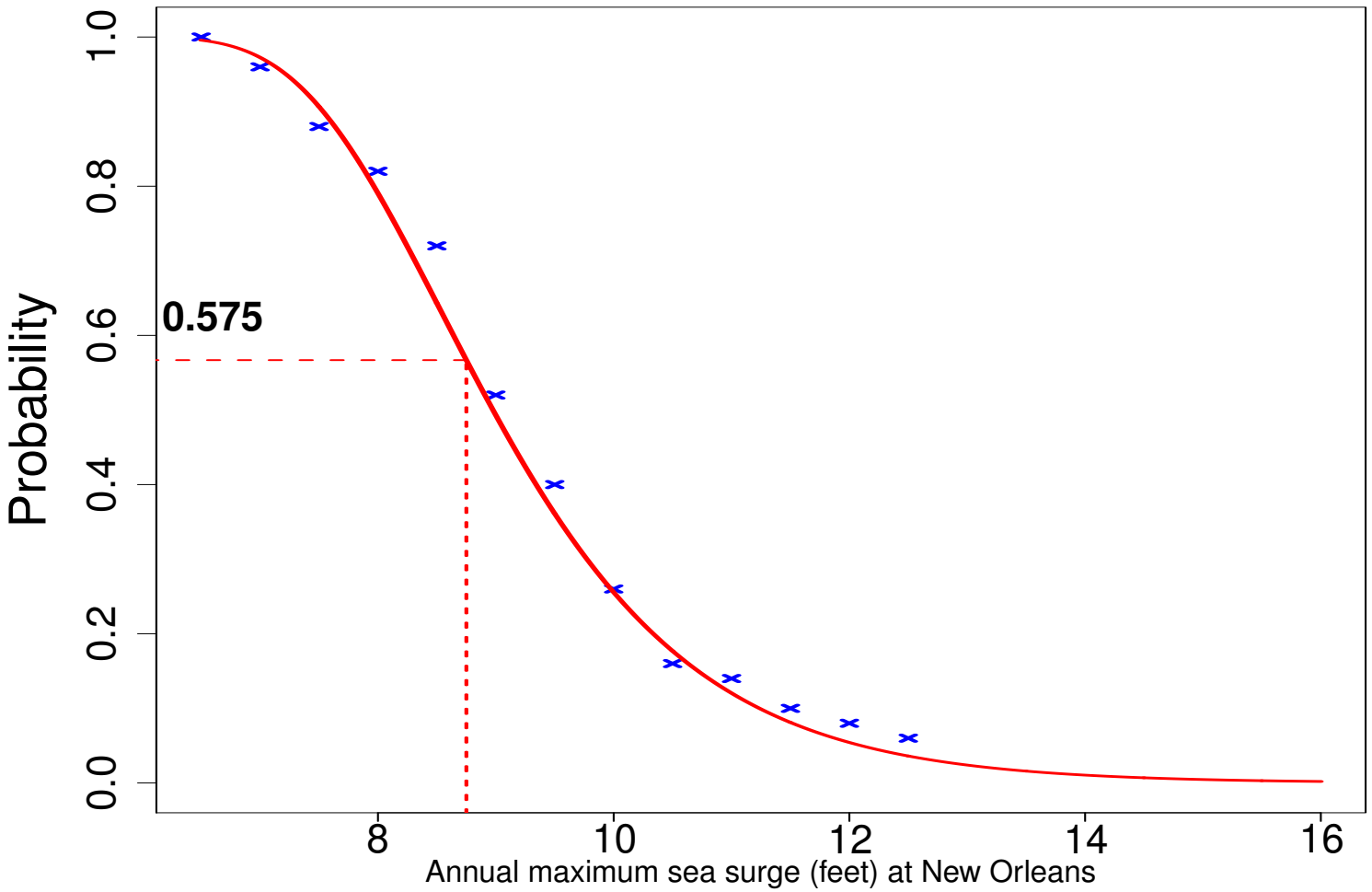

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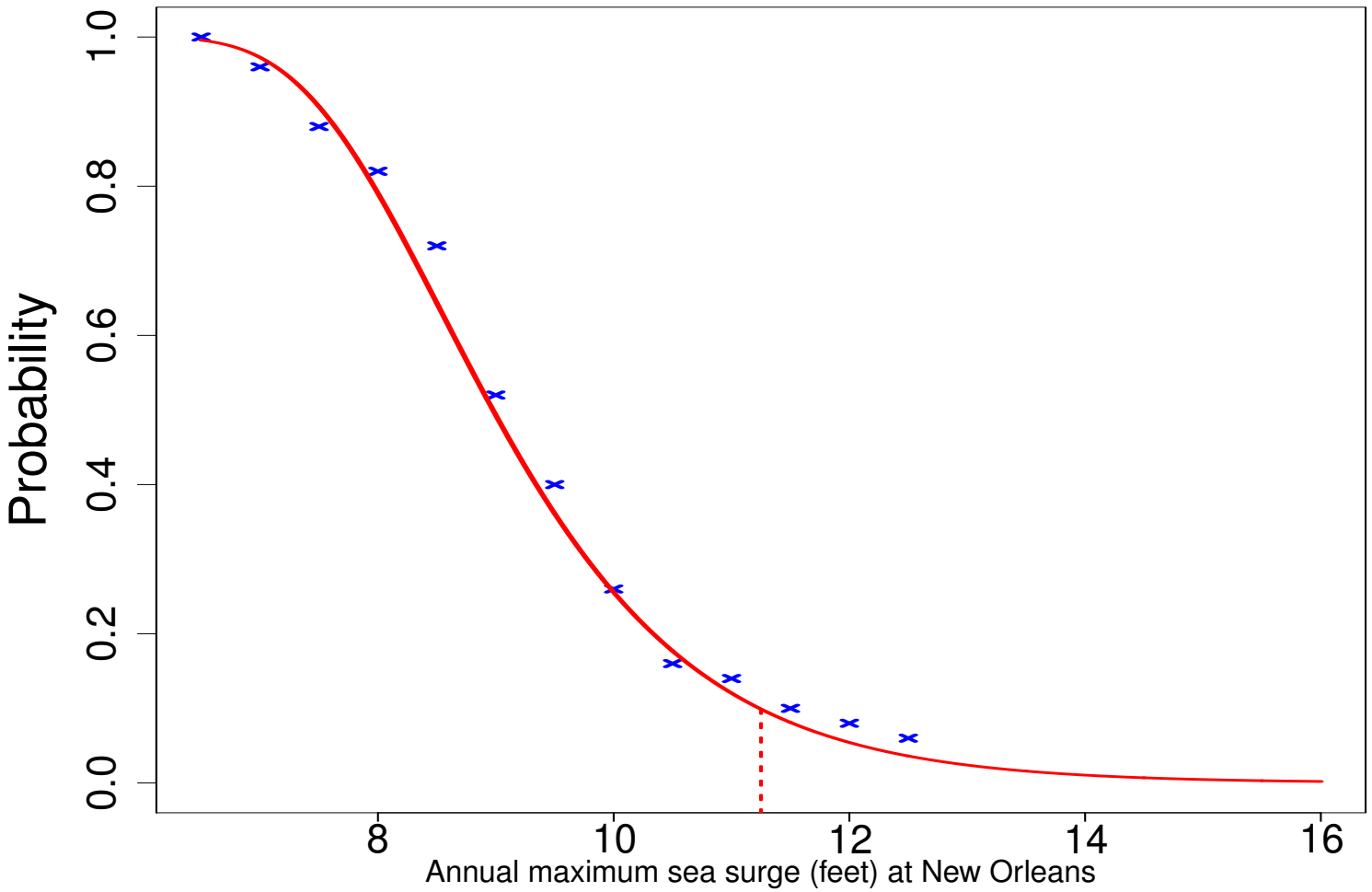

