MAS3219/8219 Geometric Group Theory: Notes for revision

1 What's examinable

This is a rough guide to the most important parts of the module to concentrate on for revision. It more or less describes the minimum you should know for the exam.

Section 1 This is background and should be familiar from the previous group theory course. You should know the basic language of group theory and be familiar with the standard examples like the symmetric and dihedral groups.

Section 2 Direct sums. You should know the definitions of internal and external direct sum and understand how to apply them in examples: which means knowing Lemma 2.5.

Section 3 Abelian groups. Know the definitions and basic examples. Don't worry about the details of change of basis but know the main theorem, Theorem 3.10.

Section 4 Semi-direct products. Know the definition of internal and external semi-direct product. Know how to check that a group is a semi-direct product. Know the statements of Theorems 4.9 and 4.11.

Section 5 Symmetries of the plane. Know how to work with these in examples: that is, know what $\mathrm{Sym}_n(\mathbb{R})$, $T_n(\mathbb{R})$ and O_n mean. Know Theorem 5.5, Lemma 5.6 and Corollary 5.7. Know the types of isometry of the plane and how to check which is which in examples: so know Theorem 5.9 and how to apply it as in Example 5.10.

Section 6 Wallpaper groups. Let the pictures amaze you, but don't bother with revising it.

Section 7 Know the language of graph theory. In particular know what directed, labelled graphs are and what trees are. Know the definition(s) of a group action: in particular know how to check that a group acts on a graph. Know what faithful and free actions are. Know the definition of Cayley graph and how to draw them in practice. Know how a group acts on its Cayley graph and Theorem 7.31 and Corollary 7.35.

Section 8 Know what reduced words are and know the definition of the free group on a set. This means knowing Theorem 8.8 in particular. Know the universal mapping theorem, Theorem 8.9; the uniqueness theorem, Theorem 8.10; the normal forms theorem, Theorem 8.12; as well as Theorem 8.19 and Corollary 8.20 (Nielsen-Schreier). Don't worry about proofs of these results, but do understand what the results mean. Know how to construct the Stallings folding from a set of generators for a subgroup of a free group: and know how to use it to find a basis.

Section 9 Know what the normal closure of a subset of a group is and what it means for a group to be given by a presentation (Definition 9.4). Know the basic examples.

Know von Dyck's substitution theorem (9.7) and how to apply it in examples. Know Theorems 9.10 and 9.12 and how to use them in examples.

That's it

2 Past exam papers

There are some topics in past exam papers that have not been covered this time, so should be ignored, for revision. Here's a list of some of these things.

2008/9 Question A3(b), "Abelianisation".
Question A4, "Tietze transformations".
Question B5, nearly all of it.
Question B7, "Todd-Coxeter enumeration".

2009/10 You can probably do Question A2, but it's not examinable this time.
You can probably do Question B4, but it's not really relevant to the course as it is this time.
You can probably do Question B5, but if I were you I wouldn't bother.

2010/11 Question B4: not needed this time.
Question B5: we have not covered normal forms.

AJD May 2011