SAgE Doctoral Training Awards

[Analytic problems arising in the design of automatic controllers]

[Newcastle University, Mathematics and Statistics]

Supervisory Team

- **Dr. Zinaida Lykova, Pure Mathematics, http://www.ncl.ac.uk/maths/staff/profile/zinaida.lykova**
- **Dr. Andrew Baggaley, Applied Mathematics and Theoretical Physics, newly hired staff, http://www.ncl.ac.uk/maths/staff/profile/andrew.baggaley**
- **Prof. Nicholas Young, Pure Mathematics, http://www.ncl.ac.uk/maths/staff/profile/nicholas.young**

Key Words **Linear systems, H infinity control, Hilbert function spaces, robust stabilization, tetrablock**

Overview

The project concerns the design of automatic controllers for engineering systems modelled by linear ordinary differential equations. Mathematically it is the problem of constructing analytic functions on the unit disc in the complex plane satisfying interpolation and boundedness conditions. The problem becomes subtle when one adds the requirement of robustness with respect to modelling uncertainty, meaning that the constructed function gives rise to a controller that stabilizes not only the initial system, but also all nearby systems. There is a substantial theory of such ``robust stabilization problems", see for example [3]. The goal of this project is to derive a method of solution (capable of numerical implementation) for a new class of robust stabilization problems, arising from systems with two inputs and two outputs in which the modelling uncertainty is characterized in terms of the structured singular value [3].

Methodology

The project will make use of basic complex variables, linear systems, operators on Hilbert space and Hilbert spaces of analytic functions. The starting point will be the technique developed in [1] for another special case of the robust stabilization problem. In this paper the problem was reduced to the minimization of a quadratic function subject to linear matrix inequalities; our intention is to demonstrate a similar reduction for a wider class of problems, and to implement it numerically. An important tool will be the function theory of the tetrablock, a domain in C^3 introduced in [2].

Timeline

Year 1: Literature critique, understanding connections between the proposed engineering and mathematical problems.

Year 2: Derive a method of solution for a new class of robust stabilization problems.

Year 3: Reduce the problems to the minimization of a quadratic function subject to linear matrix inequalities. Journal paper.

Year 4: Numerical implementation. Thesis preparation. Presentation of research results at the international conference IWOTA.

Training & Skills

The student will be based in the pure and applied research groups of the School of Mathematics and Statistics. The student will engage in regular research group meetings. He/she will participate in regular Analysis and Applied Mathematics seminars and study EPSRC funded MAGIC courses in areas relevant to his/her research. The student's Personal Training Programme will ensure that he/she will receive training to become an independent researcher. It will be highly beneficial for him/her to participate in International conferences and to have contact with international visiting collaborators of his/her supervisors. He/she will develop skills associated with collaborative research programme.

References & Further Reading

[1] J. Agler, Z.A. Lykova and N.J. Young, A case of mu-synthesis as a quadratic semidefinite program, SIAM Journal on Control and Optimization, 2013, 51(3), 2472—2508.
[2] A.A. Abouhajar, M. C. White, N.J. Young, A Schwarz lemma for a domain related to mu-synthesis, J. Geometric Analysis, 2007, 17, 717-750.
[3] G. Dullerud and F. Paganini, A course in robust control theory: a convex approach, Texts in Applied Mathematics, 36, Springer, 2000.
[4] N.J. Young, Some analysable instances of mu-synthesis, Mathematical methods in systems, optimization and control, Editors: H. Dym, M. de Oliveira, M. Putinar, Operator Theory: Advances and Applications, 222, 349—366, Birkhauser, Basel, 2012.

Further Information

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