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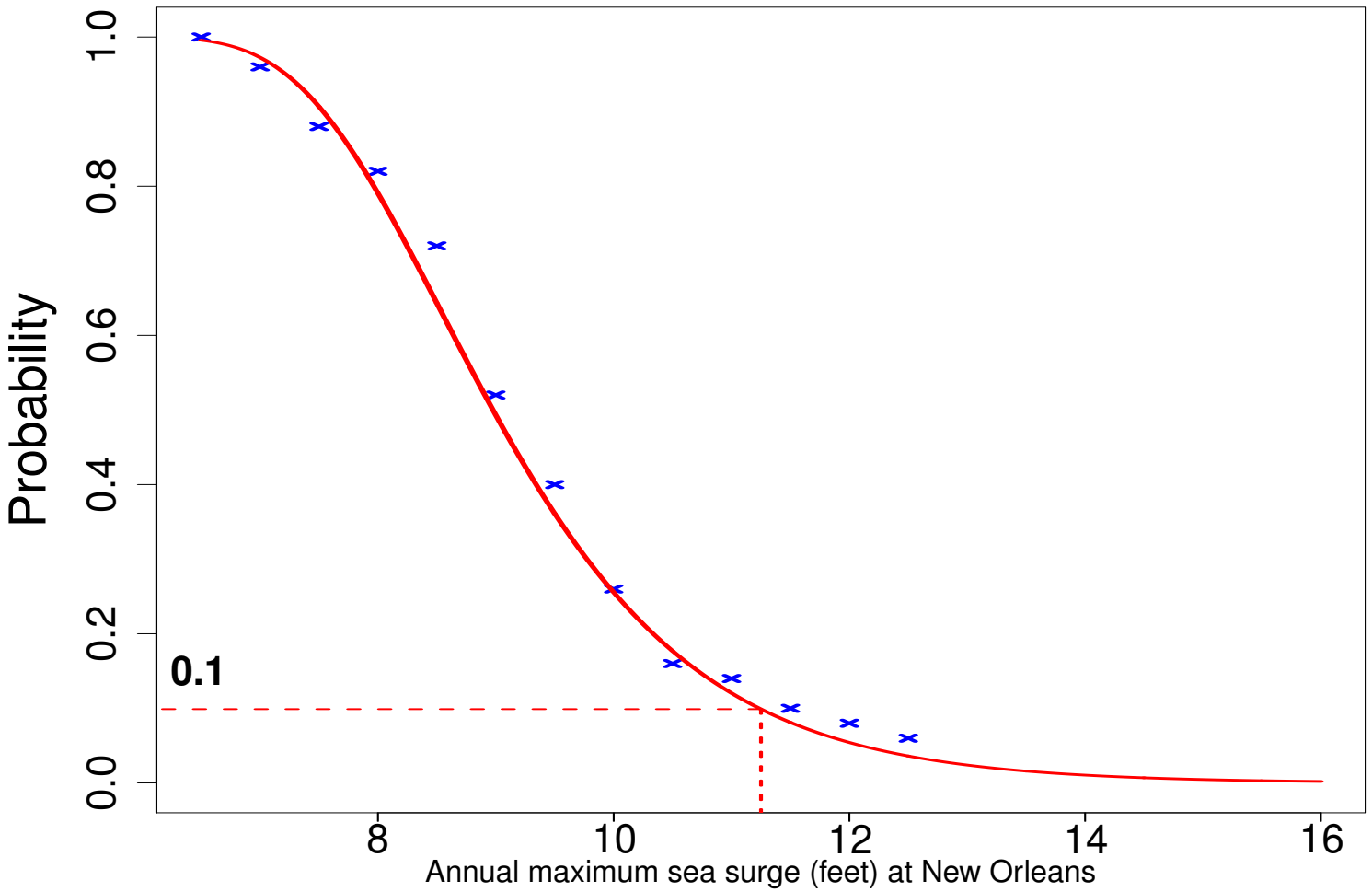

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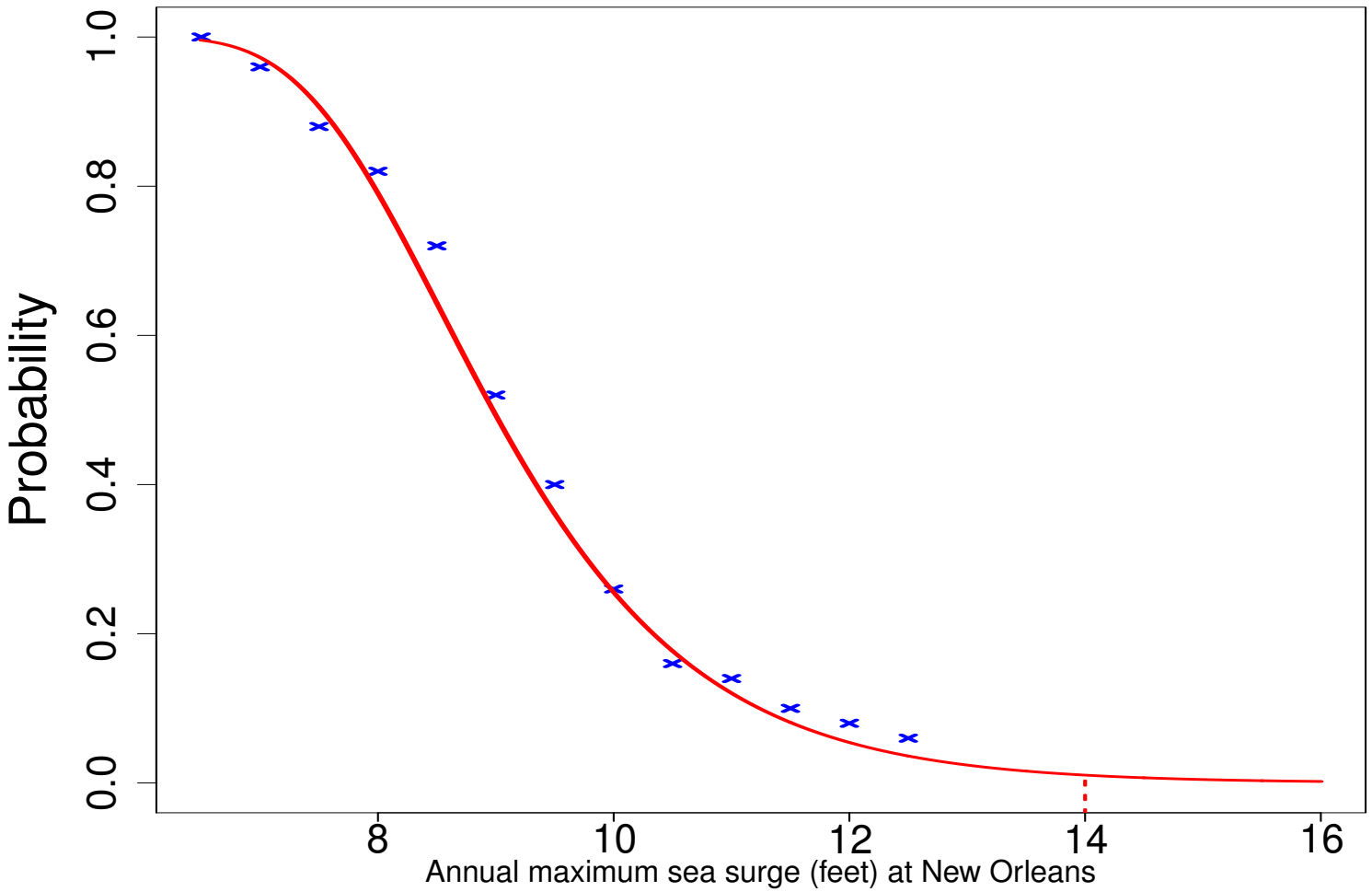

