Theorem 2.75.
Every simple planar graph G is 6–colourable.

Proof. Use induction on n, the number of vertices of G. Clearly if $n \leq 6$ the result holds. Assume the result holds for simple planar graphs of up to $n - 1 \geq 6$ vertices.

From the result of Problem Class exercise 2.15, G has a vertex v of degree at most 5. Delete v from G to form the graph $G-v$, which is 6–colourable, by the inductive hypothesis. Choose a 6–colouring for $G - v$. Now replace v. Note that, as v has degree at most 5, there is one of the 6 colours not used to colour any of the vertices adjacent to v. Colour v with this colour to give a 6–colouring of G. □

A proof of a 5–colour theorem, although somewhat harder, can be found in most introductory texts on graph theory.

We finish with a result which links vertex and edge colouring. The map of countries shown above does itself constitute a graph: put a vertex at each point where two borders meet. The resulting graph is plane, connected, regular of degree three and has no bridges or loops. Furthermore any "reasonable" map of countries constitutes a plane drawing of a graph with all these properties. The 4–colour conjecture states that the faces of such a plane graph can be coloured using 4 colours, where **colouring** means that no edge