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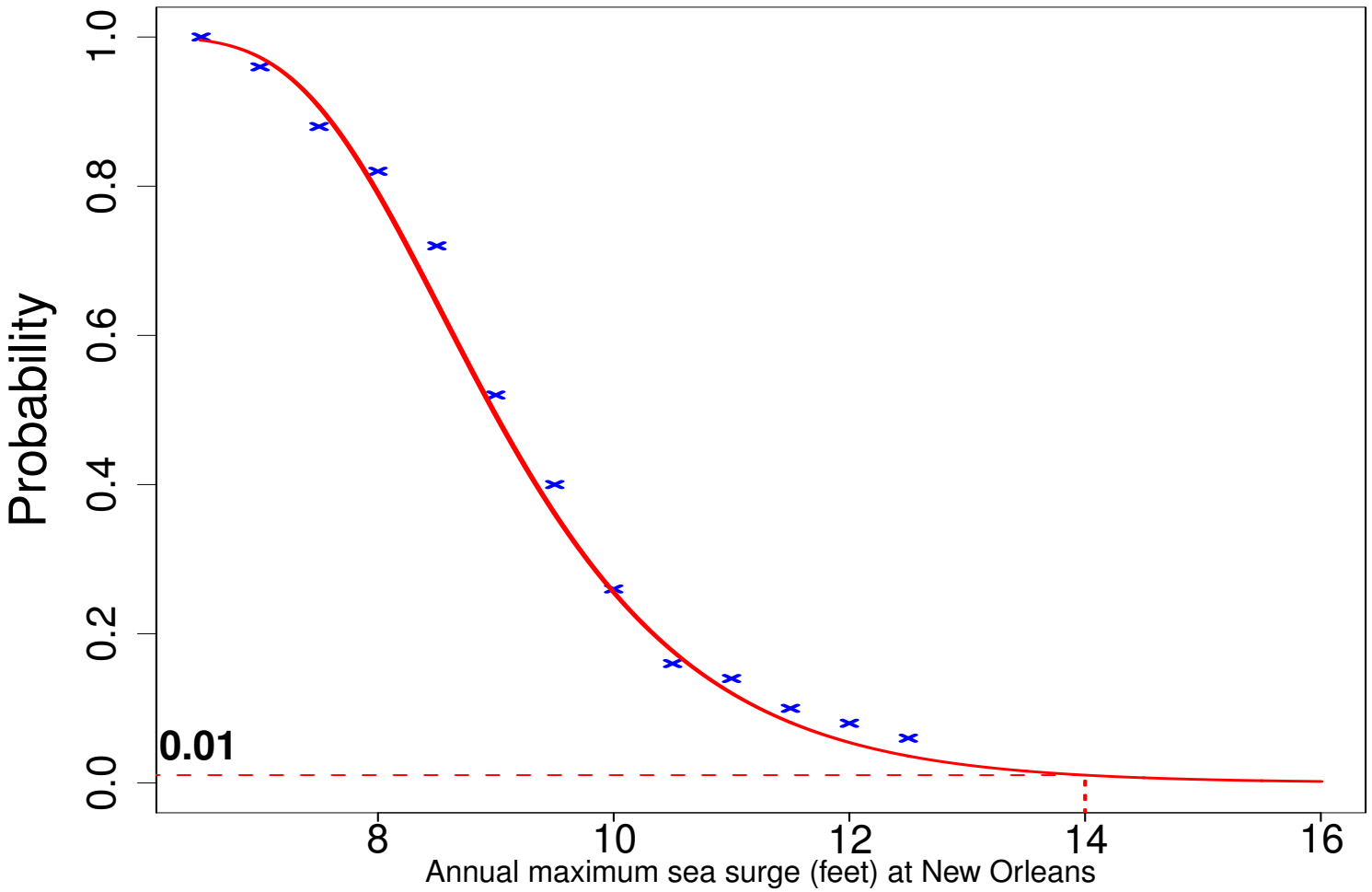

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So we get:

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Can you see why Katrina was billed the "**storm of the century**"?

The Normal distribution

The mean of our data is **9.4 feet**; the standard deviation is **1.5 feet**.

If we assume a Normal distribution for our annual maximum sea-surges, then we would write

$$X : \text{Annual maximum sea surges (in feet)}$$

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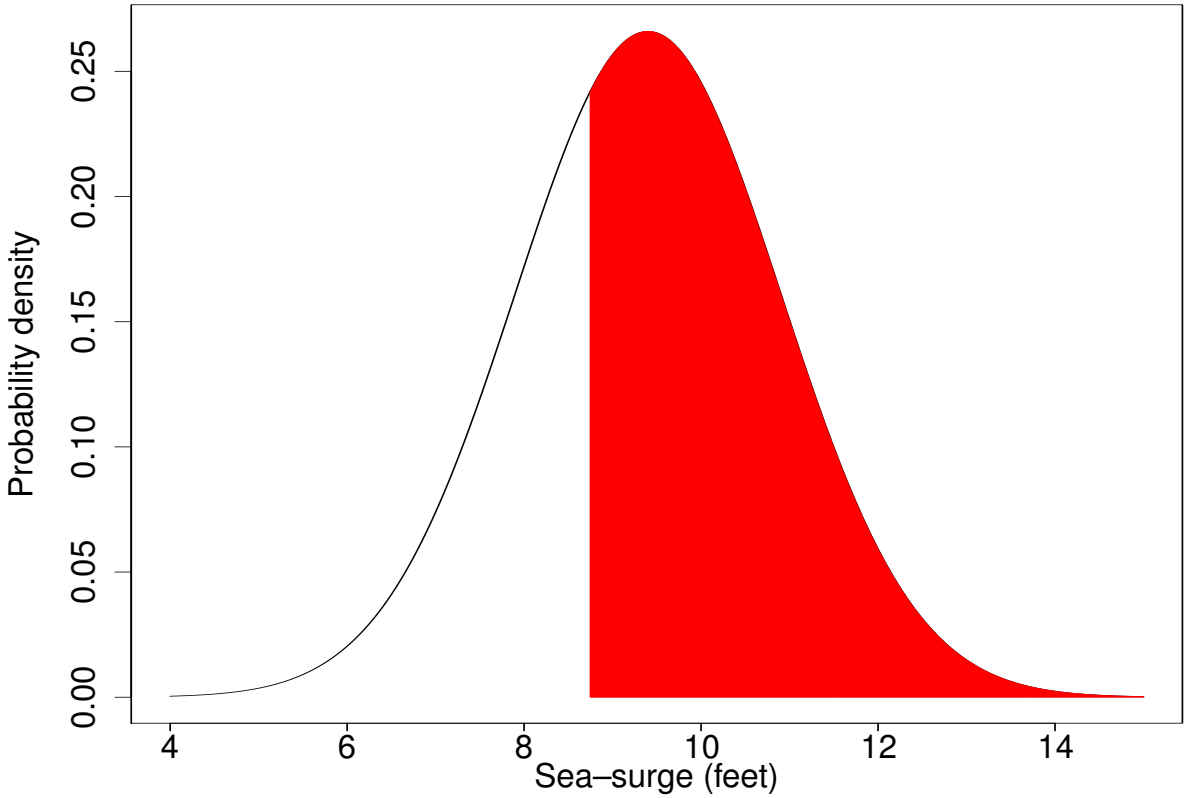
Now find the probability that the annual maximum sea–surge will exceed 8.75 feet, i.e.



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\begin{aligned}
P(X > 8.75) &= P\left(Z > \frac{8.75 - 9.4}{1.5}\right) \\
&= P(Z > -0.43) \\
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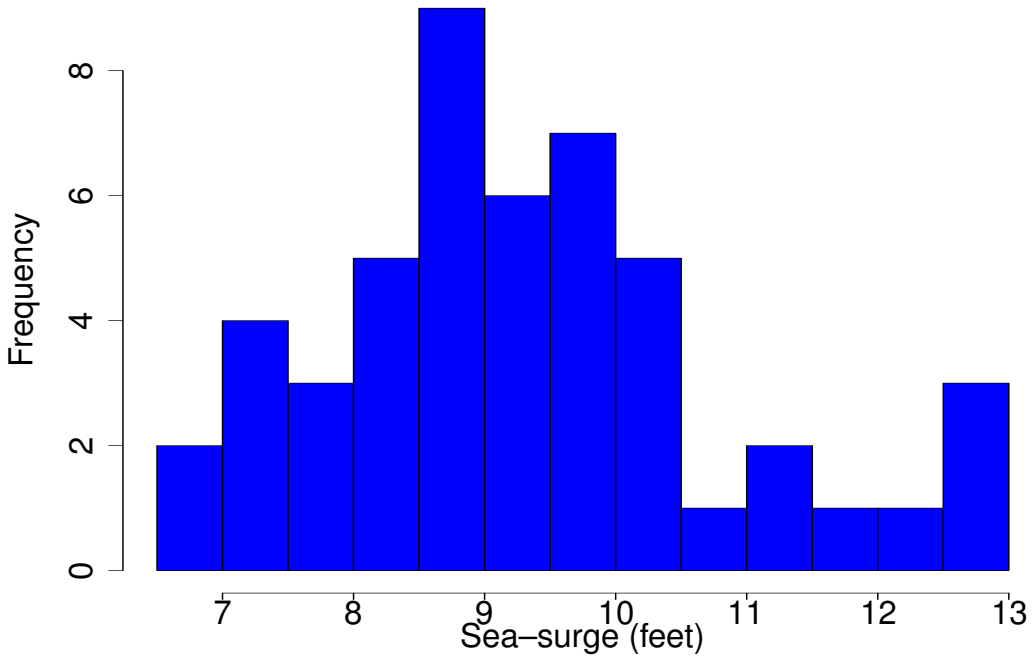
We can also find that

$$P(X > 14) = 0.001.$$



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Uncertainty!

So far, so good?!

The problem is, the 50 sea-surge annual maxima we have analysed are from **one** particular location in the Gulf of Mexico.

As you can imagine, there are **lots** of other data collection points in the Gulf of Mexico.

The Gumbel curve we have constructed will change depending on the dataset we use: it is defined by μ and σ...

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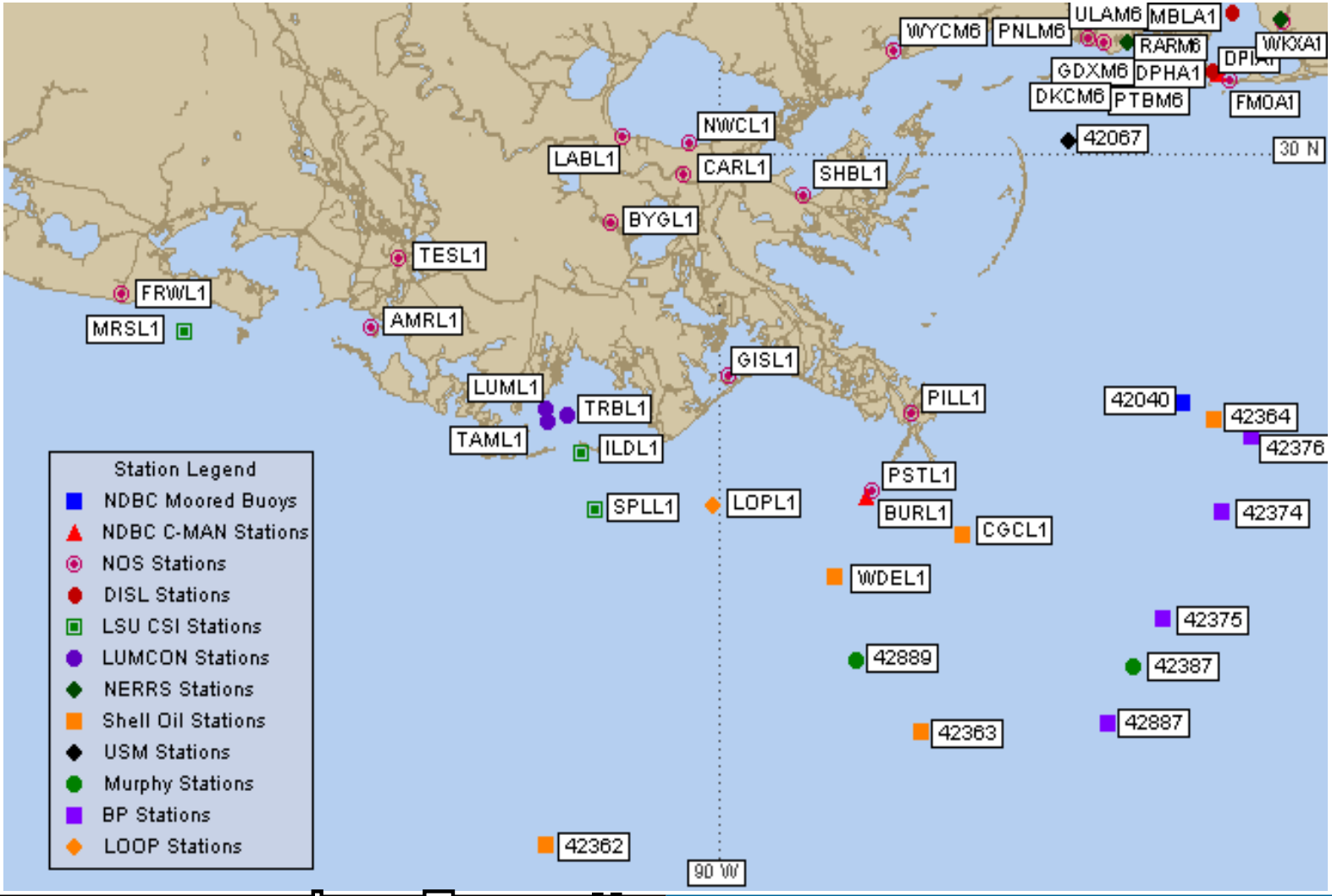

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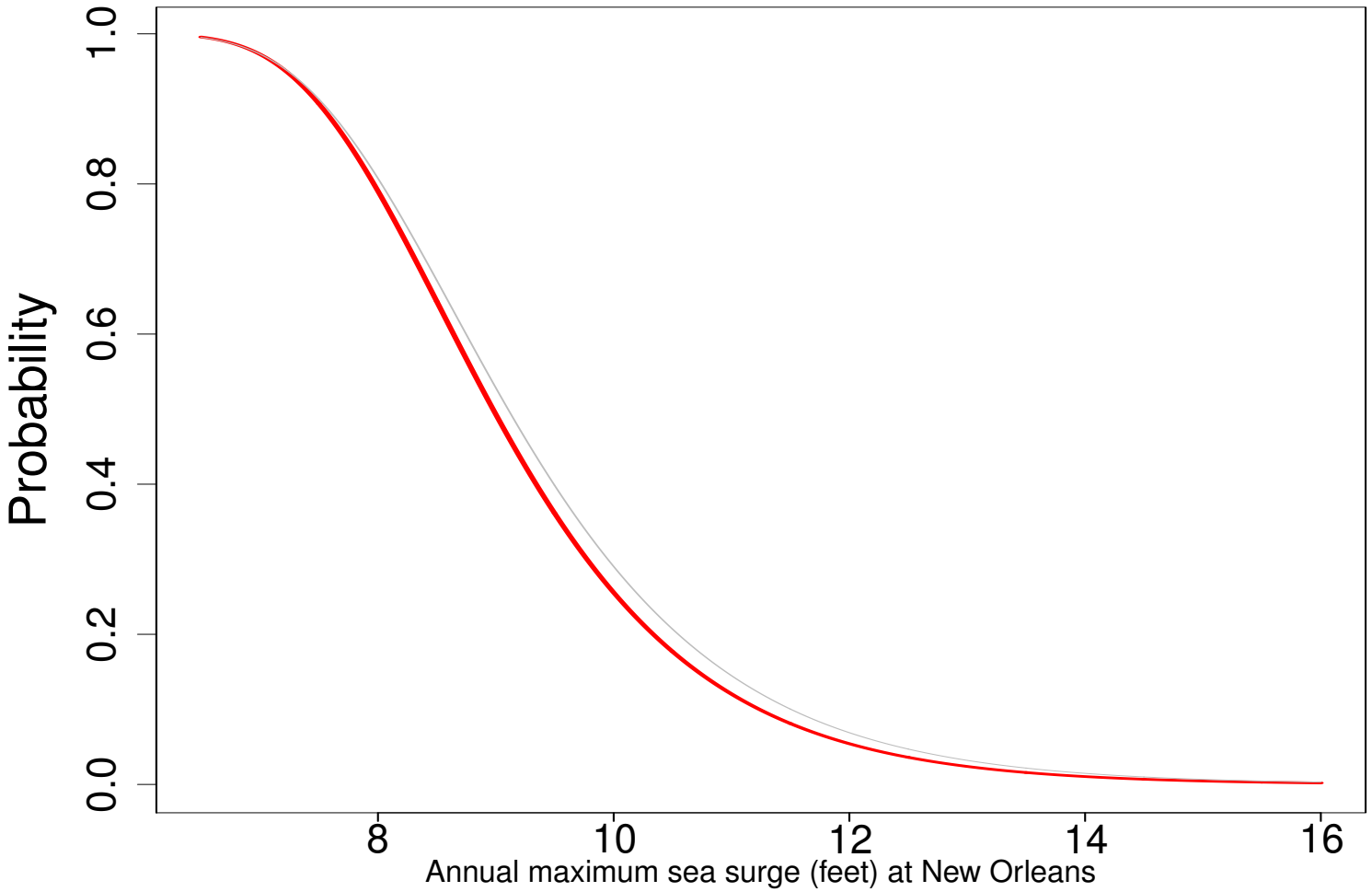

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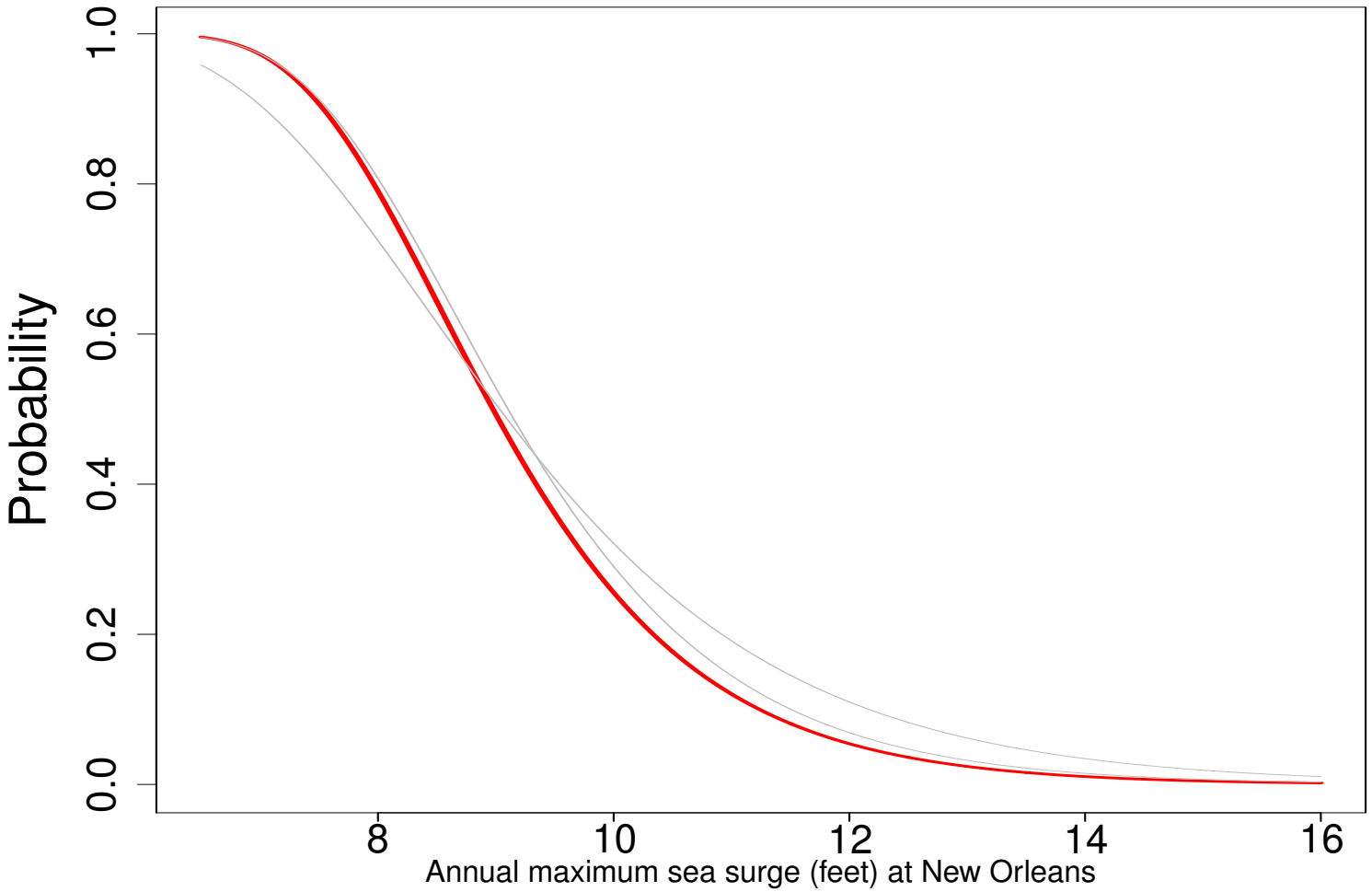

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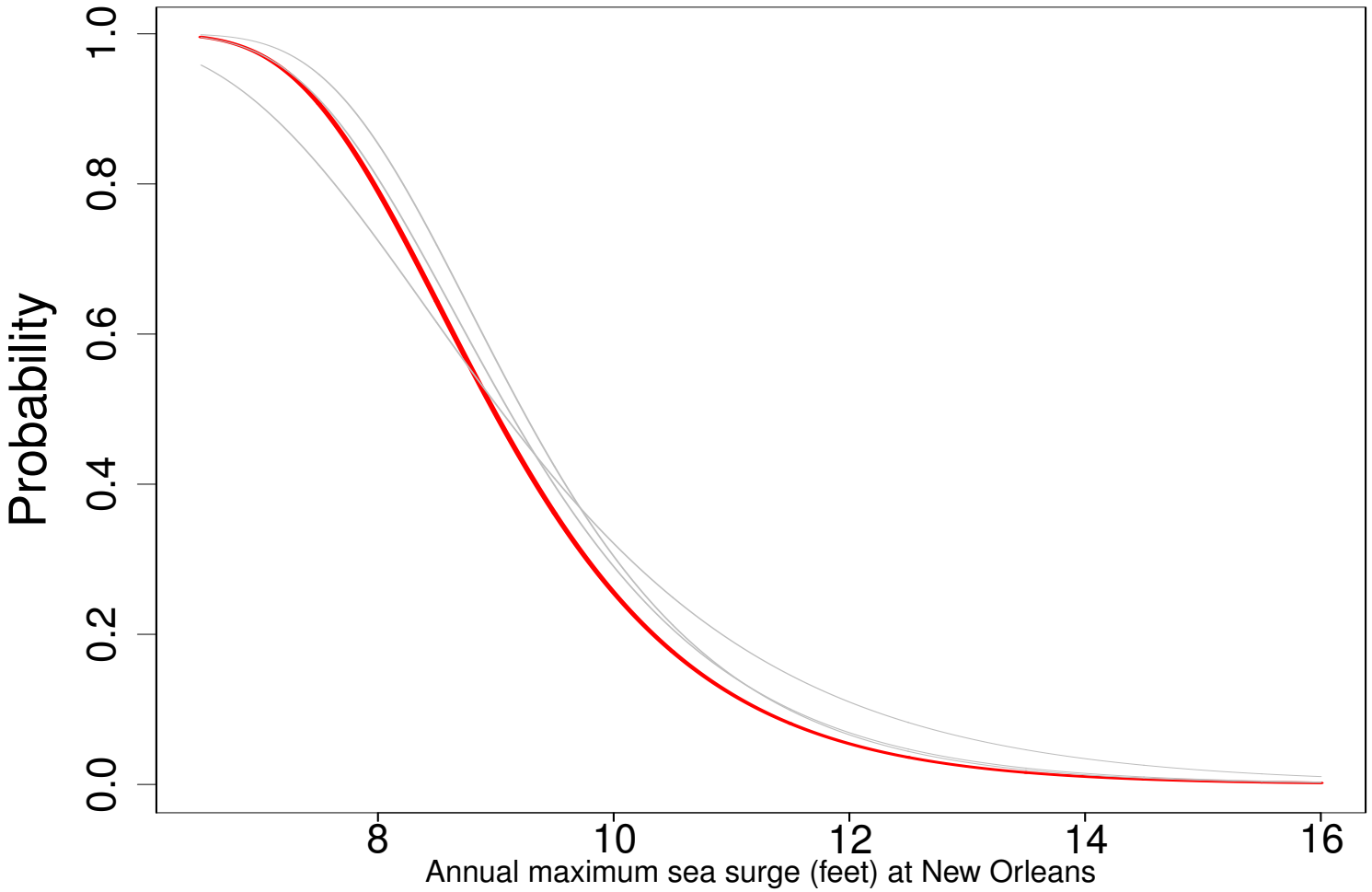

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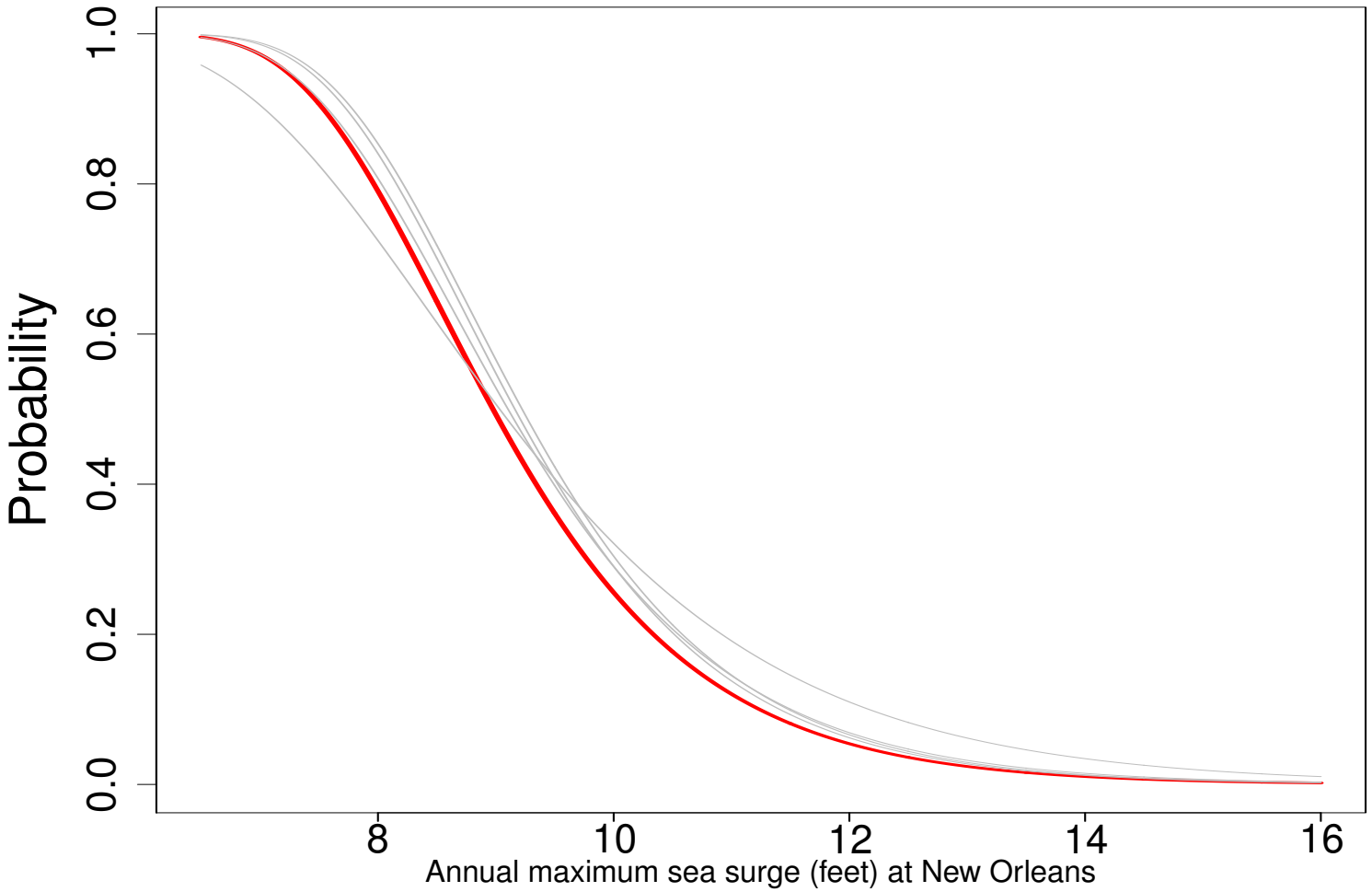

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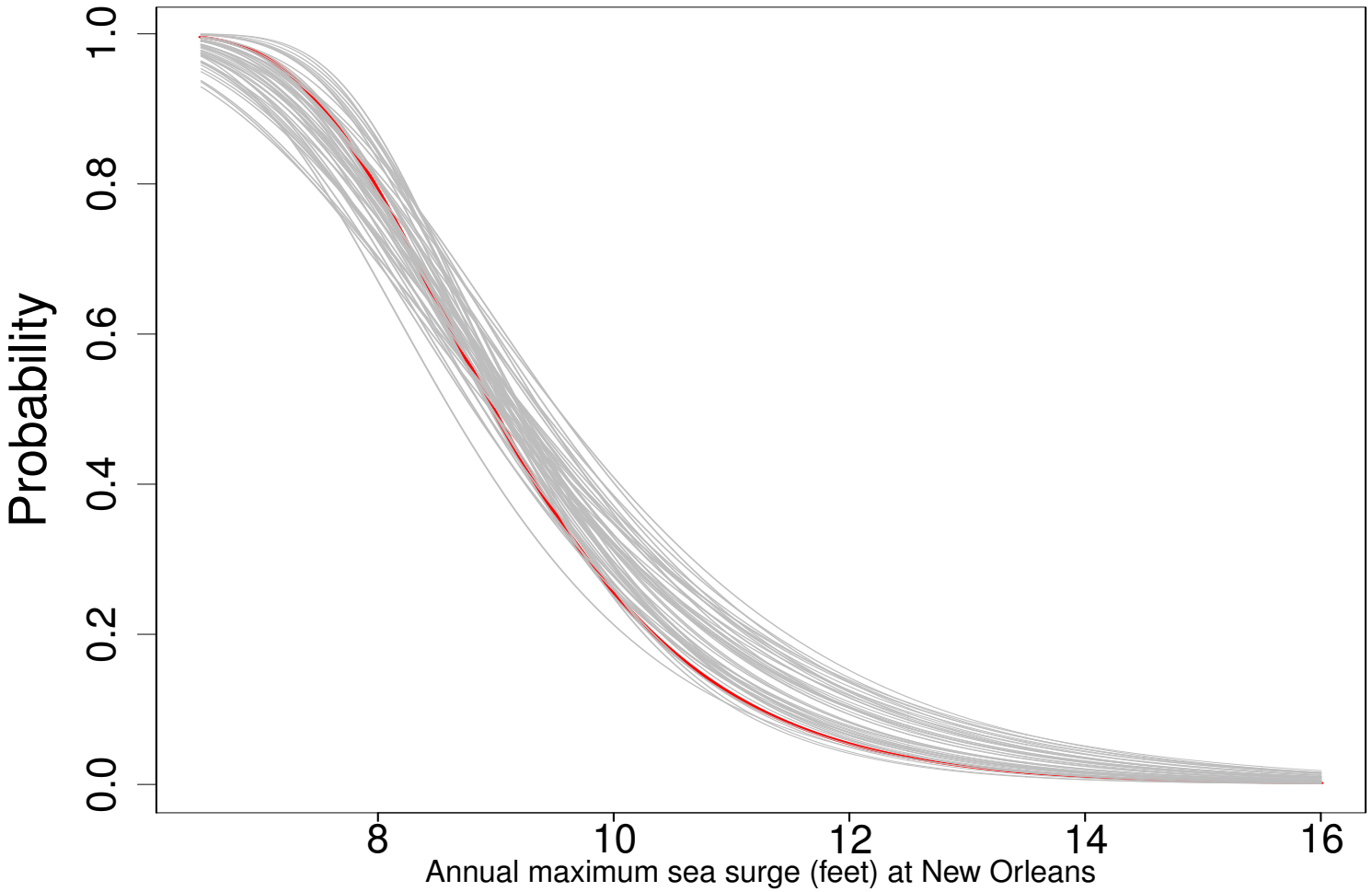

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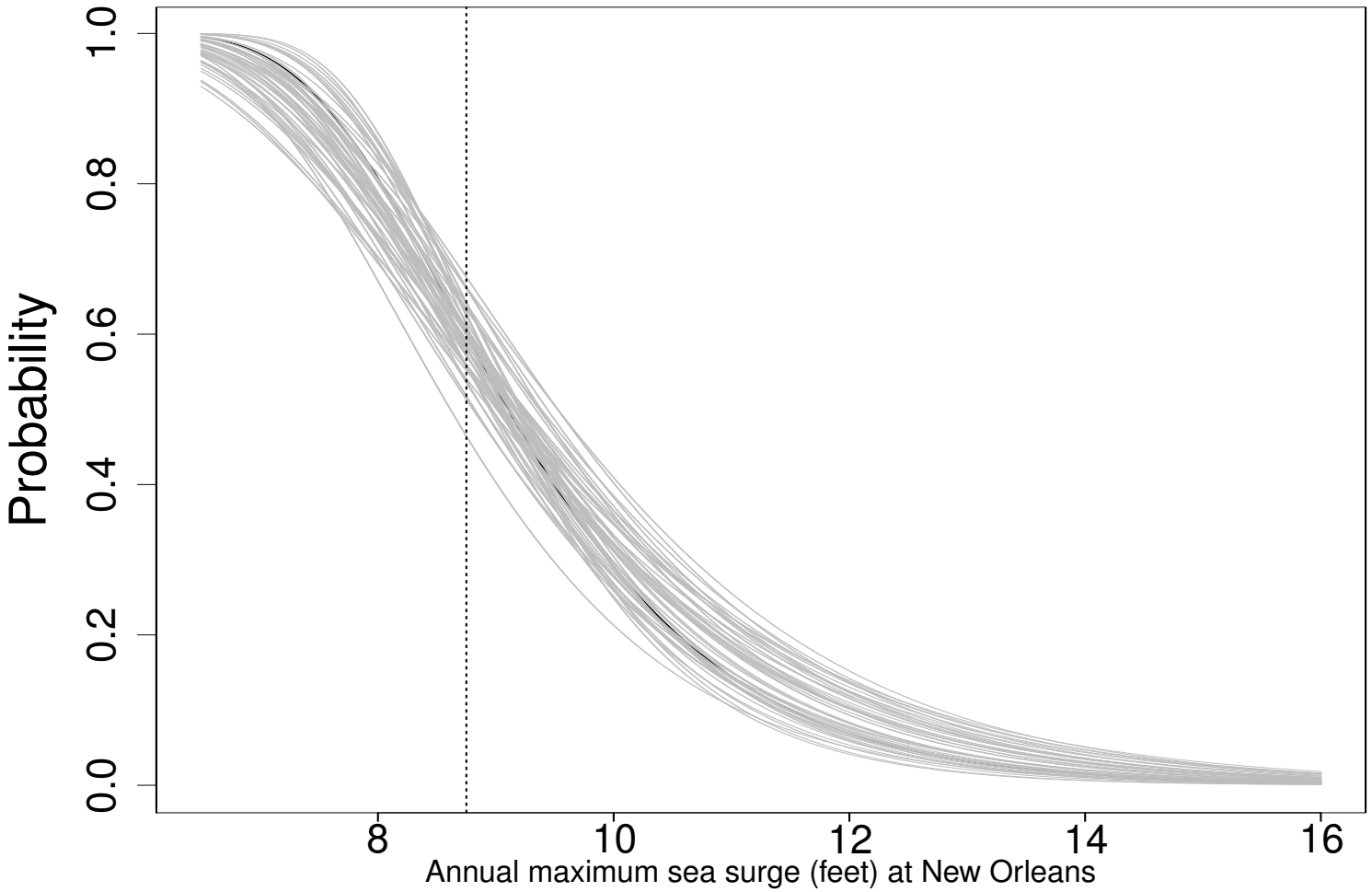

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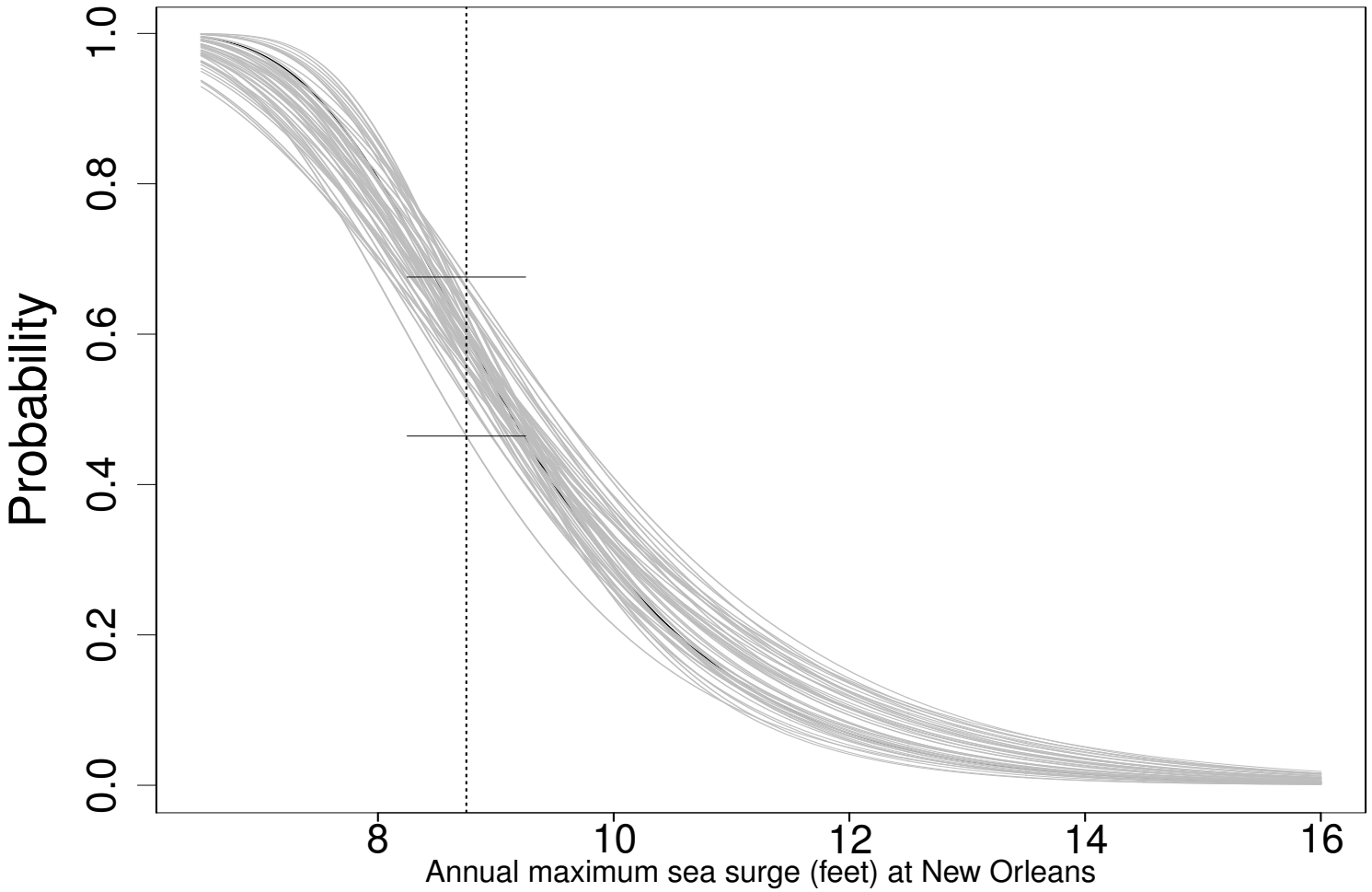

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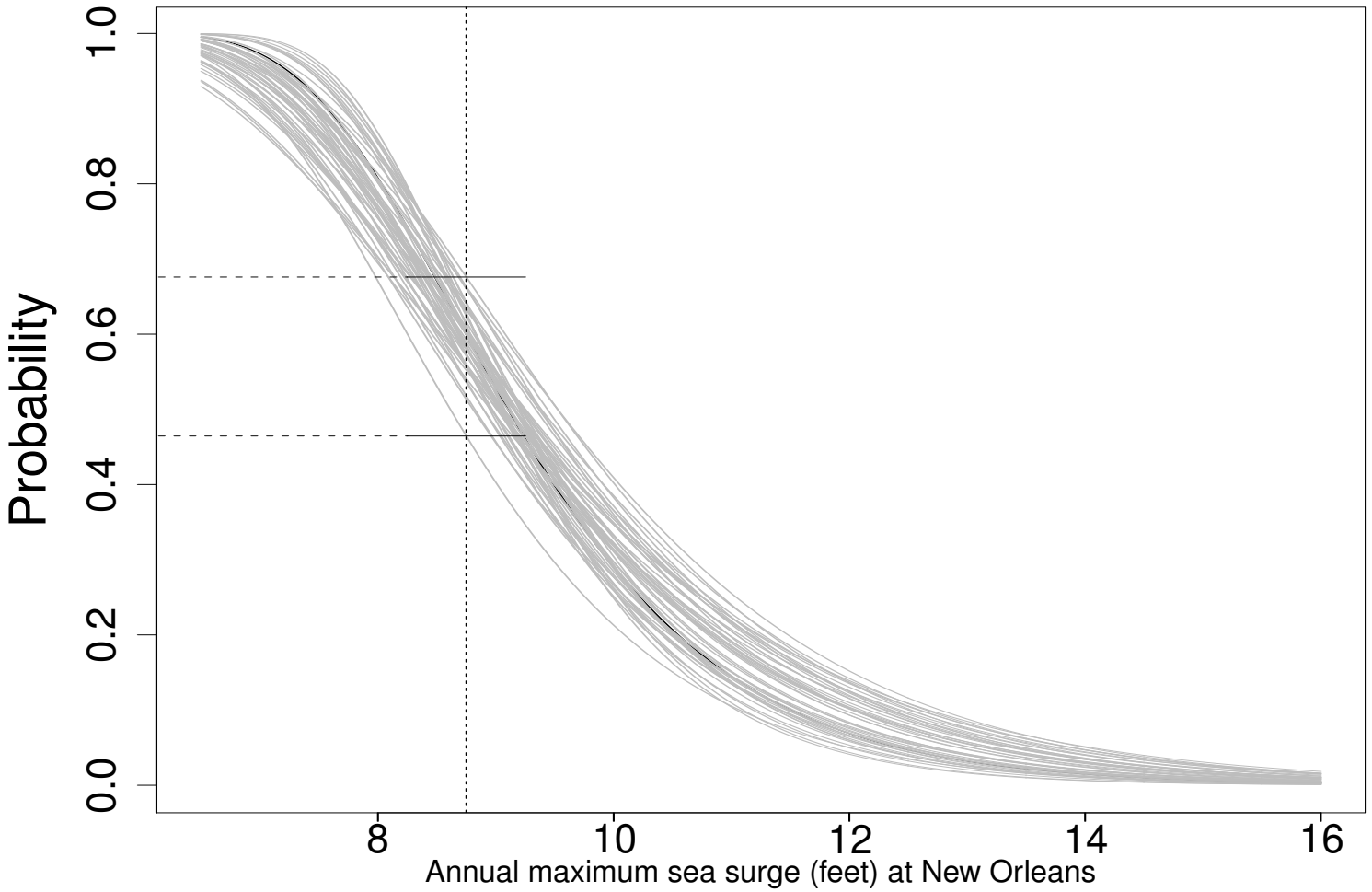

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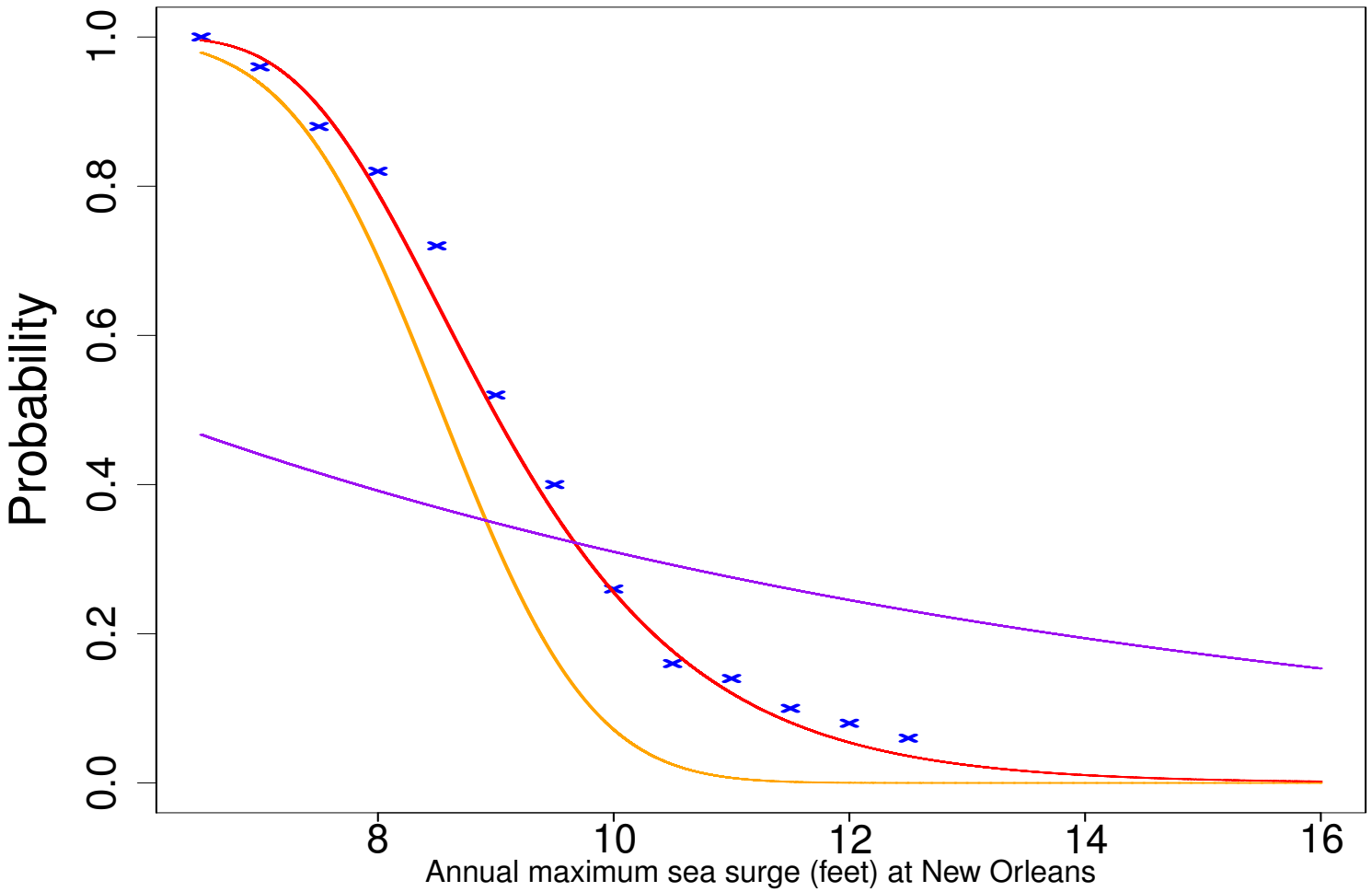

Structural design

As you might remember from the first part of this session, during Hurricane Katrina sea-surges exceeded 14 feet and parts of the sea wall system protecting the city were breached.

A new flood defence system is to be built; as the mathematician, you are asked how tall the sea wall should be to protect against the storm we might expect to see, on average, **once every 500 years**.

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Structural design

Use the Gumbel model to help estimate the height of the new sea wall.

We want x such that

$$\boldsymbol{P}(\text{Sea-surge} > x) = \frac{1}{500}.$$

Using Gumbel's formula, this gives

$$1 - \exp\left[-\exp\left\{-\left(\frac{x - 8.536}{1.2}\right)\right\}\right] = \frac{1}{500}.$$

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Part C: Application to structural design

Rearranging for x, we get:

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